

MATH 250b – Section 2

Spring Semester 2008

Tuesday, 19-Feb-08

<http://math.arizona.edu/~cushing/250b.html>

Summary from last class

$$x' = f(x) \quad (\text{autonomous equation})$$

$$x(t_0) = x_0$$

- Graphs of different solutions cannot cross
- Roots of $f(x)$ are equilibrium solutions
- Non-equilibrium solutions are monotonic

$$f(x_0) = 0 \Rightarrow x(t) = x_0 \text{ for all } t$$

$$f(x_0) > 0 \Rightarrow x(t) \text{ increases for all } t$$

$$f(x_0) < 0 \Rightarrow x(t) \text{ decreases for all } t$$

EXAMPLES

$$x' = 1 - x^2, \quad x(0) = 0 \quad \text{and} \quad x(0) = 2$$

$$f(0) = 1 > 0$$

solution
increases

$$f(2) = -3 < 0$$

solution
decreases

$$x' = \frac{x^2 - 4x}{1 + x^2}, \quad x(0) = 1 \quad \text{and} \quad x(0) = -1$$

$$f(1) = -3/2 < 0$$

solution
decreases

$$f(-1) = 5/2 > 0$$

solution
increases

$$x' = \sin x, \quad x(0) = \pi / 2$$

$$f(\pi/2) = 1 > 0$$

solution
increases

$$x' = f(x), \quad x(t_0) = x_0$$

$f(x_0) > 0 \Rightarrow x(t)$ increases for all t

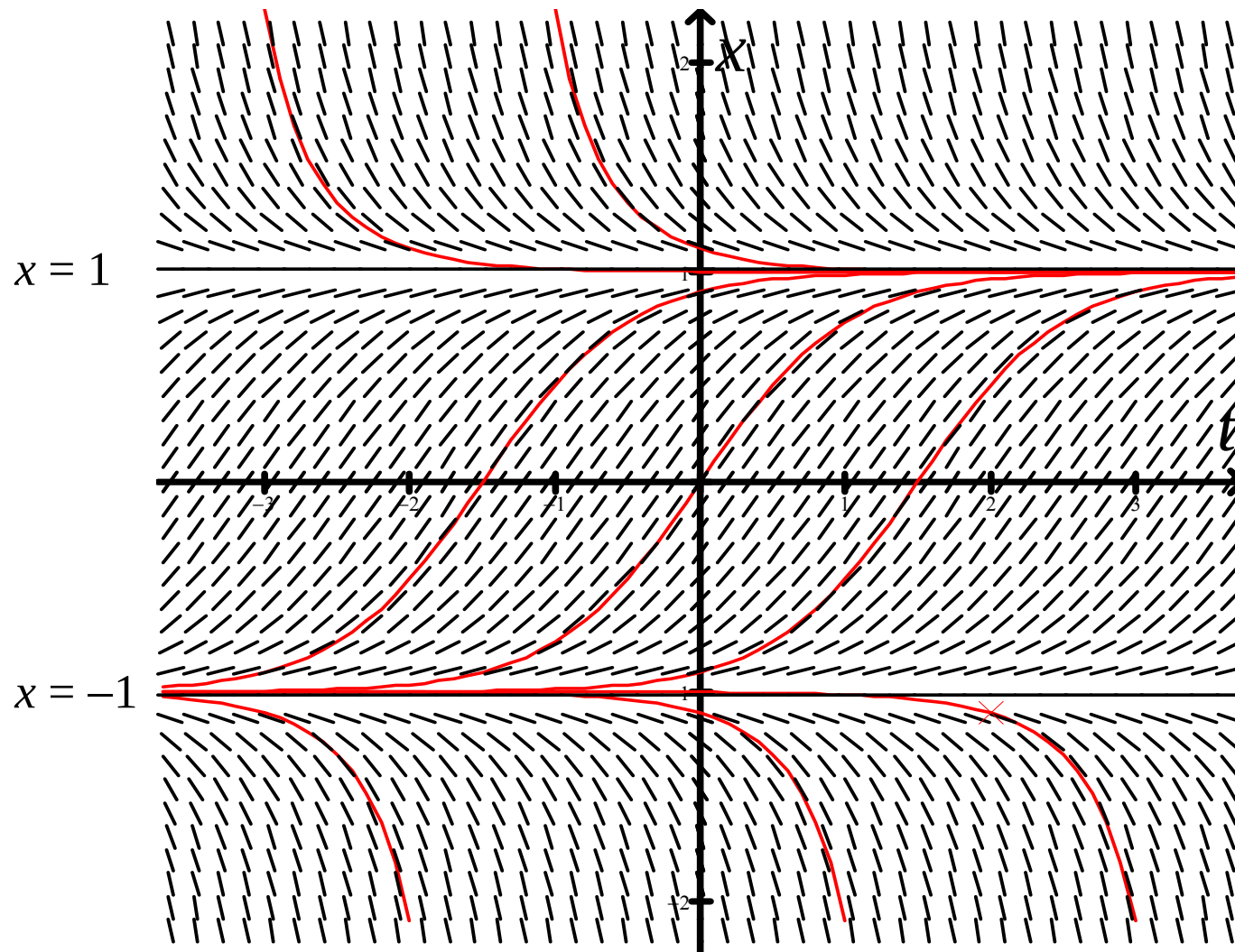
$f(x_0) < 0 \Rightarrow x(t)$ decreases for all t

What are the options for non-equilibrium solutions?

