

**MATH 413/513 (LINEAR ALGEBRA)
HOMEWORK 3**

SUMMER 2018

Due on: Friday 06-01-2018.

MATH 413: Solve questions 1 to 4.

MATH 513: Solve questions 1 to 5.

Vector spaces:

In the following, V is a vector space over the field \mathbb{F} .

- (1) Prove or give a counterexample: if U_1, U_2, W are subspaces of V such that

$$V = U_1 \oplus W \text{ and } V = U_2 \oplus W,$$

then $U_1 = U_2$.

- (2) Show that the vectors $v_1 = (1, 1, 1)$, $v_2 = (1, 2, 3)$, and $v_3 = (2, -1, 1)$ are linearly independent in \mathbb{R}^3 . Write $v = (1, -2, 5)$ as a linear combination of v_1, v_2 , and v_3 .

- (3) Define $U = \text{span}(u_1, u_2, \dots, u_n)$, where for each $i = 1, \dots, n$, $u_i \in V$. Suppose $v \in U$, prove

$$U = \text{span}(v, u_1, u_2, \dots, u_n).$$

- (4) Suppose that v_1, v_2, \dots, v_n is a linearly independent set of vectors in V . Given any $w \in V$ such that the set of vectors

$$v_1 + w, v_2 + w, \dots, v_n + w$$

is linearly dependent, prove that $w \in \text{span}(v_1, v_2, \dots, v_n)$.

- (5) Prove that the union of two subspaces of V is a subspace of V if and only if one of the subspaces is contained in the other.