

MATH 250b – Section 2

Spring Semester 2008

Tuesday, -6-Mayr-08

<http://math.arizona.edu/~cushing/250b.html>

First Order Equations

First Order Systems (& higher order equations)

First Order Equations

➤ **Linear:** $x' = p(t)x + q(t)$

➤ **Nonlinear:** $x' = f(t, x)$

First Order Systems (& higher order equations)

First Order Equations

- **Linear:** $x' = p(t)x + q(t)$
 - Homogeneous: $x' = p(t)x$
 - Non-homogeneous

- **Nonlinear:** $x' = f(t, x)$

First Order Systems (& higher order equations)

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- **Linear:** $x' = p(t)x + q(t)$
 - Homogeneous: $x' = p(t)x$
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 - VC Formula
 - Undetermined coefficients
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 - Autonomous $x' = f(x)$
 - Non-autonomous: $x' = f(t, x)$

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 - Homogeneous: $\hat{x}' = A(t)\hat{x}$
 - General solution $\hat{x} = \Phi(t)\hat{c}$
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