

MATH 250b – Section 2

Spring Semester 2008

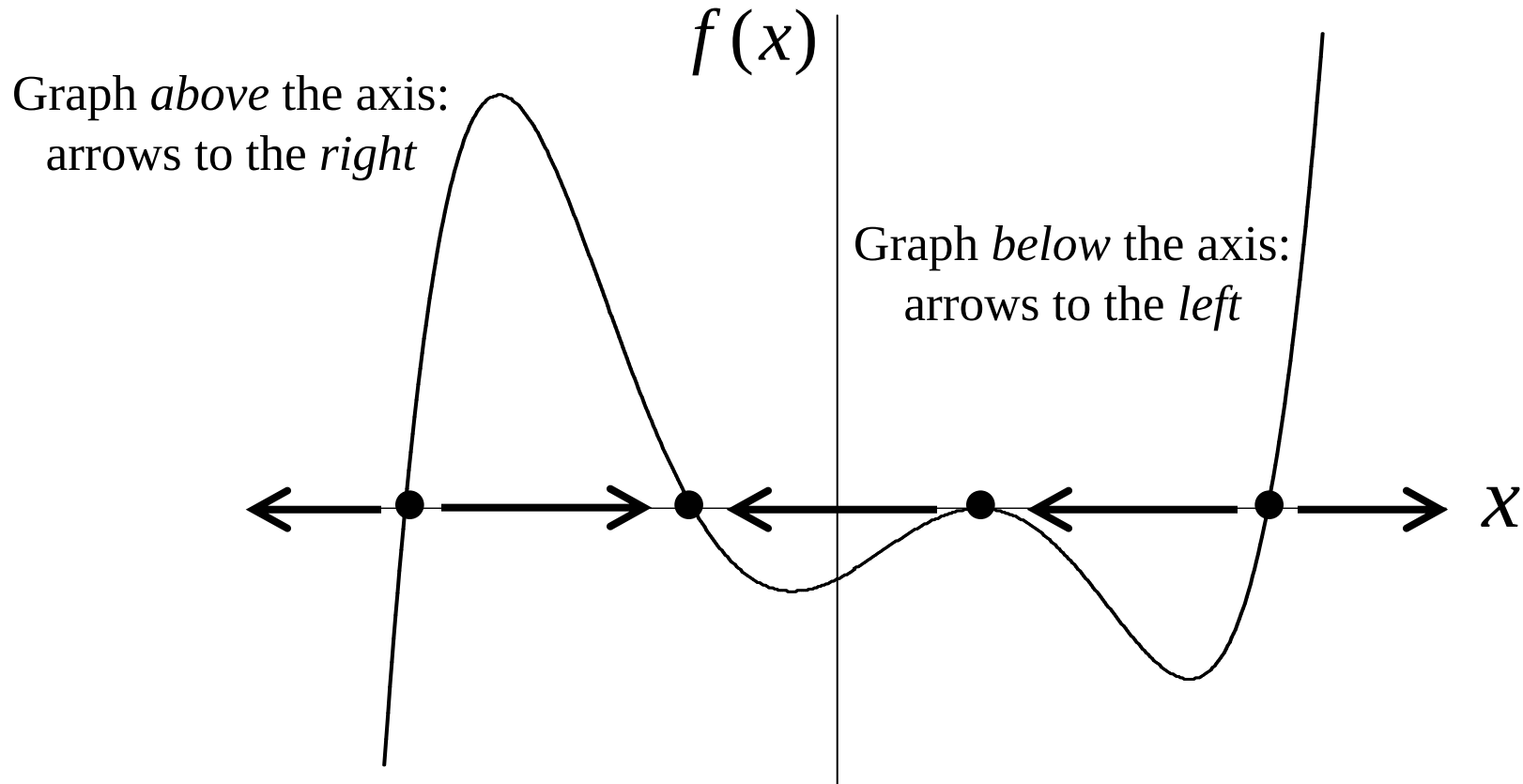
Thursday, 21-Feb-08

<http://math.arizona.edu/~cushing/250b.html>

# Construction of Phase Line Portraits for $x' = f(x)$

## 1. Draw a graph of $f(x)$

- ✓ find the roots of  $f(x)$  ( the equilibria )
- ✓ determine the sign of  $f(x)$  between consecutive roots



