

MATH 250b – Section 2

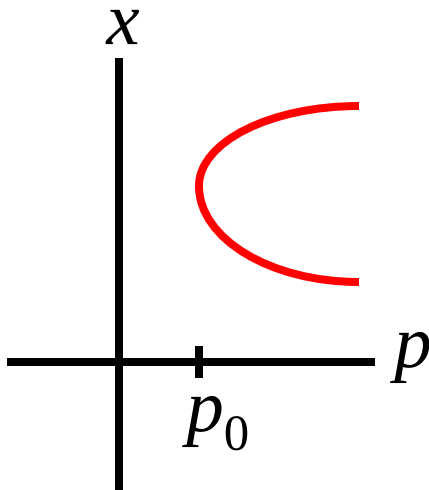
Spring Semester 2008

Tuesday, 04-Mar-08

<http://math.arizona.edu/~cushing/250b.html>

Classification of *Local* Bifurcations

$$x' = f(x, p)$$



Saddle-Node
(or *Blue Sky*)

0 on one side

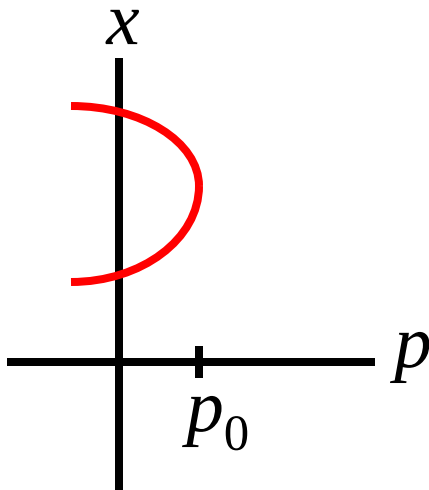
2 on the other

Prototypical Example

$$x' = x^2 - p$$

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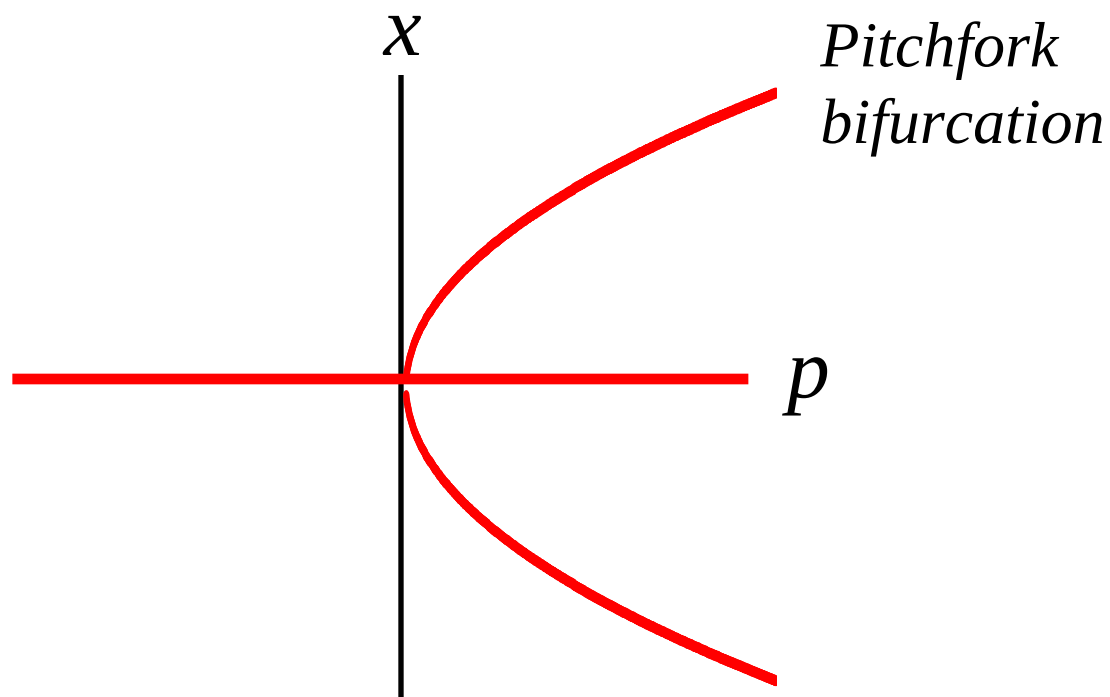
Prototypical Example

