

MATH 250b – Section 2

Spring Semester 2008

Thursday, 3-Apr-08

<http://math.arizona.edu/~cushing/250b.html>

How best to display the numerical output ?

$$x' = f(t, x, y)$$

$$y' = g(t, x, y)$$

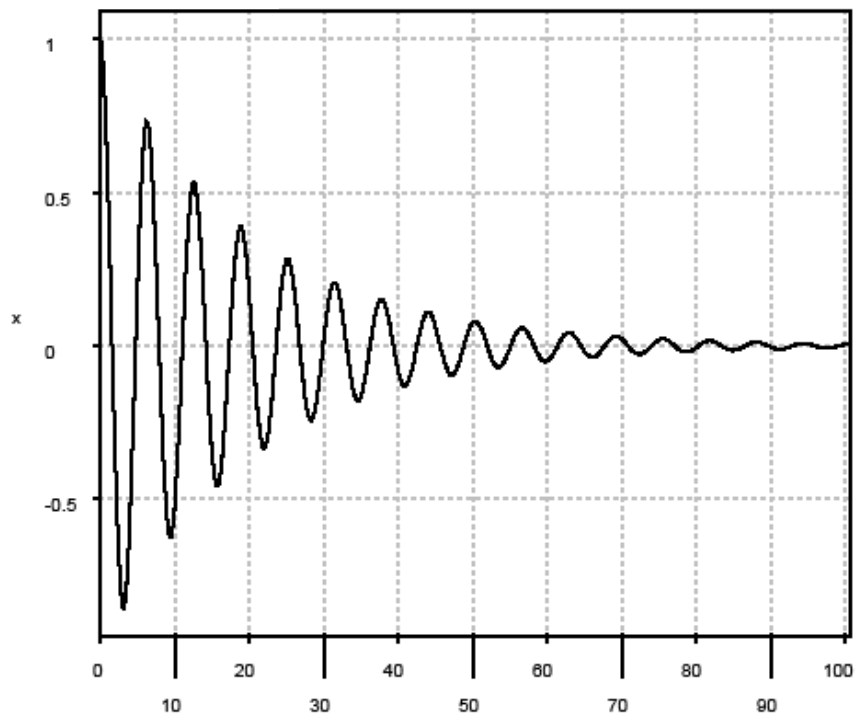
1. Individual “time series” plots of x *against* t and y *against* t .
2. 3D plot of (t, x, y)
3. For autonomous systems we can use *phase plane* plots.

$$x' = f(x, y)$$

$$y' = g(x, y)$$

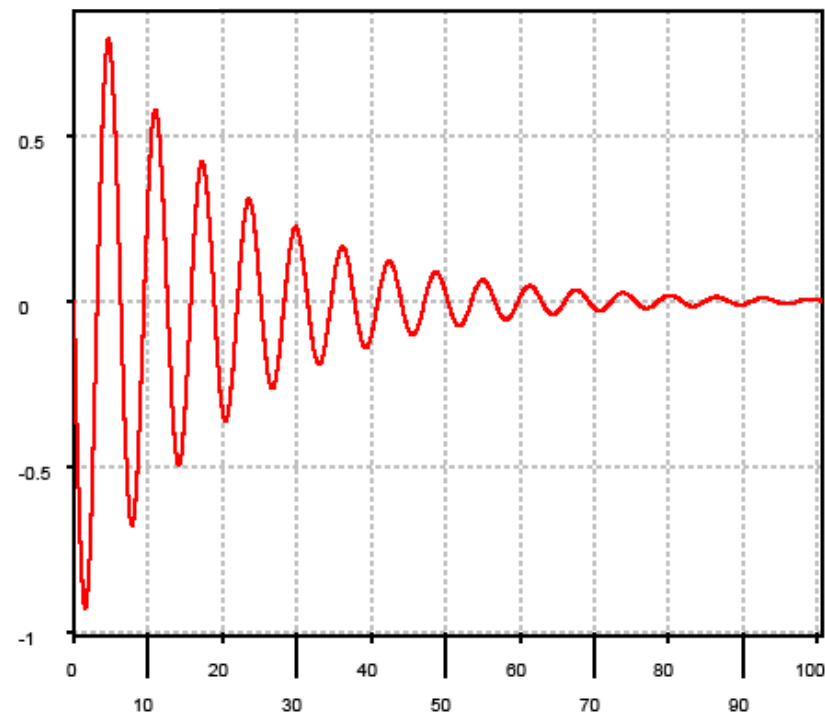
$$x'' = -x - 0.1x'$$
$$x(0) = 1, \quad x'(0) = 0$$

