

MATH 250b – Section 2

Spring Semester 2008

Thursday, 14-Feb-08

<http://math.arizona.edu/~cushing/250b.html>

Autonomous Linear Equations

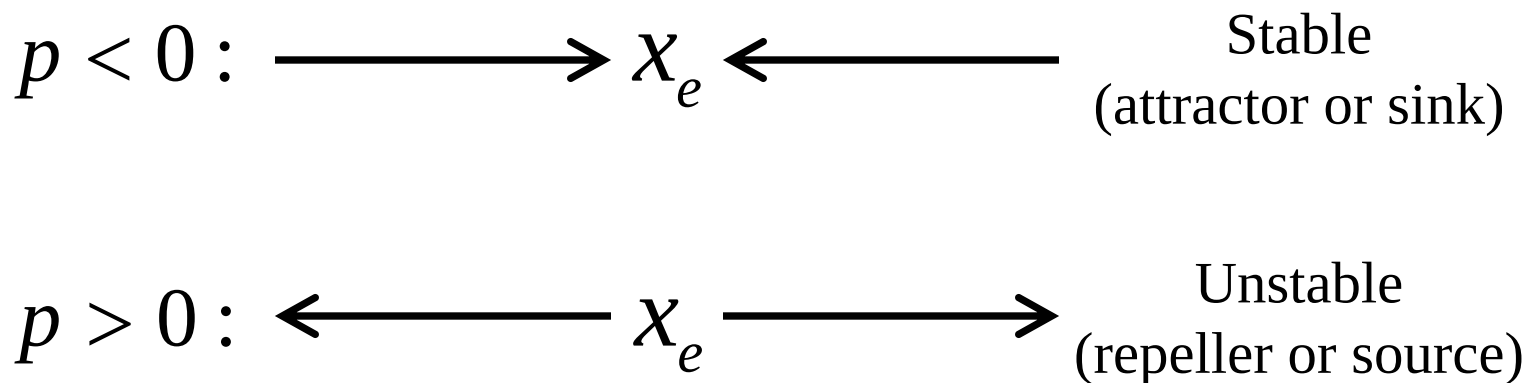
$$x' = px + q$$

p, q are constants

$$x = ce^{pt} + x_e$$

$$x_e = -q / p = \text{equilibrium}$$

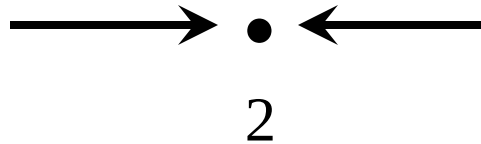
Phase Line Portraits



Using phase line portraits

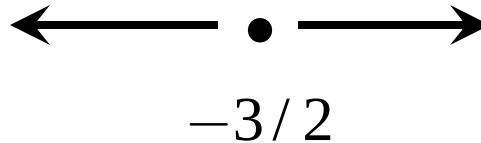
describe the properties of the solutions to the initial value problems:

$$\begin{aligned}x' &= -2x + 4 \\ x(0) &= 4\end{aligned}$$



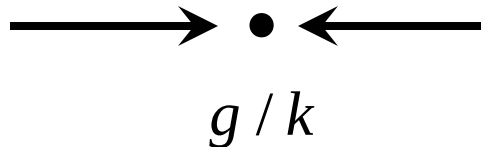
$x(t)$ is decreasing and
 $\lim_{x \rightarrow +\infty} x(t) = 2$

$$\begin{aligned}x' &= 2x + 3 \\ x(1) &= -2\end{aligned}$$



$x(t)$ is decreasing and
 $\lim_{x \rightarrow +\infty} x(t) = -\infty$

$$\begin{aligned}v' &= g - kv \\ v(0) &= 0\end{aligned}$$



$x(t)$ is increasing and
 $\lim_{x \rightarrow +\infty} x(t) = g/k$

An Application

As in Homework #3, Exercise 17, page 69

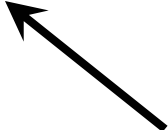
$x =$ population size

$$x' = bx - dx$$

$b =$ per unit
birth rate



$d =$ per unit
death rate



An Application

As in Homework #3, Exercise 17, page 69

$x =$ population size

$$x' = bx - dx$$

$$x' = px, \quad \text{where } p = b - d$$

$b < d \Rightarrow$ extinction

$b > d \Rightarrow$ *unbounded population explosion!*

What modification(s) of this will prevent unbounded growth ?

An Application

As in Homework #3, Exercise 17, page 69

$x =$ population size

$$x' = bx - dx$$

$b =$ *constant* per unit birth rate (as before)

... but suppose d is *proportional to* x .

Will this assumption prevent a population explosion ?

$$d = cx \quad \Rightarrow \quad x' = bx - cx^2$$

Nonlinear !

