

MATH 250b – Section 2

Spring Semester 2008

Tuesday, 25-Mar-08

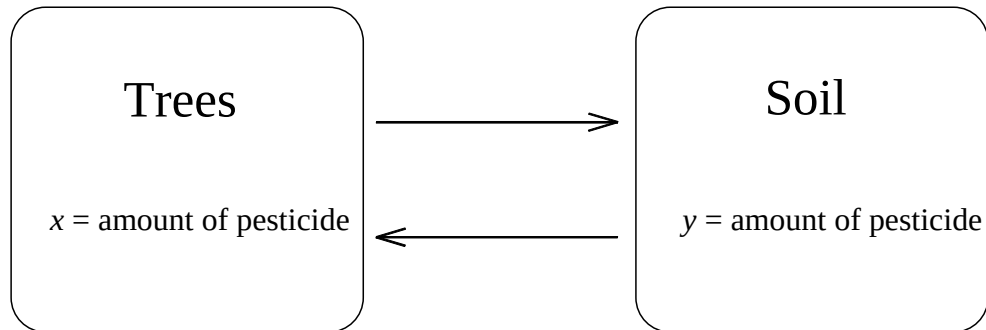
<http://math.arizona.edu/~cushing/250b.html>

First Order Differential Equations

- Solutions, initial value problems, existence & uniqueness
- Numeric & graphic approximations to solutions
- Linear equations
 - General solution $x = x_h + x_p$
 - Variation of Constants Formula
- Nonlinear autonomous equations
 - Phase line portraits: equilibria & their type
 - Linearization Principle (hyperbolicity)
 - Bifurcations
- Nonlinear nonautonomous equations
 - Separable equations

Second half of course will re-visit these topics for systems of first order equations & higher order equations

Example



An example of
“compartmental modeling”



To derive ODE's apply *rate balancing law*
to each compartment:

