

MATH 250b – Section 2

Spring Semester 2008

Thursday, 07-Feb-08

<http://math.arizona.edu/~cushing/250b.html>

Chapter 2: Linear First Order Equations

- What are “linear” equations ?
- Methods for finding solution formulas
- Learn about some basic properties of solutions
- Finish application to falling objects under the Linear Friction Law

2.1 & 2.2 Solution of Linear Equations

Review of Last Class

$$x' = f(t, x)$$

A differential equation is *linear* if $f(t, x)$ is linear in x :

$$x' = p(t)x + q(t)$$

A linear equation is *homogeneous* if $q(t) = 0$:

$$x' = p(t)x$$

The *general solution* of a homogeneous equation is

$$x(t) = ce^{P(t)}, \quad P(t) = \int p(t) dt$$

$c =$ arbitrary constant

Examples

Linear homogeneous

General solution

$$x' = x \quad \Rightarrow \quad x(t) = ce^t$$

$$x' = x \sin t \quad \Rightarrow \quad x(t) = ce^{-\cos t}$$

$$x' = px \quad \Rightarrow \quad x(t) = ce^{pt}$$

$$x' = t^{-1}x \quad \Rightarrow \quad x(t) = ct \quad \text{for } t > 0$$

2.1 & 2.2 Solution of Linear Equations

$$x' = p(t)x + q(t)$$

When $q(t)$ is present, the equation is *non - homogeneous* :

The *general solution* is

$$x(t) = ce^{P(t)} + e^{P(t)} \int e^{-P(t)} q(t) dt$$

$c =$ arbitrary constant

Structure of General Solution

$$x(t) = x_h(t) + x_p(t)$$

$x_h(t) = ce^{P(t)}$ = general solution of homogeneous equation

$x_p(t) = e^{P(t)} \int e^{-P(t)} q(t) dt =$ a particular solution

Examples

$$x' = x + 1 \quad \Rightarrow \quad x = ce^t - 1$$

$$x' = -2x + 3e^t \quad \Rightarrow \quad x = ce^{-2t} + e^t$$

$$x' = 2tx + \exp(t^2) \quad \Rightarrow \quad x = c \exp(t^2) + t \exp(t^2)$$

$$x' = 2tx + 1 \quad \Rightarrow \quad x = c \exp(t^2) + \exp(t^2) \int \exp(-t^2) dt$$

Initial Value Problems for Linear Equations

$$x' = p(t)x + q(t)$$

$$x(t_0) = x_0$$

Fundamental Theorem applies if both $p(t)$ & $q(t)$ are continuous at $t = t_0$.

- Find general solution. Then choose c to satisfy initial condition.

Example

$$x' = x + 1 \Rightarrow x = ce^t - 1$$

$$x(0) = 2 \Rightarrow x(0) = c - 1 = 2 \Rightarrow c = 3 \Rightarrow x(t) = 3e^t - 1$$

Initial Value Problems for Linear Equations

$$x' = p(t)x + q(t)$$

$$x(t_0) = x_0$$

Fundamental Theorem applies if both $p(t)$ & $q(t)$ are continuous at $t = t_0$.

- Find general solution. Then choose c to satisfy initial condition.

or

- use the formula :

$$x(t) = x_0 e^{P(t)} + e^{P(t)} \int_{t_0}^t e^{-P(u)} q(u) du$$

$$P(t) = \int_{t_0}^t p(s) ds$$

Example

$$x' = x + 1$$

$$x(0) = 2$$

$$P(t) = \int_0^t 1 du = t$$

$$x(t) = 2e^t + e^t \int_0^t e^{-u} \cdot 1 du = 3e^t - 1$$