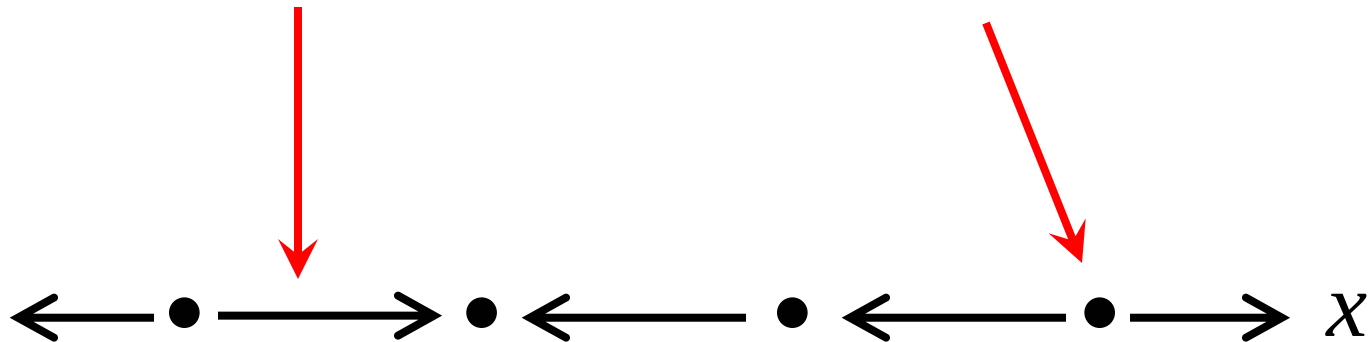
# Construction of Phase Line Portraits for $x' = f(x)$

## 1. Draw a graph of $f(x)$

- ✓ find the roots of  $f(x)$  ( the equilibria )
- ✓ determine the sign of  $f(x)$  between consecutive roots

Oriented line segments  
are called  
*orbits*

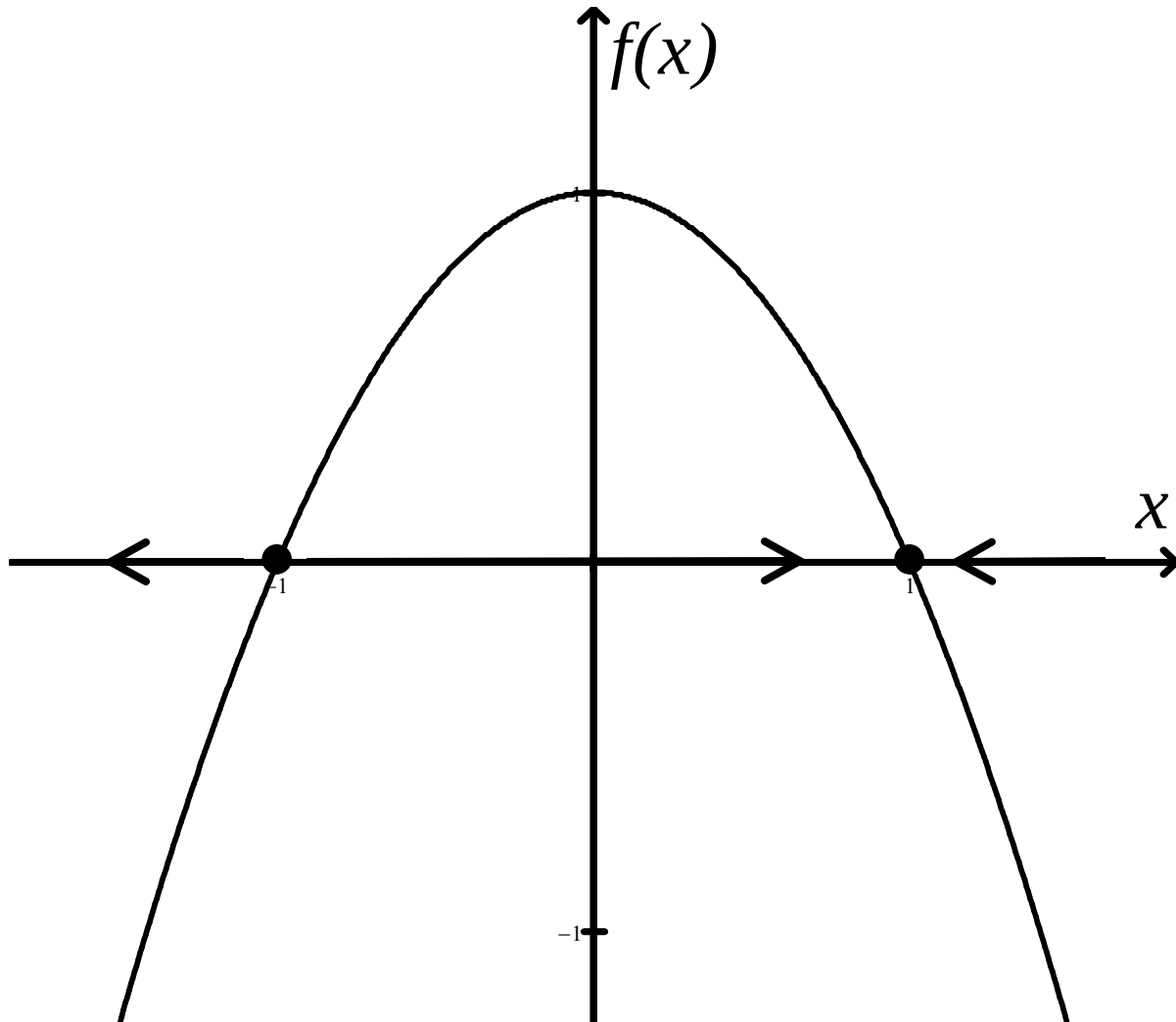
Equilibrium points  
are called  
*point orbits*