x



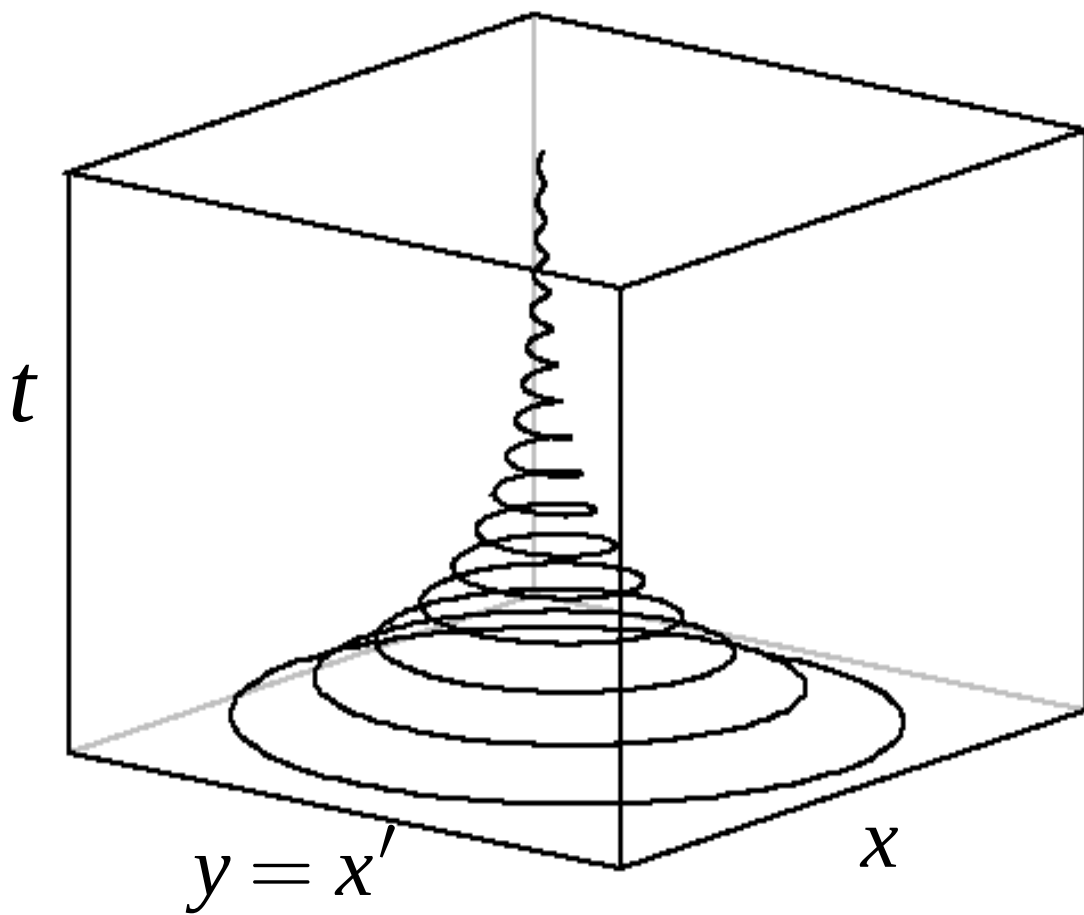
t

x'



