

## Homework: Chapter 5

Name: \_\_\_\_\_

Signature: \_\_\_\_\_

### SHOW ALL YOUR WORK!

Consider the following  $n \times n$  matrix where  $n \geq 2$  (threes on the diagonal and ones elsewhere).

$$A = \begin{bmatrix} 3 & 1 & 1 & \dots & 1 & 1 \\ 1 & 3 & 1 & \dots & 1 & 1 \\ 1 & 1 & 3 & \dots & 1 & 1 \\ & & & \ddots & & \\ 1 & 1 & 1 & \dots & 3 & 1 \\ 1 & 1 & 1 & \dots & 1 & 3 \end{bmatrix}_{n \times n}$$

- (a) Check that  $\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$  is an eigenvector of  $A$ , find its corresponding eigenvalue  $\lambda_1$ .
- (b) Check that  $\lambda_2 = 2$  is an eigenvalue of  $A$  and find its geometric multiplicity.
- (c) Explain why there are no other eigenvalues and write down the characteristic equation in a factorized form.
- (d) Diagonalize  $A$ , if possible.
- (e) Find the determinant of  $A$ .