$$x' = x^2 - p$$

Other Examples

$$x' = px - x^3$$

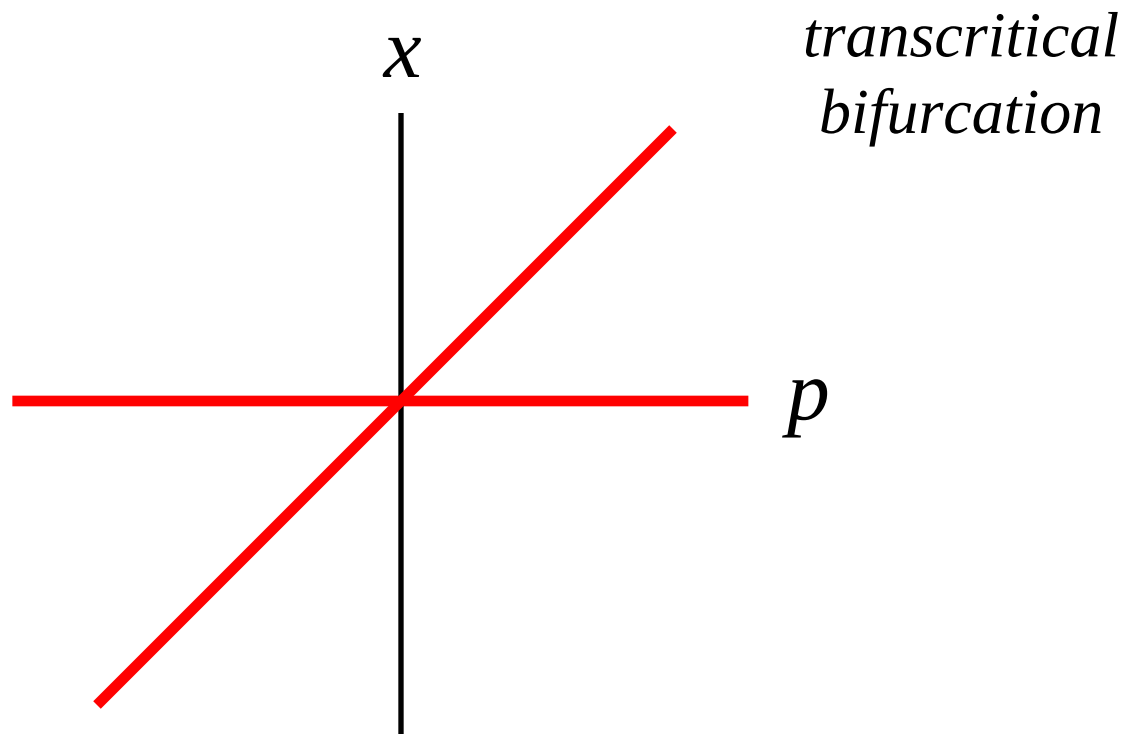
$$x = 0 \text{ and } p = x^2$$



Other Examples

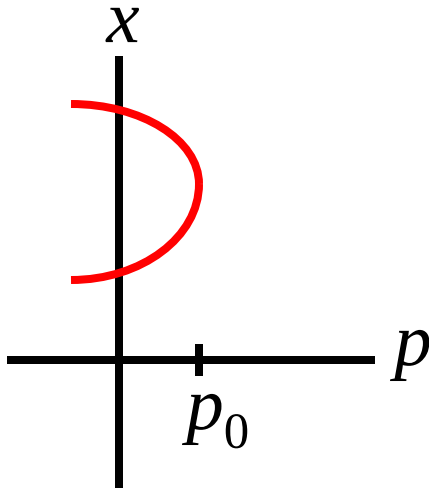
$$x' = px - x^2$$

$$x = 0 \text{ and } p = x$$



Classification of *Local* Bifurcations

$$x' = f(x, p)$$

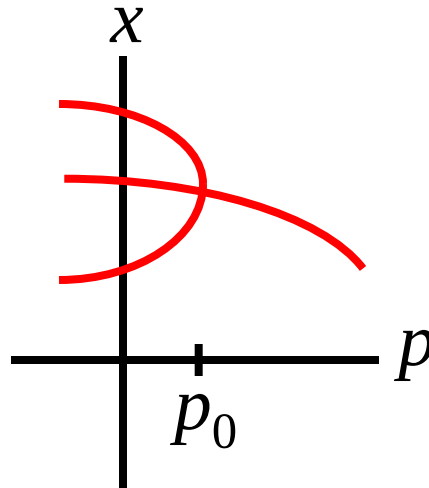


Saddle-Node
(or *Blue Sky*)