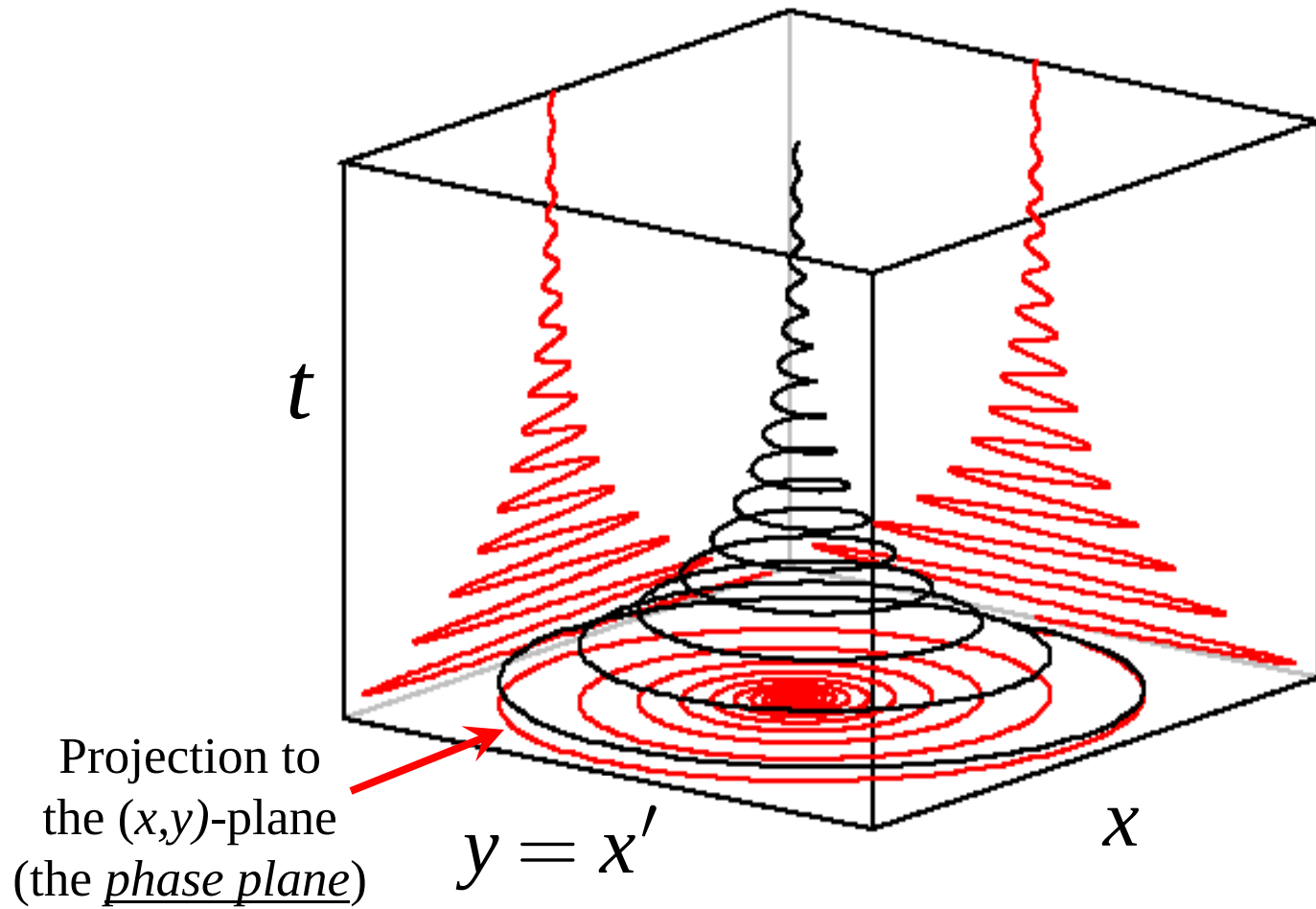
t

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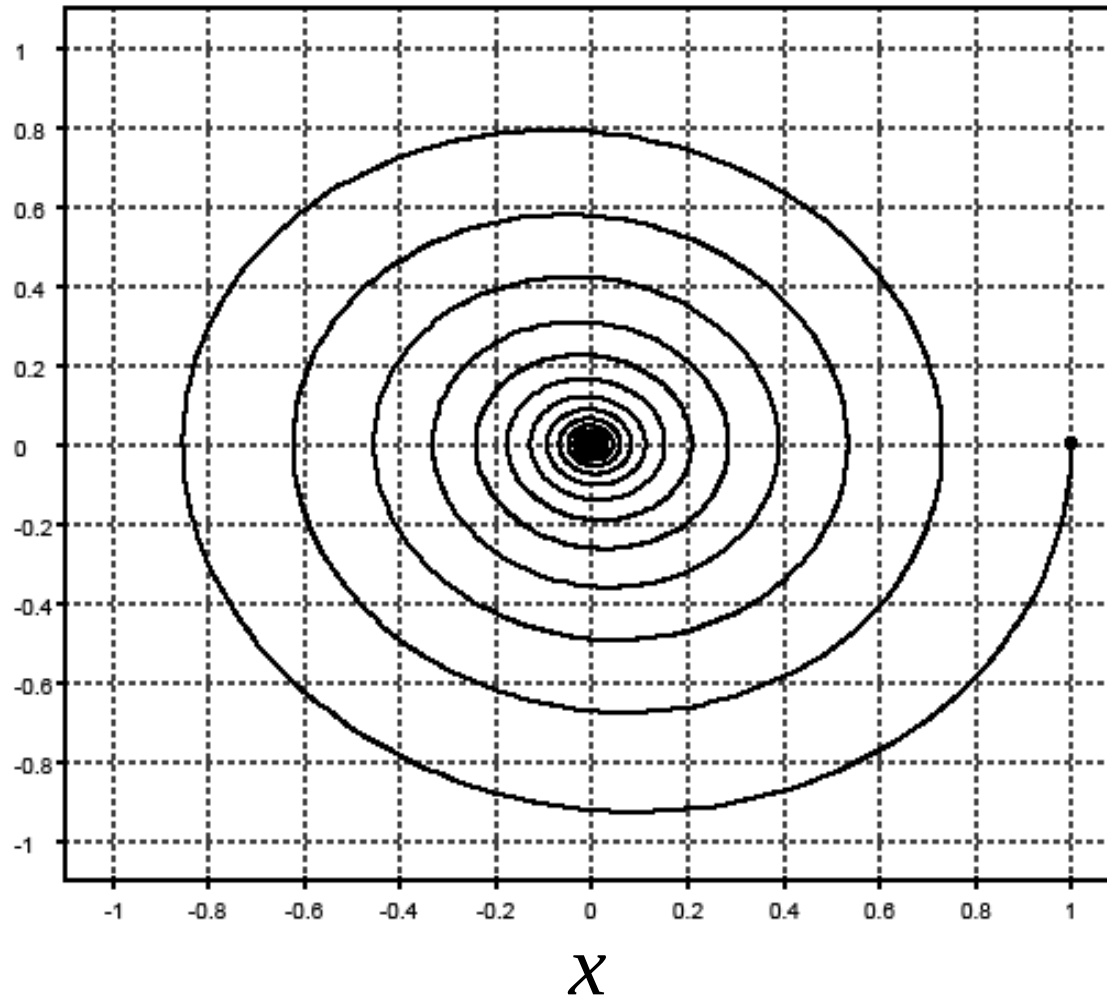


$$x'' = -x - 0.1x'$$

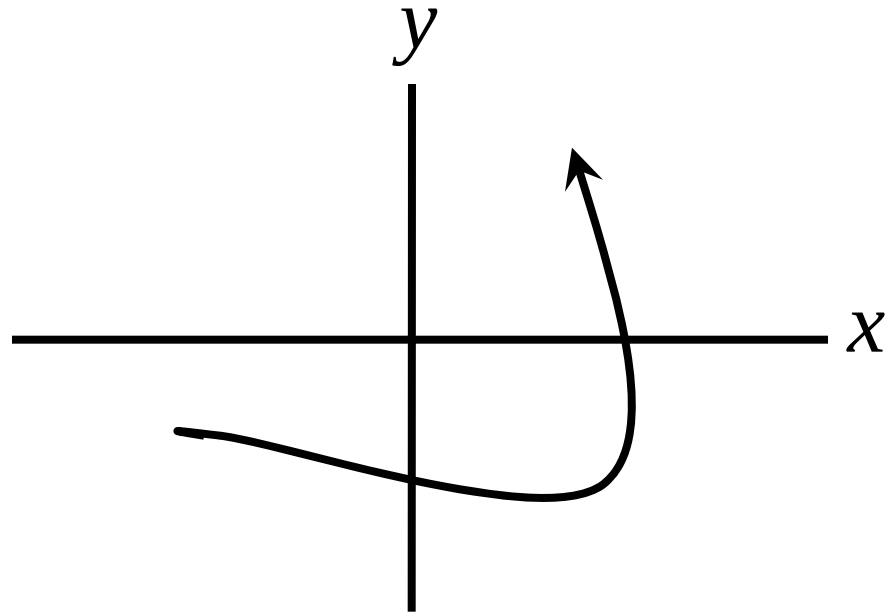
$$x(0) = 1, \quad x'(0) = 0$$

Phase Plane

$y = x'$



If $x(t), y(t)$ is a solution pair,
the associated *orbit* is the set of points
 $\{(x(t), y(t))\}$
together with an
orientation in the direction of increasing t