Chapter 3.1 Autonomous Nonlinear Equations

$$x' = f(x)$$



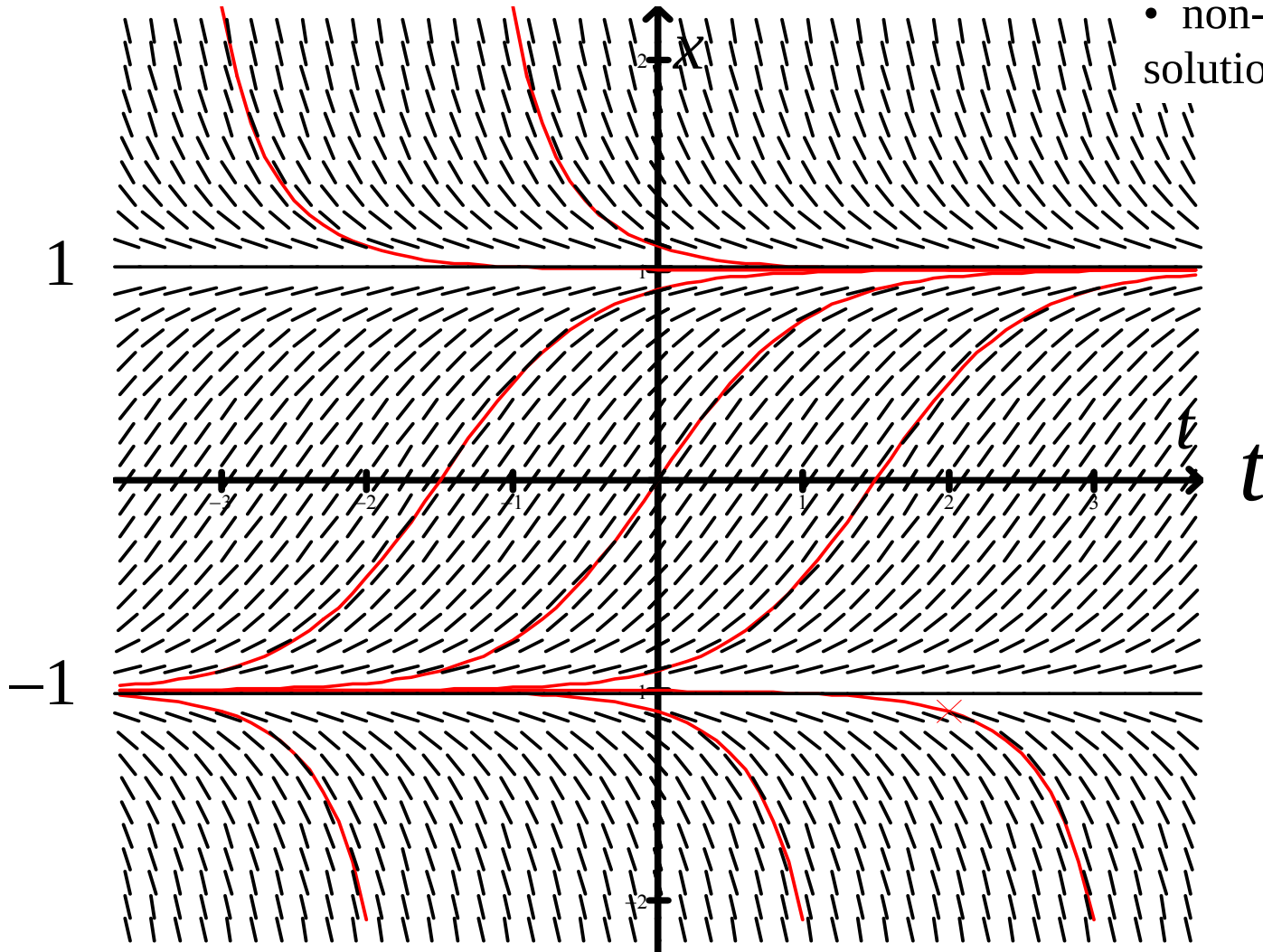
No explicit dependence on the independent variable t !

EXAMPLES

$$x' = 1 - x^2$$

Note

- equilibrium solutions
- non-equilibrium solutions are monotonic



NOTES

- (1) Solution graphs cannot intersect
- (2) Roots of $f(x)$ are equilibrium solutions
- (3) $x' = f(x), x(t_0) = x_0$

$$f(x_0) = 0 \Rightarrow x(t) = x_0 \text{ for all } t$$

$$f(x_0) > 0 \Rightarrow x(t) \text{ increases for all } t$$

$$f(x_0) < 0 \Rightarrow x(t) \text{ decreases for all } t$$

(THEOREM 1 in text)