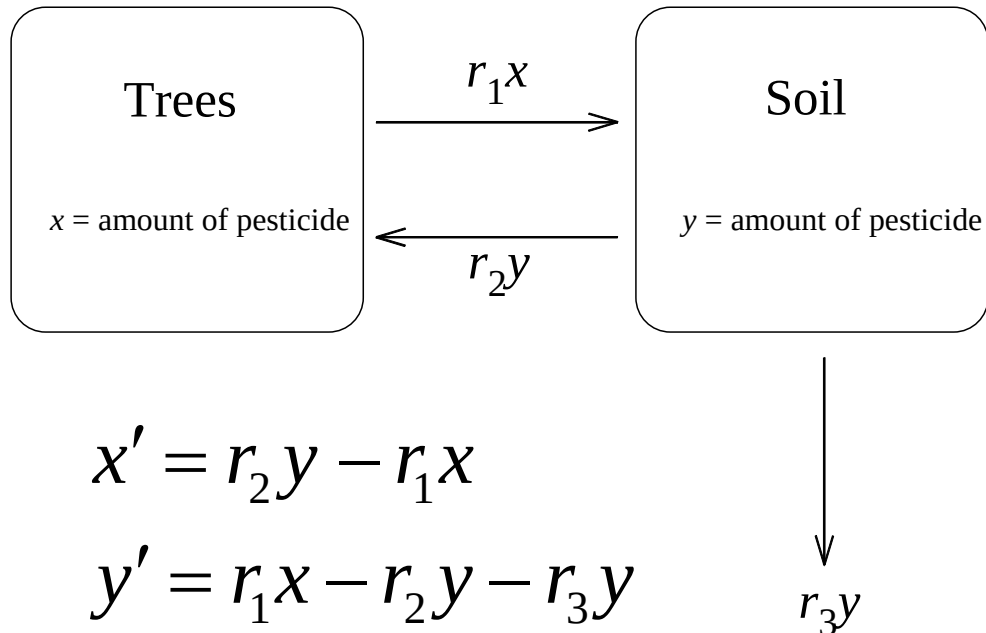
BASIC RATE BALANCING LAW

rate of change = input rate – output rate

Example

Assumption

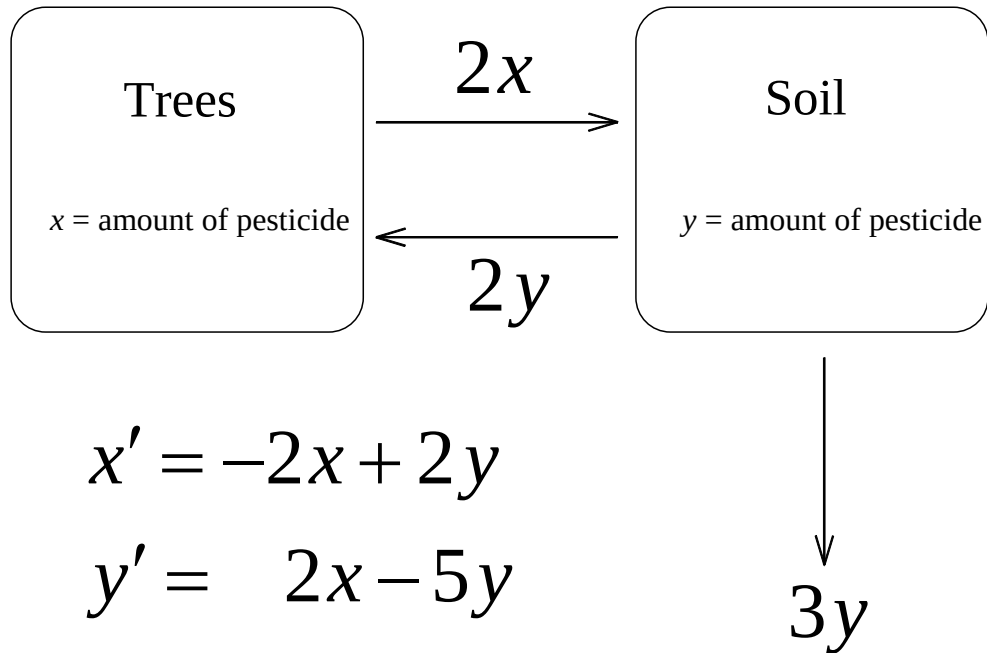
Rates of exchange
are proportional to
amounts present.



r_i = per unit rate of change
units of r_i = 1/time

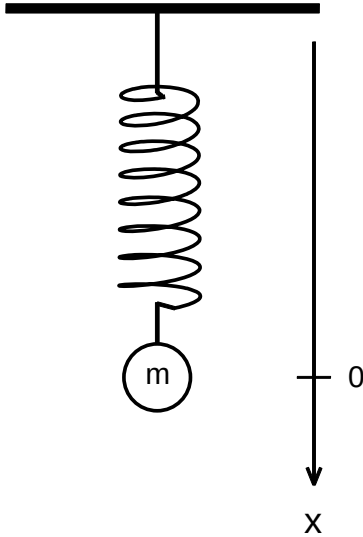
A *system* of first order equations for
two state variables x, y
(a *two dimensional* system)

Example



A *system* of first order equations for
two state variables x, y
(a *two dimensional* system)

Example



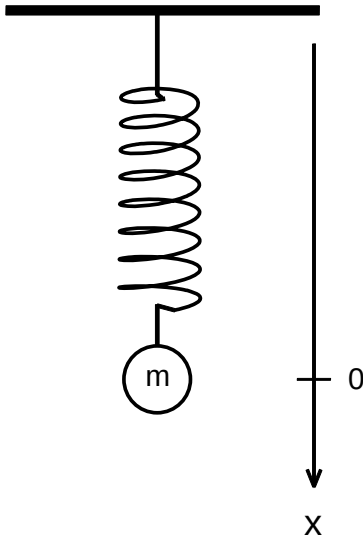
$$mx'' = F$$

Newton's Law

$x(t)$ = displacement from rest position

m = mass of object

Example



$$m\ddot{x} = -kx - c\dot{x}$$

Newton's Law

Hooke's Law

Linear Friction Law

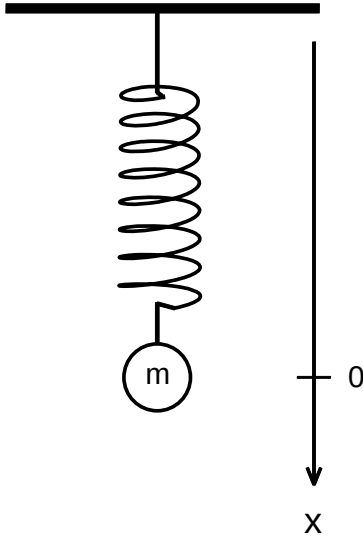
$x(t)$ = displacement from rest position

m = mass of object

k = spring constant (Hooke's constant)

c = coefficient of friction

Example



$$m\ddot{x} = -kx - c\dot{x}$$

A Second Order Differential Equation

$$m\ddot{x} + kx + c\dot{x} = 0$$

An higher equation *can always be re-written*
as an equivalent first order system

Example

$$mx'' + kx + cx' = 0$$

- Define $y = x'$
- Write a first order system for x and y :

(Note: $y' = x''$)

$$\begin{cases} x' = y \\ y' = -\frac{k}{m}x - \frac{c}{m}y \end{cases}$$