0 on one side
2 on the other

Prototypical Example

$$x' = x^2 - p$$

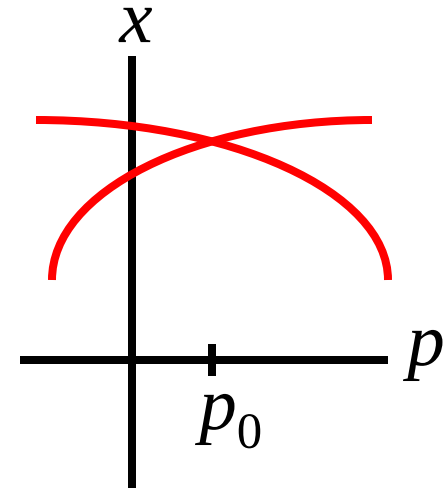


Pitchfork

1 on one side 3
on the other

Prototypical Example

$$x' = px - x^3$$



Transcritical

2 on both sides
1 at p_0

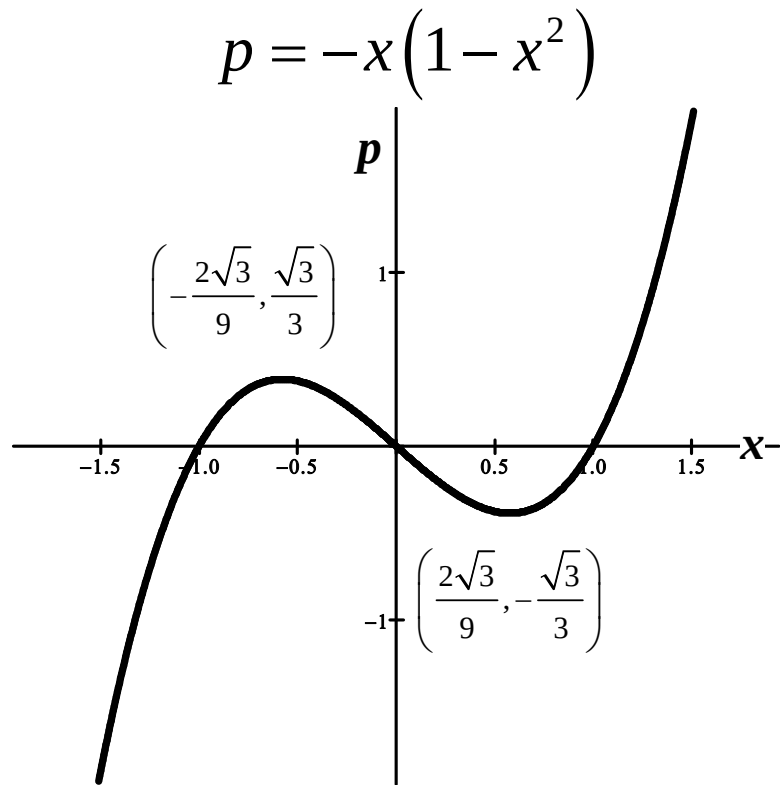
Prototypical Example

$$x' = px - x^2$$

EXAMPLE

$$x' = x \left[x(1 - x^2) + p \right]$$

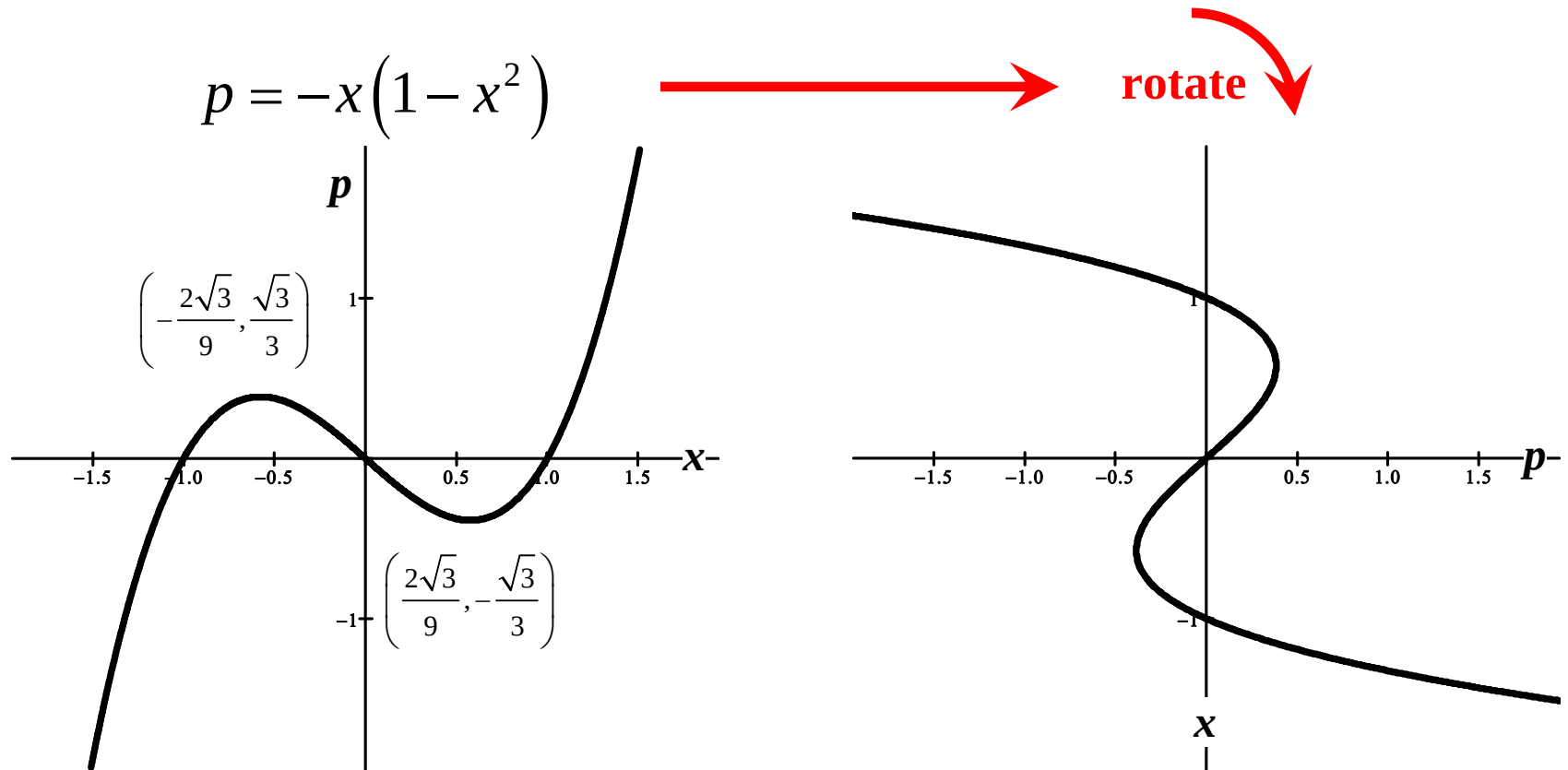
Equilibria: $x = 0$ and $x(1 - x^2) + p = 0$