Some basic facts about the phase plane orbits

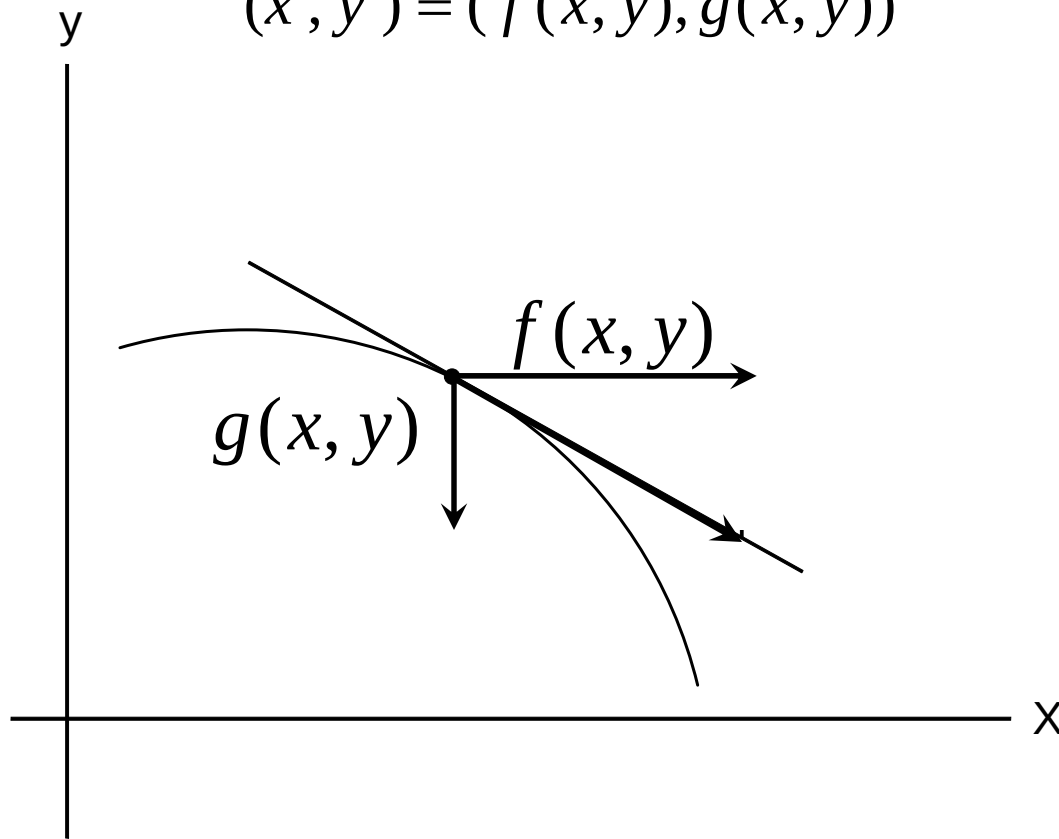
- Translations of solutions are solutions
- Orbits are associated with solutions all of which are translates of each other
- There is an orbit through each point in the plane (where the fundamental theorem applies)
- Orbits associated with different solutions (non-translates) cannot intersect
- The collection of all orbits is called the phase plane portrait