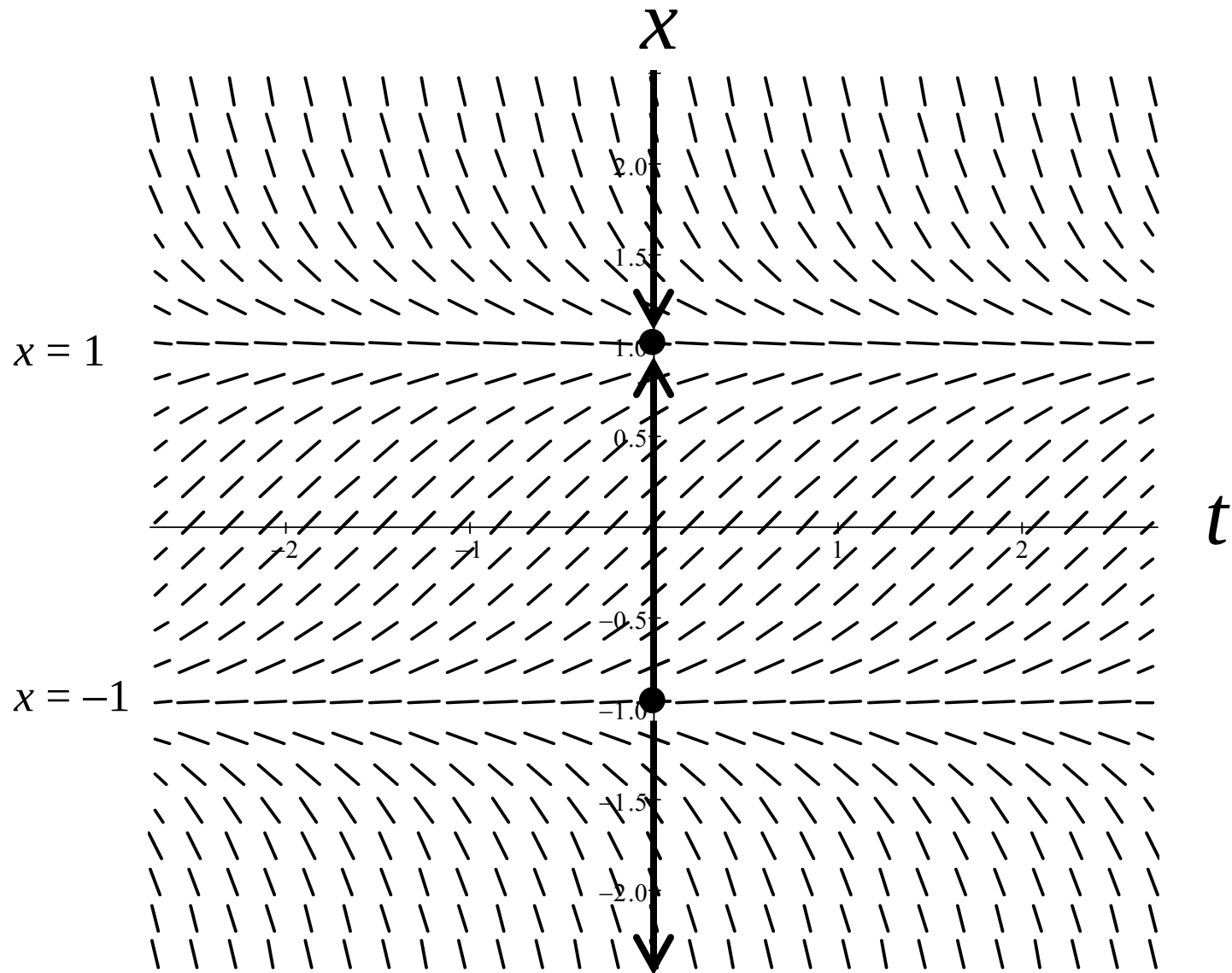
- (1) unbounded
- (2) approaches an equilibrium as $t \rightarrow +\infty$

(THEOREM 2 in text)

$$x' = 1 - x^2$$



$$x' = 1 - x^2$$



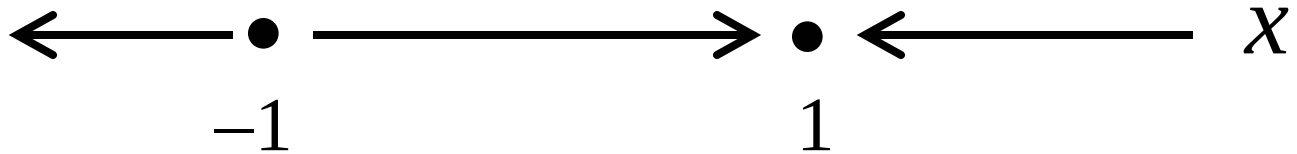
$$x' = 1 - x^2$$

x

*asymptotic dynamics
summarized by a
phase line portrait*



$$x' = 1 - x^2$$



asymptotic dynamics
are summarized by this
phase line portrait