EXAMPLE

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Equilibria: $x = 0$ and $x(1 - x^2) + p = 0$

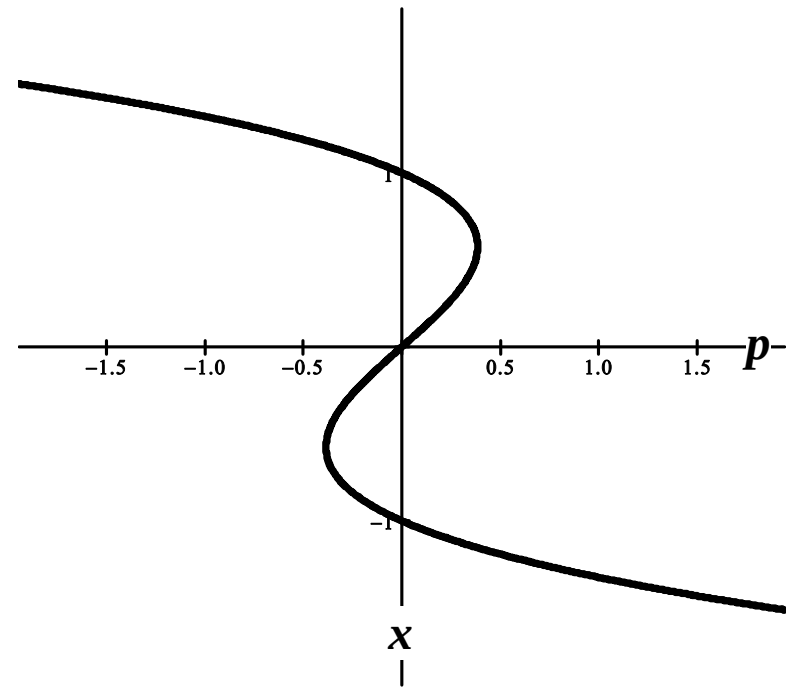
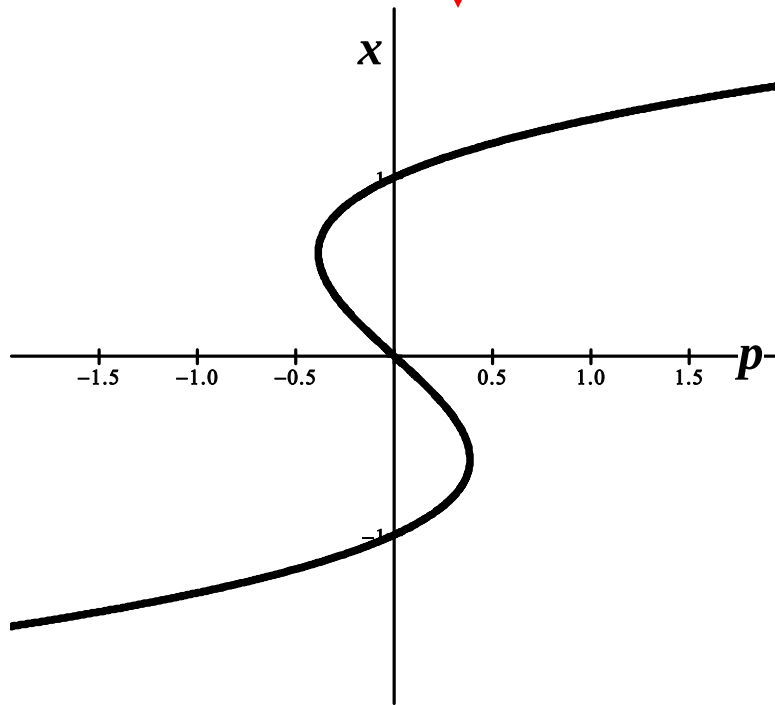


EXAMPLE

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Equilibria: $x = 0$ and $x(1 - x^2) + p = 0$