Direction (vector) Fields

At each point (x, y)

the *tangent to the orbit* has the direction

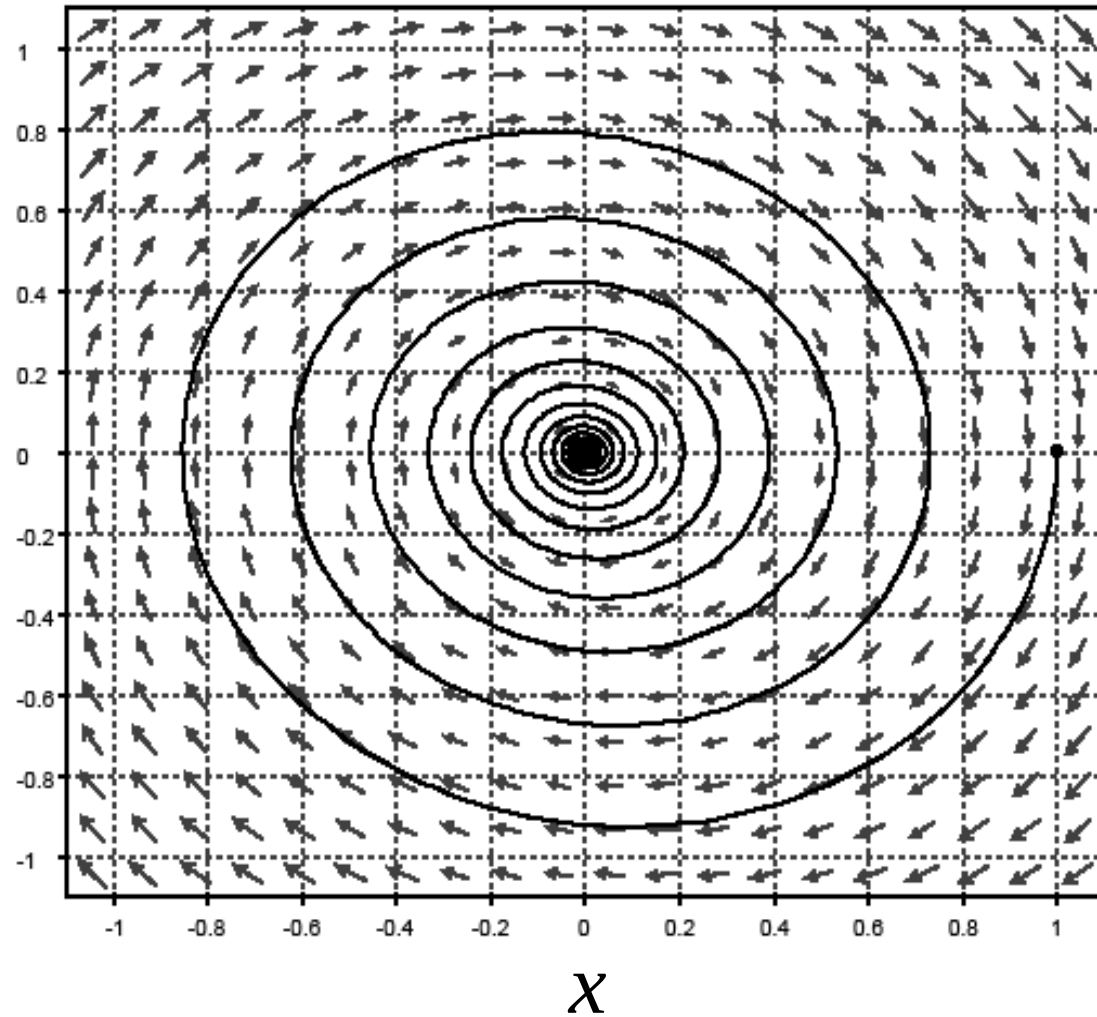
$$(x', y') = (f(x, y), g(x, y))$$



$$x'' = -x - 0.1x'$$

$$x(0) = 1, \quad x'(0) = 0$$

$$y = x'$$



Isoclines

$$x' = f(x, y)$$

$$y' = g(x, y)$$

Vectors in phase plane are *vertical* at those points (x, y)

$$x' = 0 \quad \text{that is, where} \quad f(x, y) = 0$$

Vectors in phase plane are *horizontal* at those points (x, y)

$$y' = 0 \quad \text{that is, where} \quad g(x, y) = 0$$

$f(x, y) = 0$ defines a curve called the *vertical null isocline*

$g(x, y) = 0$ defines a curve called the *horizontal null isocline*

Example

Pesticide example

$$x' = -2x + 2y$$

$$y' = 2x - 5y$$

$-2x + 2y = 0 \Rightarrow$ vertical nullcline is a straight line $y = x$

$2x - 5y = 0 \Rightarrow$ horizontal nullcline is a straight line $y = (2 / 5)x$

Isoclines

$$x' = f(x, y)$$

$$y' = g(x, y)$$

Isoclines divide phase plane into regions

In each region, all vector field directions
have the same “compass direction”

