Homework: Chapter 5

Name: _____ **Signature:** _____

SHOW ALL YOUR WORK!

Consider the following $n \times n$ matrix where $n \geq 2$ (threes on the diagonal and ones elsewhere).

$$A = \begin{bmatrix} 3 & 1 & 1 & \dots & 1 & 1 \\ 1 & 3 & 1 & \dots & 1 & 1 \\ 1 & 1 & 3 & \dots & 1 & 1 \\ & & & \ddots & & \\ 1 & 1 & 1 & \dots & 3 & 1 \\ 1 & 1 & 1 & \dots & 1 & 3 \end{bmatrix}_{n \times n}$$

(a) Check that $\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$ is an eigenvector of A, find its corresponding eigenvalue λ_1.

(b) Check that $\lambda_2 = 2$ is an eigenvalue of A and find its geometric multiplicity.

(c) Explain why there are no other eigenvalues and write down the characteristic equation in a factorized form.

(d) Diagonalize A, if possible.

(e) Find the determinant of A.