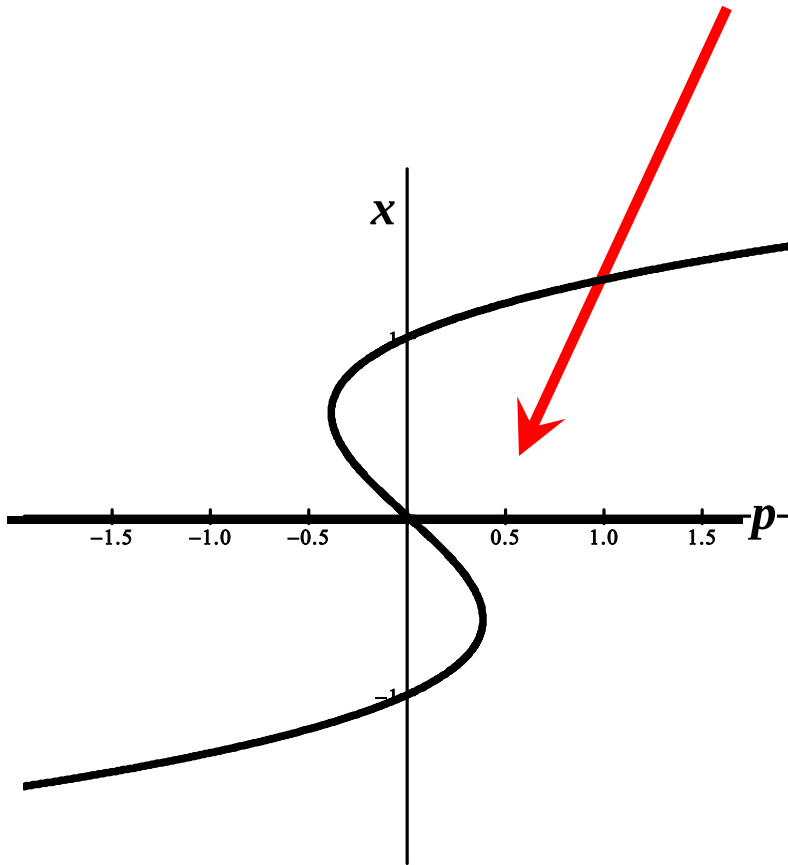
reflect



EXAMPLE

$$x' = x \left[x(1 - x^2) + p \right]$$

Equilibria: $x = 0$ and $x(1 - x^2) + p = 0$



EXAMPLE

$$x' = x \left[x(1 - x^2) + p \right]$$

Two saddle-node bifurcations

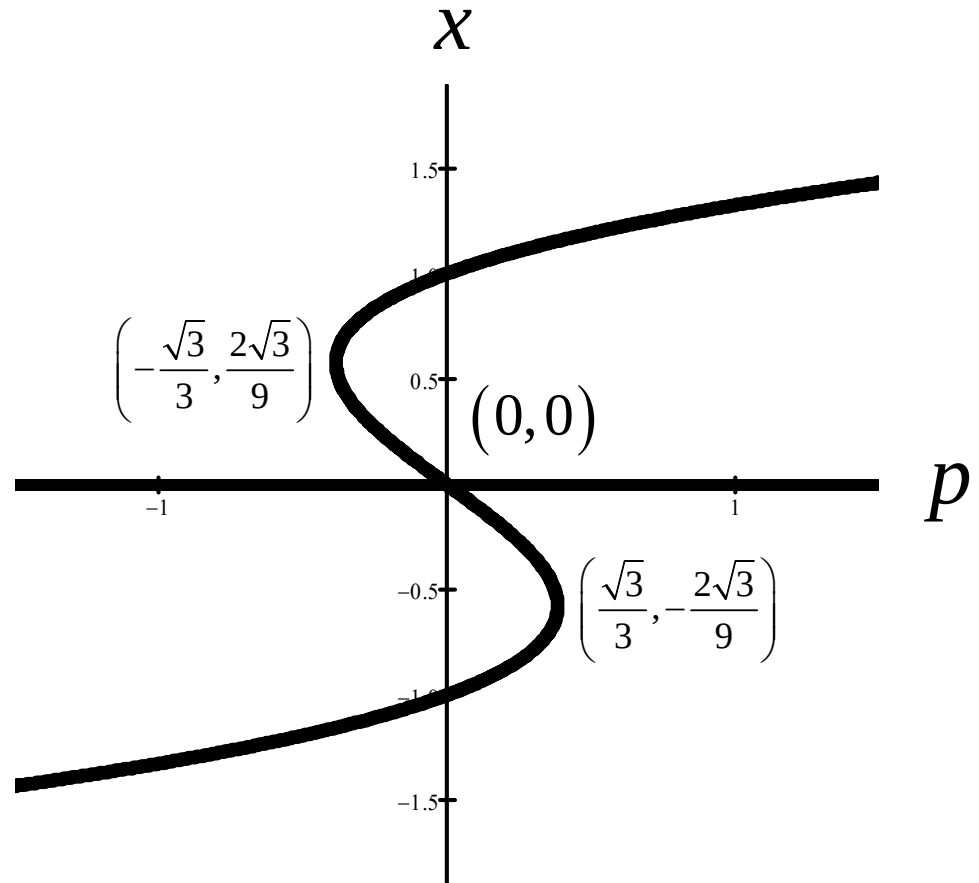
occurring at

$$p_0 = \frac{\sqrt{3}}{3} \quad \text{and} \quad p_0 = -\frac{\sqrt{3}}{3}$$

and

a transcritical bifurcation

occurring at $p_0 = 0$



