

MATH 250b – Section 2

Spring Semester 2008

Thursday, 13-Mar-08

<http://math.arizona.edu/~cushing/250b.html>

x	y	residuals $q(x) - y$
x_1	y_1	$q(x_1) - y_1$
x_2	y_2	$q(x_2) - y_2$
x_3	y_3	$q(x_3) - y_3$
$\vdots$	$\vdots$	$\vdots$
x_n	y_n	$q(x_n) - y_n$

GOAL

To find the “best” fit
using a quadratic polynomial

$$q(x) = ax^2 + bx + c.$$

Find coefficients a , b , and c
so as to minimize the differences between

$$q(x_i) = ax_i^2 + bx_i + c \quad \text{and the data } y_i.$$

Namely, minimize the sum of squared residuals:

$$E = (q(x_1) - y_1)^2 + (q(x_2) - y_2)^2 + \cdots + (q(x_n) - y_n)^2$$

METHOD

$$q(x) = ax^2 + bx + c$$

$$E = (q(x_1) - y_1)^2 + (q(x_2) - y_2)^2 + \cdots + (q(x_n) - y_n)^2$$

Solve

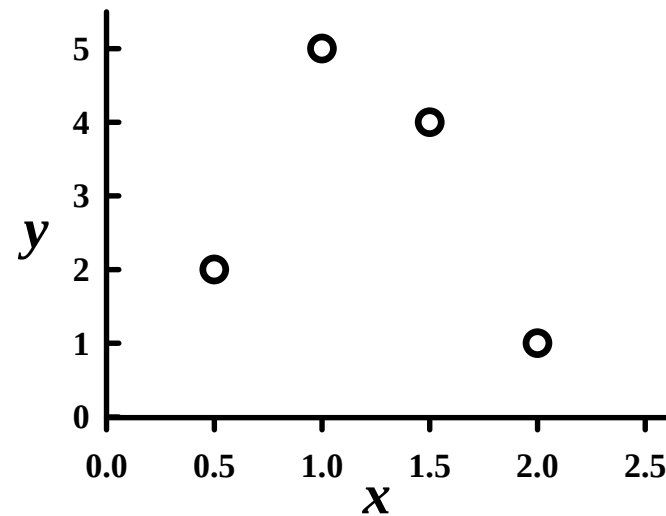
$$\frac{dE}{da} = 0, \quad \frac{dE}{db} = 0, \quad \frac{dE}{dc} = 0$$

for a, b and c

x	y
0.5	2
1	5
1.5	4
2	1

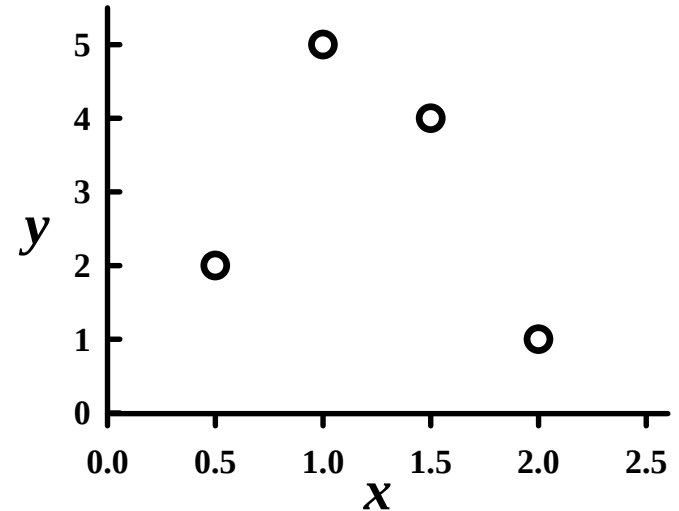
EXAMPLE

$$q(x) = ax^2 + bx + c$$



x	y
0.5	2
1	5
1.5	4
2	1

EXAMPLE



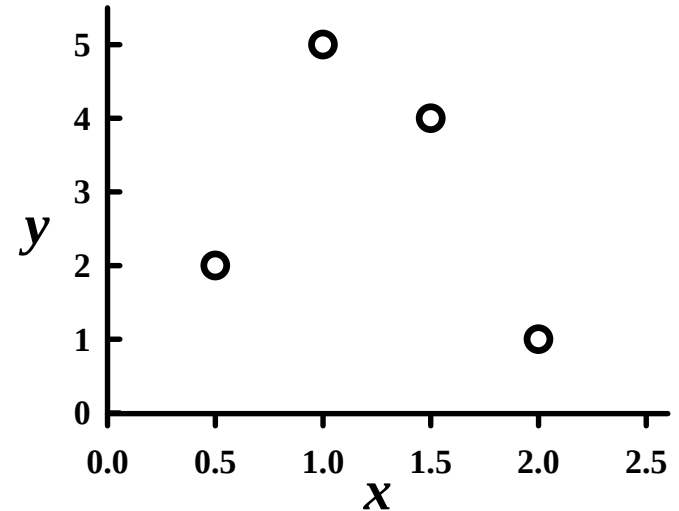
$$q(x) = ax^2 + bx + c$$

$$E = (0.25a + 0.5b + c - 2)^2 + (a + b + c - 5)^2 \\ + (2.25a + 1.5b + c - 4)^2 + (4a + 2b + c - 1)^2$$

Calculate $\frac{dE}{da}$, $\frac{dE}{db}$, $\frac{dE}{dc}$ and set each equal to 0.

