MATH 413/513 (LINEAR ALGEBRA)

HOMEWORK 5 - SUMMER 2018

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<u>Due on:</u> Tuesday 06-12-2018.

In the following, V is a finite dimensional vector space over the field $\mathbb{F}$.

(1) Let U and V be five-dimensional subspaces of $\mathbb{R}^9$. Prove that $U \cap V \neq \{0\}$.

(2) Show that no linear map $T : \mathbb{F}^5 \to \mathbb{F}^2$ can have its null space the set

$$\{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{F}^5 \mid x_1 = 2x_2, x_3 = x_4 = x_5\}.$$

(3) Suppose that $S, T \in \mathcal{L}(V)$ such that $\text{range}(S) \subseteq \text{null}(T)$. Prove that $(ST)^2 = 0$.

(4) Let U, V, and W be finite dimensional vector spaces over $\mathbb{F}$ with $S \in \mathcal{L}(U, V)$ and $T \in \mathcal{L}(V, W)$. Prove that

$$\dim(\text{null}(T \circ S)) \leq \dim(\text{null}(T)) + \dim(\text{null}(S)).$$

(5) Suppose $\phi \in \mathcal{L}(V, \mathbb{F})$. Suppose $u \in V$ is not in $\text{null}(\phi)$. Prove that

$$V = \text{null}(\phi) \oplus \text{span}\{u\}.$$