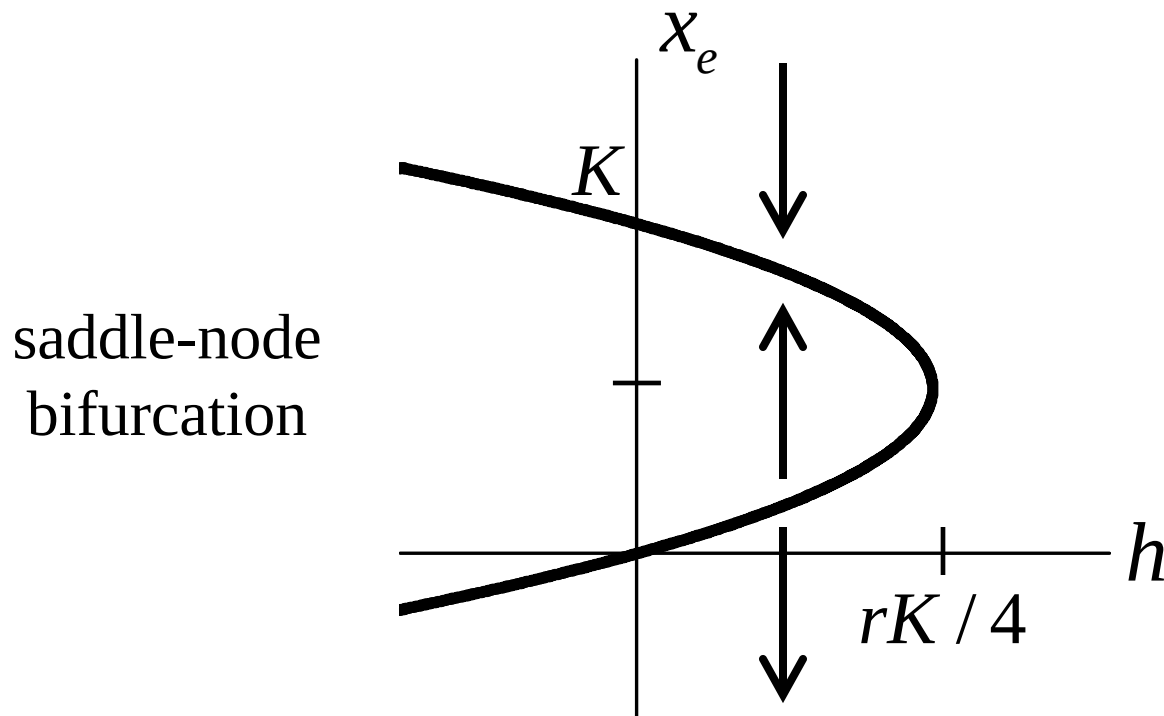
So far our bifurcation diagrams depict only the *existence & number* of equilibria.

There are several ways to also depict the *stability properties* of equilibria.

Harvested Logistic Growth

$$x' = rx \left(1 - \frac{x}{K} \right) - h$$

$$r > 0, \quad K > 0$$



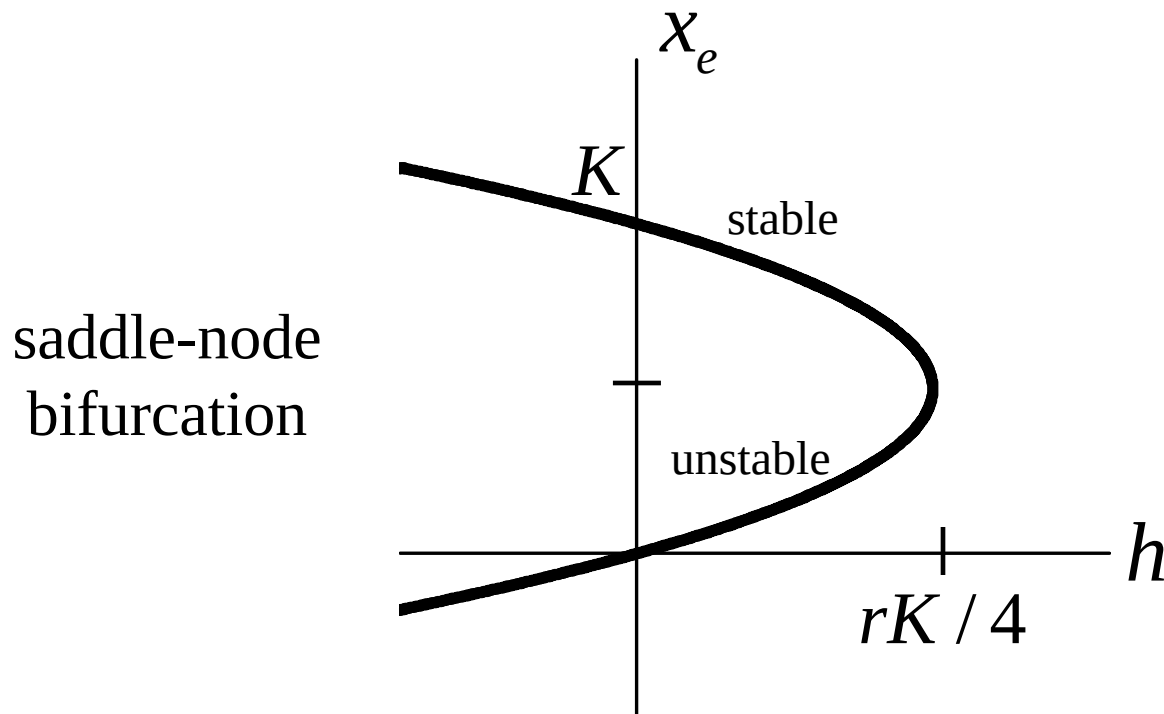
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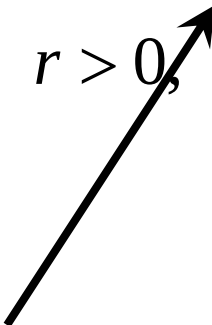
AN APPLICATION

