Test 3 | **Calculus I** | **December 5, 2019**

MATH 125-07 | **Dr. Abdul-Rahman**

Name: _____ **Signature:** _____

1. [25 points] If two electric resistances, R_1 and R_2, are connected in parallel, their combined resistance, R, is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Suppose R_1 is held constant at 10 ohms, and that R_2 is increasing at 2 ohms per minute when R_2 is 20 ohms. How fast is R changing at that moment?

2. [30 points] In each of the following, determine whether the limit exists, and where possible evaluate it.

   (a) $\lim_{x\to 0} \dfrac{\arctan x}{x}$

   (b) $\lim_{n\to\infty} \sqrt[n]{n}$

   (c) $\lim_{x\to\infty} \dfrac{e^x}{e^x + e^{-x}}$

   (d) $\lim_{x\to 0} \dfrac{\sin(x^2)}{\sin^2 x}$

   (e) $\lim_{x\to 0^+} \dfrac{1}{x} - \dfrac{1}{\sin x}$

   (f) $\lim_{x\to 0^+} x \ln x$

[Extra page for question 2]

3. [20 points] Evaluate the following integrals

   (a) $\int \dfrac{e^x + 1}{x + e^x + 10}\, dx$

   (b) $\int \tan 2x \ \mathrm{dx}$

   (c) $\int \dfrac{e^{\sqrt{t}}}{\sqrt{t}}\, dt$

   (d) $\int \dfrac{1}{x \ln(x)}\, dx$

4. [30 points] Evaluate the following integrals

   (a) $\displaystyle\int_0^1 6x^5 + 2x + 1 \; dx$

   (b) $\displaystyle\int_0^1 \frac{1}{\sqrt{2x+5}} \; dx$

   (c) $\displaystyle\int_0^6 \sqrt{36 - x^2} \; dx$

5. [15 points] Suppose that f is an **odd function** and that $\int_2^4 f(x)\ dx = 10$, find the following

   (a) $\int_{-2}^4 f(x)\ dx$

   (b) $\int_2^{-4} f(x)\ dx$

   (c) $\int_2^4 (2f(x) + x)\ dx$

6. [15 points] Find the solution of the initial value problem

$$\frac{dy}{dx} = x^5 + x^6, \quad y(1) = 2.$$

7. [20 points] Let

$$E(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\ dt$$

(a) Find intervals where the graph of $E(x)$ is increasing and decreasing.

(b) Find intervals where the graph of $E(x)$ is concave up and concave down.

(c) Find $\dfrac{d}{dx} \displaystyle\int_0^{x^2} e^{-t^2}\ dt$

(d) Find $\lim\limits_{x \to 0} \dfrac{E(x)}{x}$.