Phase line portrait

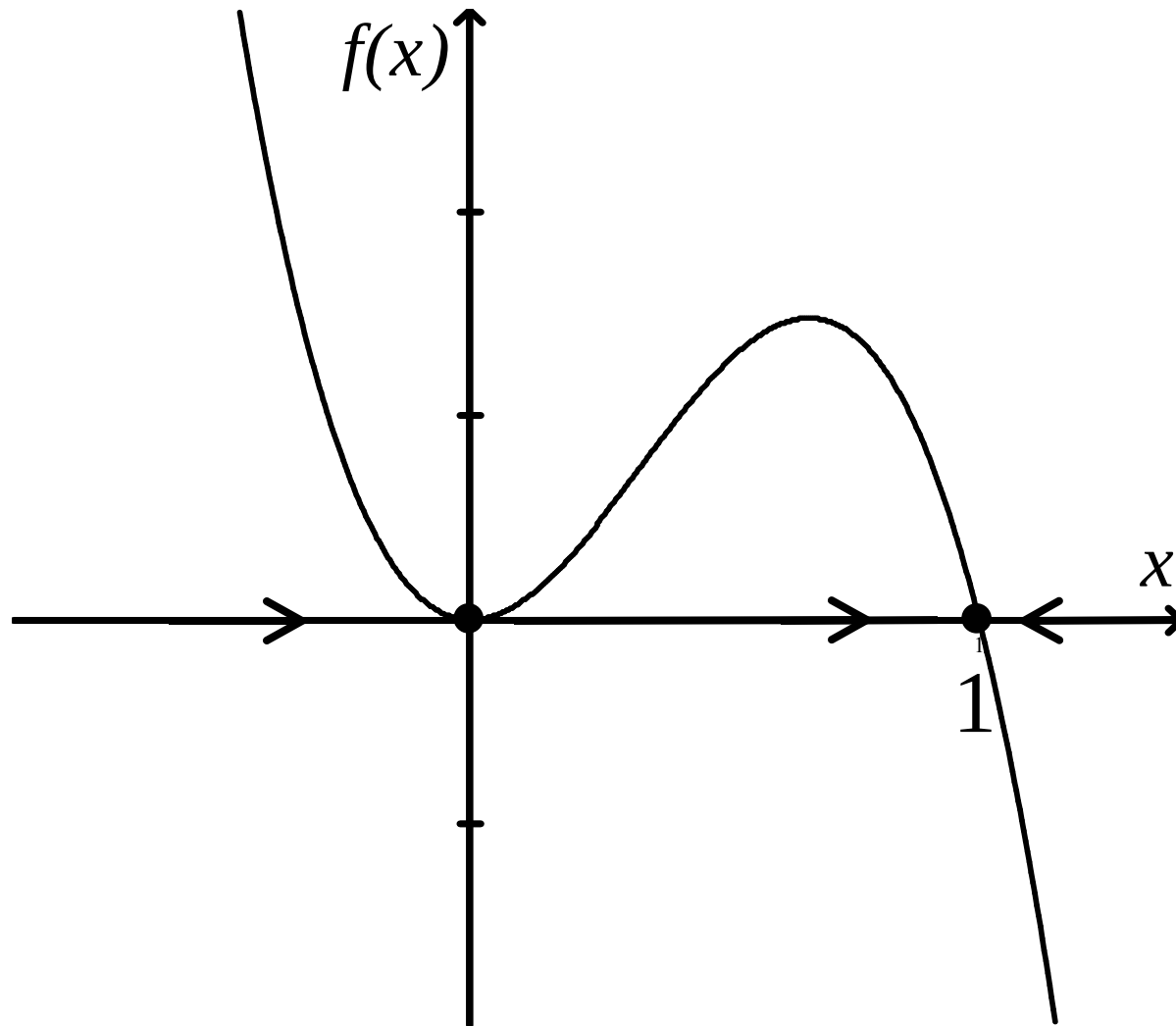
# Example

$$x' = 1 - x^2 \Rightarrow f(x) = 1 - x^2$$



# Example

$$x' = x^2 (1 - x) \Rightarrow f(x) = x^2 (1 - x)$$



## Recall Population Dynamic Application

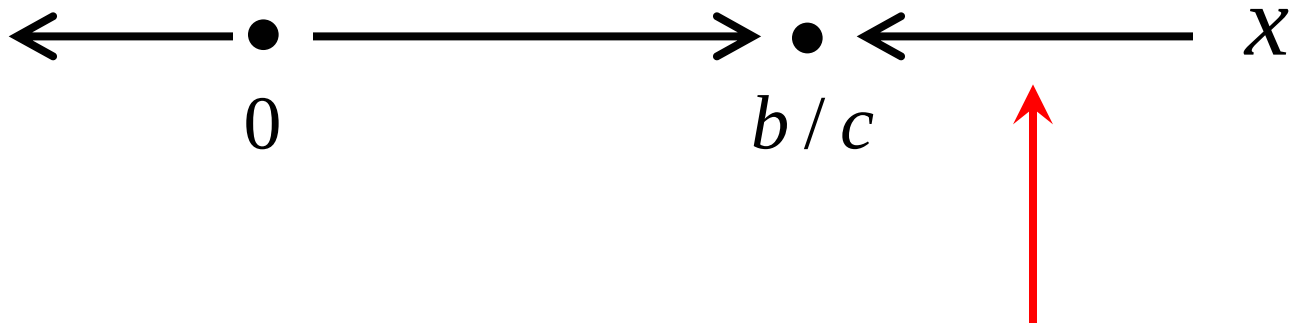
Constant per capita birth rate  $b$

Per capita death rate proportional to population size  $x$

