

**MATH 413/513 (LINEAR ALGEBRA)  
HOMEWORK 2**

SUMMER 2018

**Due on: Friday 05-25-2018.**

**MATH 413:** Solve questions 1 to 4.

**MATH 513:** Solve questions 1 to 5.

**Vector spaces:**

In the following,  $V$  is a vector space over the field  $\mathbb{F}$ .

- (1) Let  $p(z)$  be a polynomial with real coefficient, and let  $\alpha \in \mathbb{C}$ . Prove that  $p(\alpha) = 0$  if and only if  $p(\bar{\alpha}) = 0$ .
- (2) Prove that  $-(-v) = v$  for every  $v \in V$ .
- (3) For  $v, w \in V$ , explain why there exists a unique  $x \in V$  such that  $v + \alpha x = w$ , for any nonzero  $\alpha \in \mathbb{F}$ .
- (4) The empty set is not a vector space, explain!
- (5) Show that the additive inverse in the definition of the vector space can be replaced with the condition that

$$0v = 0, \text{ for all } v \in V.$$