The Spruce Budworm Model

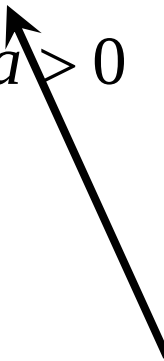
$$x' = rx \left(1 - \frac{x}{K} \right) - c \frac{x^2}{a + x^2}$$

$$r > 0, K > 0, c > 0, a > 0$$

A logistically
growing population
of budworms



A removal rate
caused by
predators & parasites



AN APPLICATION

The Spruce Budworm Model

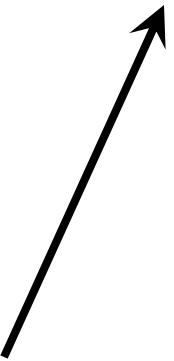
$$x' = rx \left(1 - \frac{x}{K} \right) - c \frac{x^2}{a + x^2}$$

$$r > 0, \quad K > 0, \quad c > 0, \quad a > 0$$

An Example

$$r > 0, \quad K = 1, \quad c = 1, \quad a = 0.01$$

$$x' = rx(1 - x) - \frac{x^2}{0.01 + x^2}$$



A parameter determined by
environmental conditions,
e.g. the “health” of the forest

AN APPLICATION

The Spruce Budworm Model

$$x' = rx(1-x) - \frac{x^2}{0.01 + x^2}$$

Equilibrium Equation:

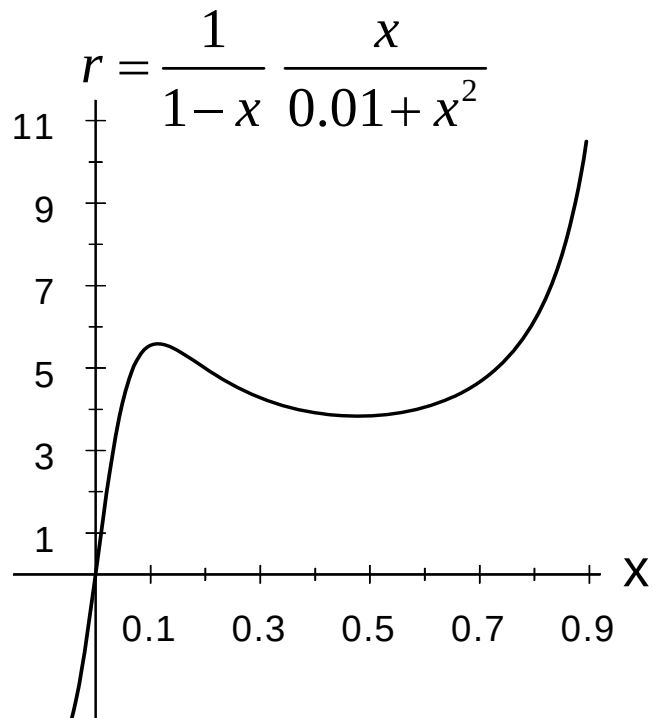
$$rx(1-x) - \frac{x^2}{0.01 + x^2} = 0$$