Example

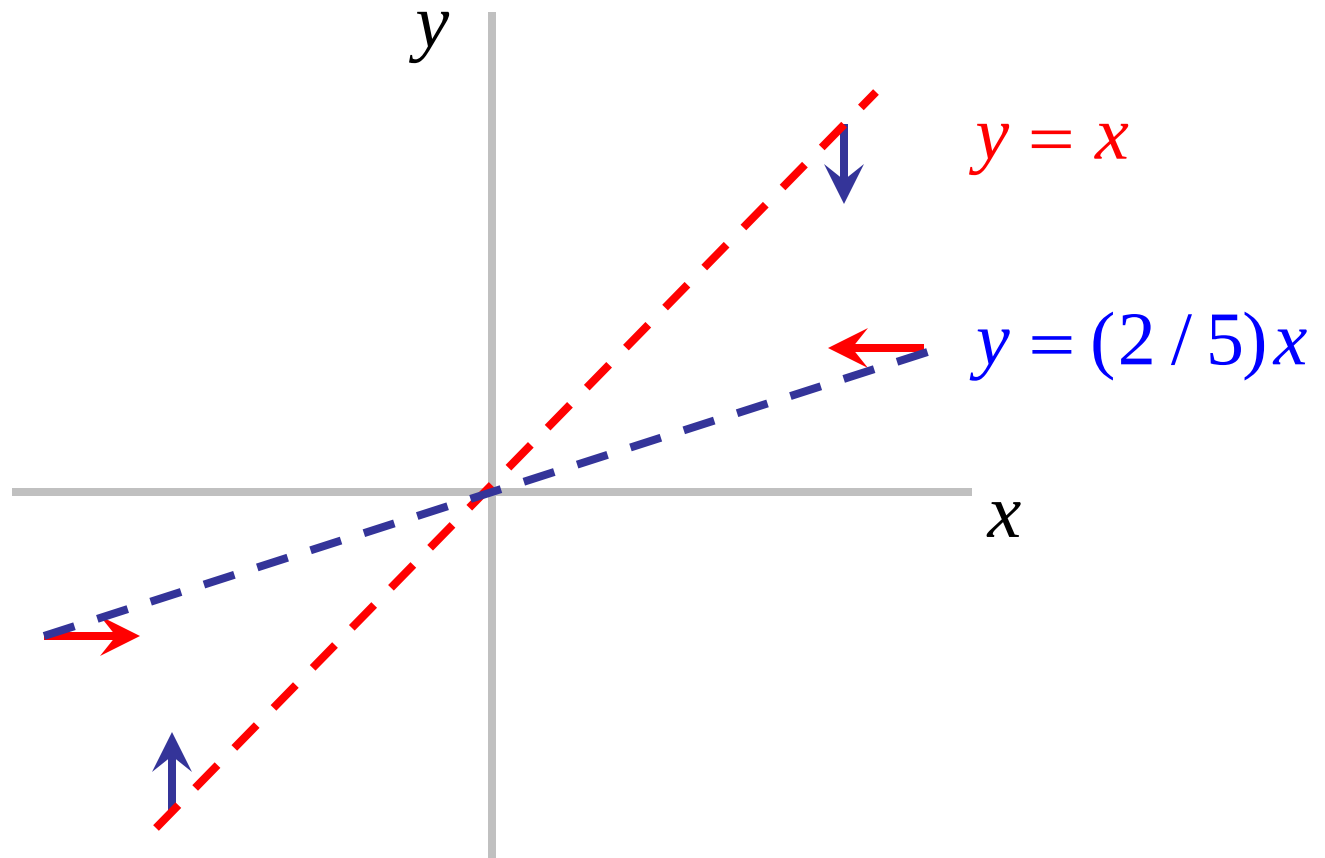
Pesticide example

$$x' = -2x + 2y$$

vertical nullcline is $y = x$

$$y' = 2x - 5y$$

horizontal nullcline $y = (2/5)x$



Example

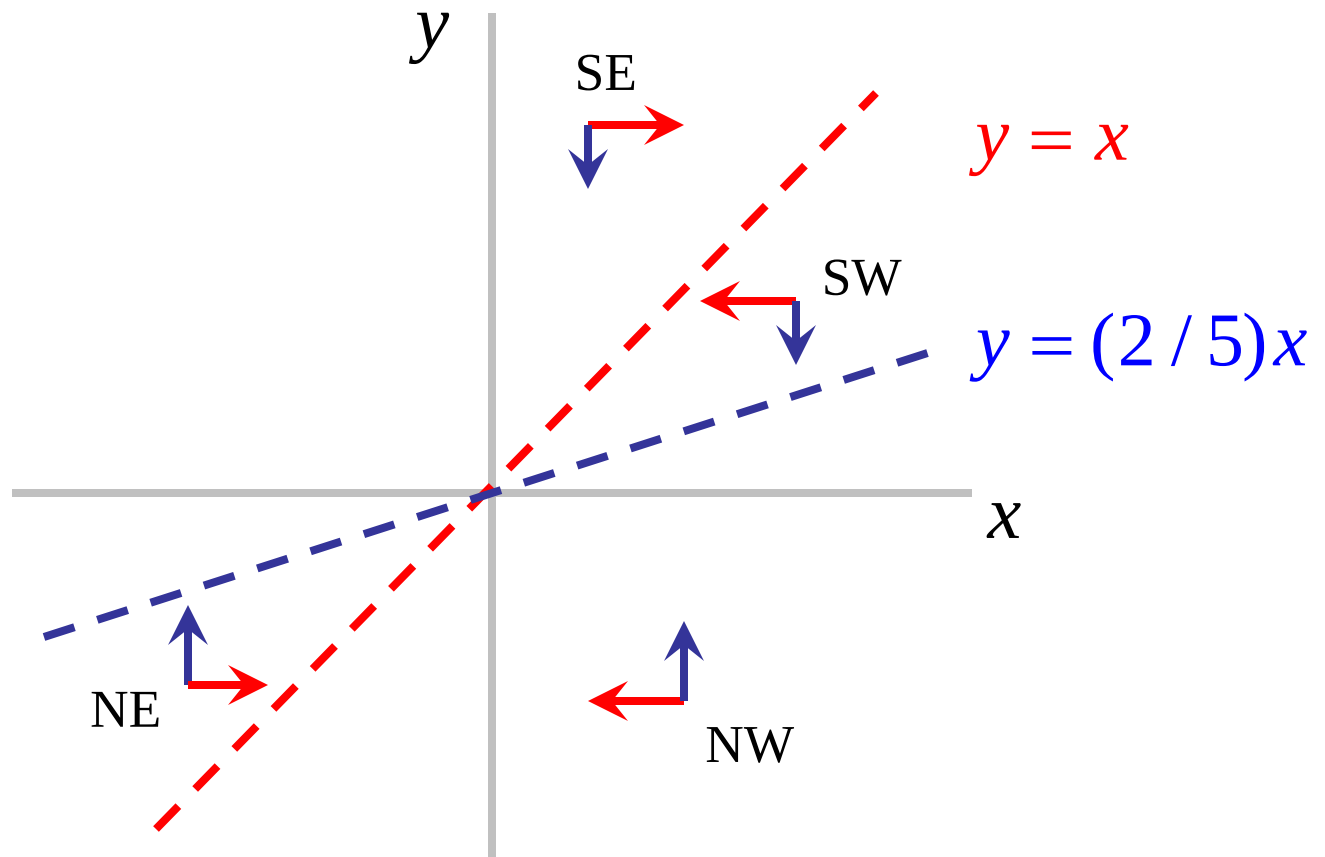
Pesticide example

$$x' = -2x + 2y$$

vertical nullcline is $y = x$

$$y' = 2x - 5y$$

horizontal nullcline $y = (2/5)x$



Example

Frictionless Mass-Spring (harmonic oscillator)

$$x' = y$$

vertical nullcline is $y = 0$ (x -axis)

$$y' = -x$$

horizontal nullcline $x = 0$ (y -axis)