Does the second assumption prevent unbounded population growth ?

$$x' = bx - cx^2$$

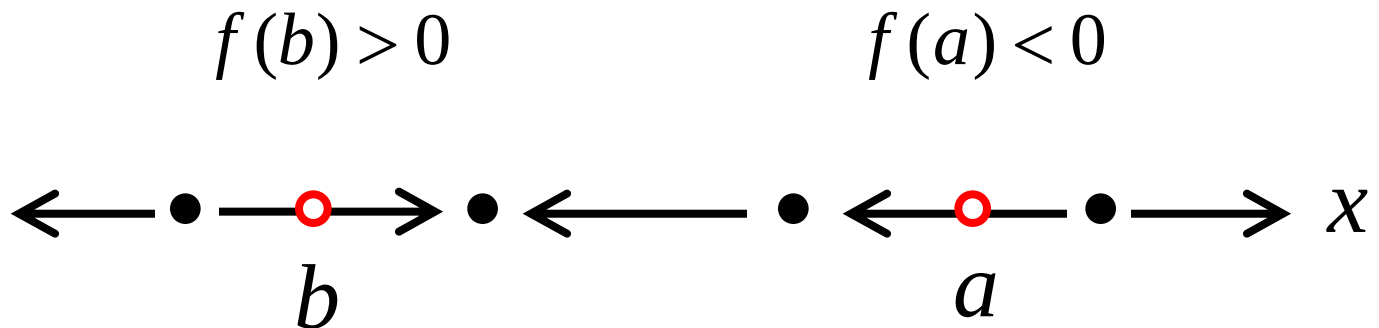
$$b, c > 0$$



Answer: Yes

## Construction of Phase Line Portraits for $x' = f(x)$

1. Draw a graph of  $f(x)$
2. Test points
  - ✓ find the roots of  $f(x)$  (the equilibria)
  - ✓ evaluate  $f(x)$  at convenient test points between consecutive roots



