MATH 413/513 (LINEAR ALGEBRA)
HOMEWORK 1

SUMMER 2018

Due on: Tuesday 05-22-2018.

MATH 413: Solve questions 1 to 4.
MATH 513: Solve questions 1 to 5.

Chapters 1 and 2:

(1) Write the following system of linear equations as an equation for a single function $f : \mathbb{R}^n \to \mathbb{R}^m$ for appropriate choices of $m, n \in \mathbb{Z}_+$,

$$\begin{aligned} x + 2y - 3z &= 4 \\ x + 3y + z &= 11 \\ 2x + 5y - 4z &= 0 \\ x + y + z &= 22 \end{aligned}$$

(2) Solve the equation $z^3 - 4i = 0$ where $z \in \mathbb{C}$.

(3) Find $r > 0$ and $\theta \in [0, 2\pi)$ such that $(1 - i)/\sqrt{2} = re^{i\theta}$.

(4) Let $z, w \in \mathbb{C}$. Prove the parallelogram law

$$|z - w|^2 + |z + w|^2 = 2(|z|^2 + |w|^2).$$

(5) Let $z, w \in \mathbb{C}$ with $\overline{z}w \neq 1$ such that either $|z| = 1$ or $|w| = 1$. Prove that

$$\left| \frac{z - w}{1 - \overline{z}w} \right| = 1$$