$$x = 0 \quad \text{and} \quad r = \frac{1}{1-x} \frac{x}{0.01 + x^2}$$

AN APPLICATION

The Spruce Budworm Model

$$x' = rx(1-x) - \frac{x^2}{0.01+x^2}$$

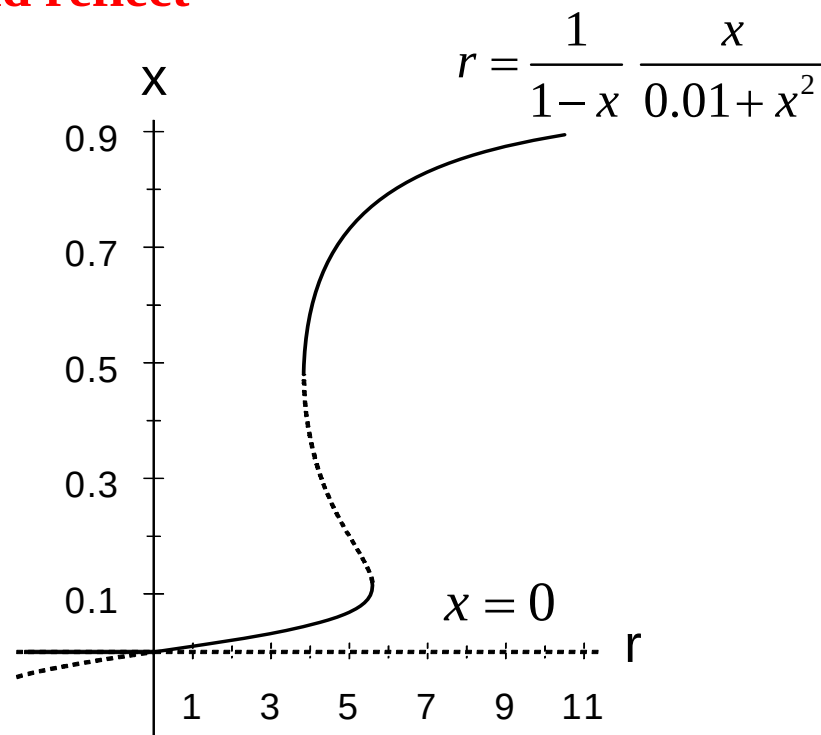
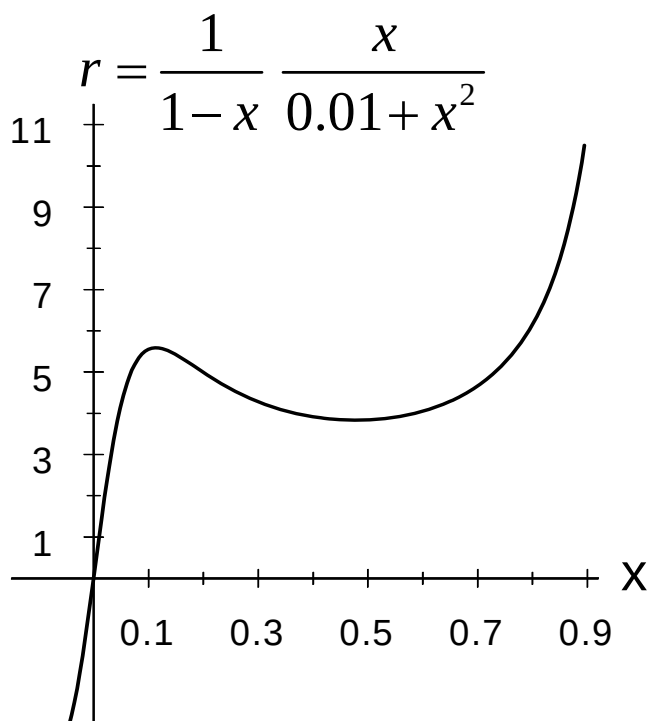


AN APPLICATION

The Spruce Budworm Model

$$x' = rx(1-x) - \frac{x^2}{0.01+x^2}$$

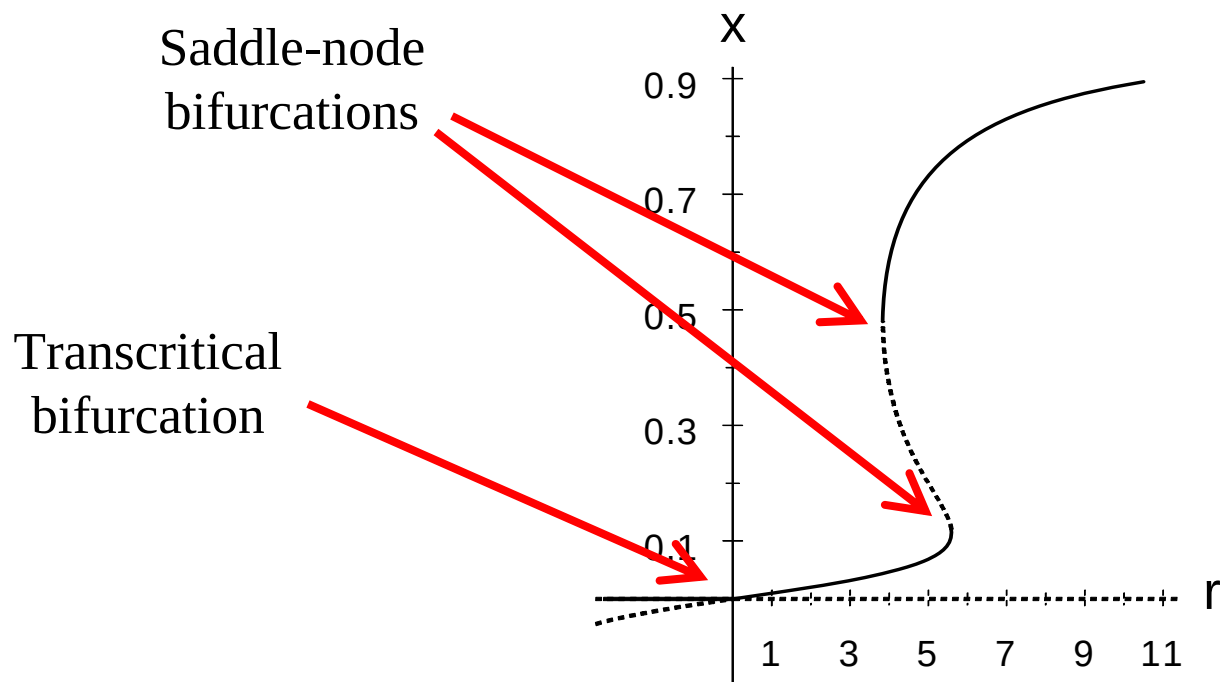
rotate and reflect



AN APPLICATION

The Spruce Budworm Model

$$x' = rx(1-x) - \frac{x^2}{0.01+x^2}$$



AN APPLICATION

The Spruce Budworm Model

$$x' = rx(1-x) - \frac{x^2}{0.01+x^2}$$