### Example

$$x' = \frac{x^3 + 2x^2}{1 + x^4}$$

$$f(x) = \frac{x^3 + 2x^2}{1 + x^4} \quad \text{has roots } x = -2 \quad \text{and} \quad 0$$

$$f(-3) = \frac{-9}{82} < 0$$

$$f(-1) = \frac{1}{2} > 0$$

$$f(1) = \frac{3}{2} > 0$$



# Classification of Equilibria

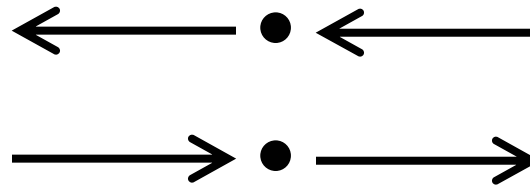
Sink or attractor  
(stable equilibrium)



Source or repellor  
(unstable equilibrium)

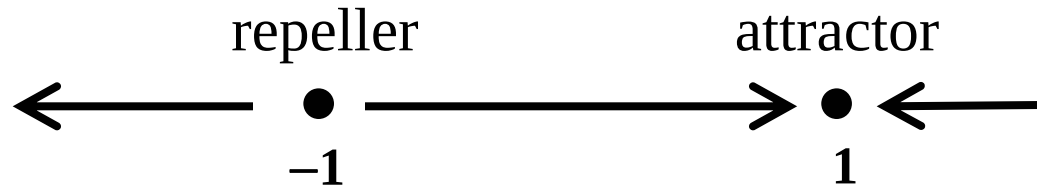


Shunt  
(semi-stable equilibrium)



## Summary of Examples

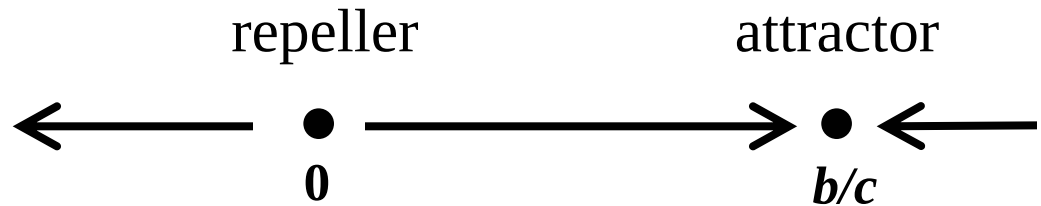
$$x' = 1 - x^2$$



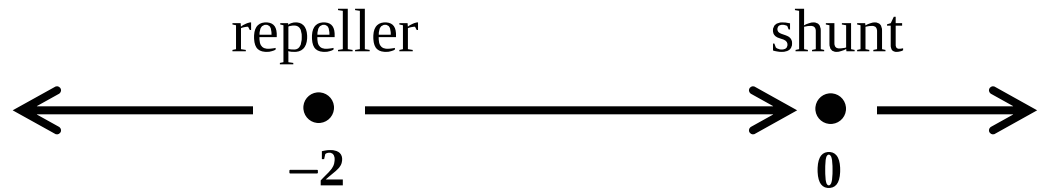
$$x' = x^2(1 - x)$$



$$x' = bx - cx^2$$
$$b, c > 0$$



$$x' = \frac{x^3 + 2x^2}{1 + x^4}$$



# Classification of Equilibria and Geometry of $f(x)$

( Theorem 3 in text )

*Attractor* (stable equilibrium)       $\longrightarrow \bullet \longleftarrow$

This occurs when the graph of  $f(x)$  to *decreases* through the root

*Repeller* (unstable equilibrium)       $\longleftarrow \bullet \longrightarrow$

This occurs when the graph of  $f(x)$  to *increases* through the root

*Shunt* (semi-stable equilibrium)       $\begin{array}{c} \longleftarrow \bullet \longleftarrow \\ \longrightarrow \bullet \longrightarrow \end{array}$

This occurs when graph of  $f(x)$  has a *max or min* at the root