Test 2 (A) | **Introduction to Linear Algebra MA 313** | **Dr. Abdul-Rahman October 19, 2018**

Name: ______________________ **Signature:** ______________________

SHOW ALL YOUR WORK!

1. [10 points] Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$, defined as

$$T(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2)$$

for every real numbers $x_1, x_2 \in \mathbb{R}$.

(a) Find the standard matrix A of the linear transformation T.

(b) Is T one-to-one? onto? Explain!

2. [10 points] Find the standard matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that rotates points (about the origin) through $\frac{3\pi}{2}$ radians (counterclockwise).

3. [20 points] Find the inverses of the following matrices

$$A = \begin{bmatrix} -1 & 2 \\ 3 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

4. [20 points] Given

$$A = \begin{bmatrix} -2 & -7 & -9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}$$

(a) Find the determinant of A.

(b) Find the **third** column of A^{-1}.

5. [20 points] Given that

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

with $\det(A) = 3$, Find each of the following (explain)

(a) $\det\left(2A^{-1}(A^T)^2\right)$.

(b) $\begin{vmatrix} a+2d & b+2e & c+2f \\ d & e & f \\ g & h & i \end{vmatrix}$.

(c) $\begin{vmatrix} a & b & c \\ 3a+4d & 3b+4e & 3c+4f \\ -g & -h & -i \end{vmatrix}$.

(d) $\begin{vmatrix} e & d & f \\ b & a & c \\ h & g & i \end{vmatrix}$.

6. [20 points] Solve **FOUR** from the following five questions. Explain your answers!

(a) Show that $S := \{f \in C[a,b]; f(a) = f(b)\}$ is a subspace of the vector space $C[a,b]$ of all continuous functions on the interval $[a,b]$.

(b) Let

$$W := \left\{ \begin{bmatrix} s+3t \\ s-t \\ s+t \end{bmatrix}; s,t \in \mathbb{R} \right\}.$$

Is W a subspace of $\mathbb{R}^3$? Explain.

(c) Determine if the set H of all polynomials in $\mathbb{P}_n$ such that $\mathbf{p}(0) = 0$, is a subspace of the vector space $\mathbb{P}_n$ of all polynomials of degree less than or equal to n.

(d) Let H be the set of all matrices of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, where a and b are real numbers. Determine if H is a subspace of the vector space $M_{2\times 2}$ of all 2×2 matrices.

(e) Let

$$U := \left\{ \begin{bmatrix} 2x-1 \\ x+1 \end{bmatrix}; x \in \mathbb{R} \right\}.$$

Is U a subspace of $\mathbb{R}^2$? Explain.