

MATH 413/513 (LINEAR ALGEBRA)

HOMEWORK 4 - SUMMER 2018

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Due on: Wednesday 06-6-2018.

MATH 413: Solve questions 1 to 4.

MATH 513: Solve questions 1 to 5.

Vector spaces:

In the following, V is a finite dimensional vector space over the field \mathbb{F} .

(1) Prove or disprove: the list of vectors $(\sin^2(x), \cos(2x), \alpha)$ is linearly independent in $\mathcal{C}(\mathbb{R})$.

(2) Find the dimension of the following subspace of \mathbb{R}^3

$$U = \{(x_1, x_2, x_3, x_4) \mid x_4 = x_1 + x_2\}.$$

(3) Let $\dim(V) = n$ for some $n \in \mathbb{Z}_+$. Prove that there are n one-dimensional subspaces U_1, U_2, \dots, U_n of V such that

$$V = U_1 \oplus U_2 \oplus \dots \oplus U_n.$$

(4) Let $U = \{p \in \mathbb{F}_4[z] : p(6) = 0\}$.

(a) Find a basis of U .

(b) Extend the basis in part (a) to a basis of $\mathbb{F}_4[z]$.

(c) Find a subspace W of $\mathbb{F}_4[z]$ such that $\mathbb{F}_4[z] = U \oplus W$.

(5) Let $\mathbb{F}_m[z]$ denote the vector space of all polynomials with degree less than or equal to $n \in \mathbb{Z}_+$ and having coefficient over \mathbb{F} , and suppose that $p_0, p_1, \dots, p_m \in \mathbb{F}_m[z]$ satisfy $p_j(1) = 0$. Prove that (p_0, p_1, \dots, p_m) is a linearly dependent list of vectors in $\mathbb{F}_m[z]$.