x	y
0.5	2
1	5
1.5	4
2	1

EXAMPLE



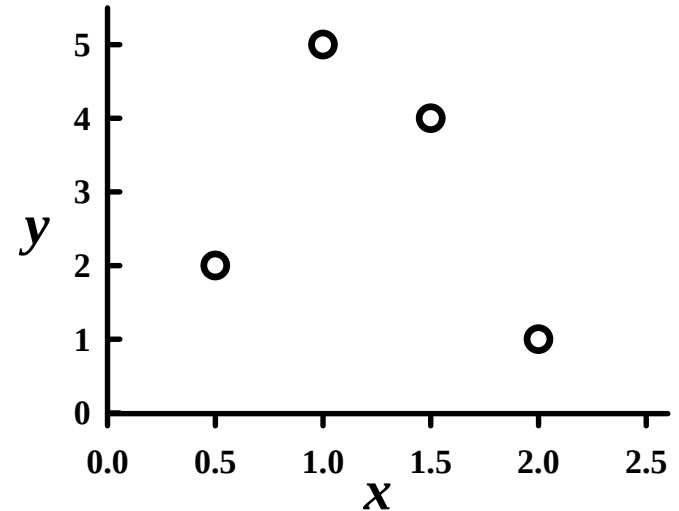
$$q(x) = ax^2 + bx + c$$

$$E = (0.25a + 0.5b + c - 2)^2 + (a + b + c - 5)^2 \\ + (2.25a + 1.5b + c - 4)^2 + (4a + 2b + c - 1)^2$$

$$\frac{dE}{da} = 2(0.25a + 0.5b + c - 2)0.25 + 2(a + b + c - 5) \\ + 2(2.25a + 1.5b + c - 4)2.25 + 2(4a + 2b + c - 1)4$$

x	y
0.5	2
1	5
1.5	4
2	1

EXAMPLE



$$q(x) = ax^2 + bx + c$$

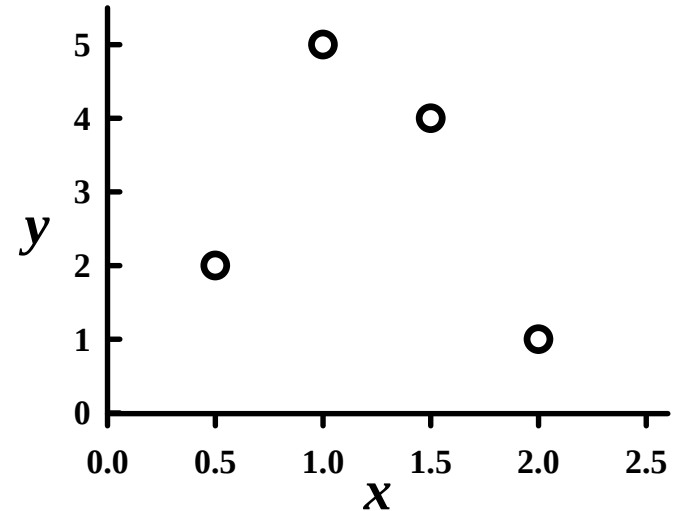
$$E = (0.25a + 0.5b + c - 2)^2 + (a + b + c - 5)^2 \\ + (2.25a + 1.5b + c - 4)^2 + (4a + 2b + c - 1)^2$$

$$\frac{dE}{da} = 44.25a + 25b + 15c - 37 = 0$$

In a similar way, calculate $\frac{dE}{db}$ and $\frac{dE}{dc}$ and set each equal to 0.

EXAMPLE

x	y
0.5	2
1	5
1.5	4
2	1



$$q(x) = ax^2 + bx + c$$

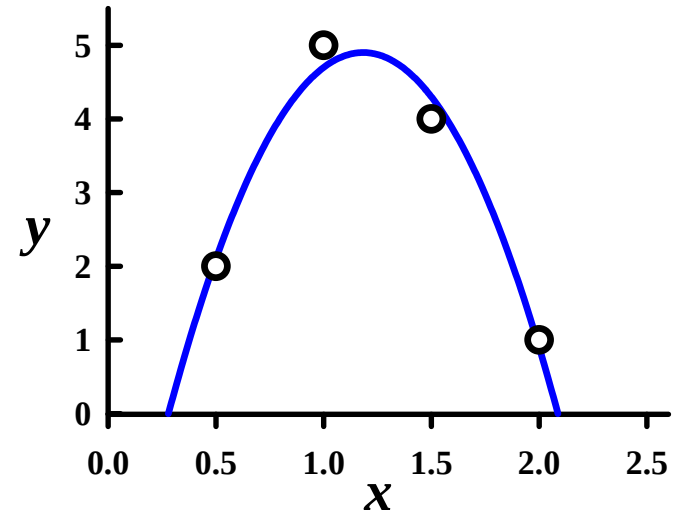
$$\left. \begin{aligned} \frac{dE}{da} &= 44.25a + 25b + 15c - 37 = 0 \\ \frac{dE}{db} &= 25a + 15b + 10c - 28 = 0 \\ \frac{dE}{dc} &= 15a + 10b + 8c - 24 = 0 \end{aligned} \right\}$$

Three equations
to solve for
 a , b , and c .

$$a = -7, \quad b = 17.3, \quad c = -5.75$$

x	y
0.5	2
1	5
1.5	4
2	1

EXAMPLE



$$q(x) = -7x^2 + 17.3x - 5.75$$

Best quadratic fit (in the least squares sense)