Test 1 (B) **Introduction to Linear Algebra** **September 17, 2018**

MA 313

Name: ________________________ **Signature:** ________________________

SHOW ALL YOUR WORK!

1. [15 points] Let

$$A = \begin{bmatrix} 1 & -4 & 7 & -5 \\ 0 & 1 & -4 & 3 \\ 2 & -6 & 6 & -4 \end{bmatrix}$$

(a) What must n and m be in order to define $T : \mathbb{R}^n \to \mathbb{R}^m$ by $T(\mathbf{x}) = A\mathbf{x}$?

(b) Find all $\mathbf{x} \in \mathbb{R}^4$ that are mapped into the zero vector by the transformation T in part (a).

(c) Is $\mathbf{b} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ in the range of the transformation T in part (a)?

2. [15 points] Find the value(s) of h for which the vectors are linearly independent.

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix},$$

3. [15 points] Suppose the 3×3 matrix A has the three columns $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ where $\mathbf{a}_3 = 2\mathbf{a}_2 - \mathbf{a}_1$.

   (a) Are the columns of A linearly independent? Why?

   (b) Find a non-trivial solution for $A\mathbf{x} = \mathbf{0}$.

4. [10 points] How many pivot columns must a

   (a) 6×5 matrix have if its columns are linearly independent? Why?

   (b) 5×6 matrix have if its columns span $\mathbb{R}^5$? Why?

5. [10 points] Do the columns of A span $\mathbb{R}^3$? Explain.

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & -1 \\ 5 & 8 & 7 \end{bmatrix}$$

6. [20 points] Describe all solutions of $A\mathbf{x} = \mathbf{b}$ in parametric form, where

$$A = \begin{bmatrix} 1 & 5 & 2 & -6 & 9 & 0 \\ 0 & 0 & 1 & -7 & 4 & -8 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

7. [20 points] Mark each statement as **True** or **False**, explain.

(a) (   ) If a system of linear equations has no free variables, then it has a unique solution.

(b) (   ) If a system $A\mathbf{x} = \mathbf{b}$ has more than one solution, then so does the system $A\mathbf{x} = \mathbf{0}$.

(c) (   ) If matrices A and B are row equivalent, they have the same reduced row echelon form.

(d) (   ) If none of the vectors in $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in $\mathbb{R}^3$ is a multiple of one of the other vectors, then S is linearly independent.

(e) (   ) If $S = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ are linearly dependent then each vector in the set S is in the span of the other vectors.

(f) (   ) If $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, then $\mathbf{u}$ is a linear combination of $\mathbf{w}$ and $\mathbf{v}$.

(g) (   ) If A is a 3×5 matrix and T is a transformation defined by $T(\mathbf{x}) = A\mathbf{x}$, then the domain of T is $\mathbb{R}^3$.

(h) (   ) The columns of any 4×5 matrix are linearly dependent.

(i) (   ) If a set contains fewer vectors then there are entries in the vectors, then the set is linearly independent.

(j) (   ) The set $\text{span}\{\mathbf{u}, \mathbf{v}\}$ is always visualized as a plane through the origin.