

MATH 250b – Section 2

Spring Semester 2008

Thursday, 31-Jan-08

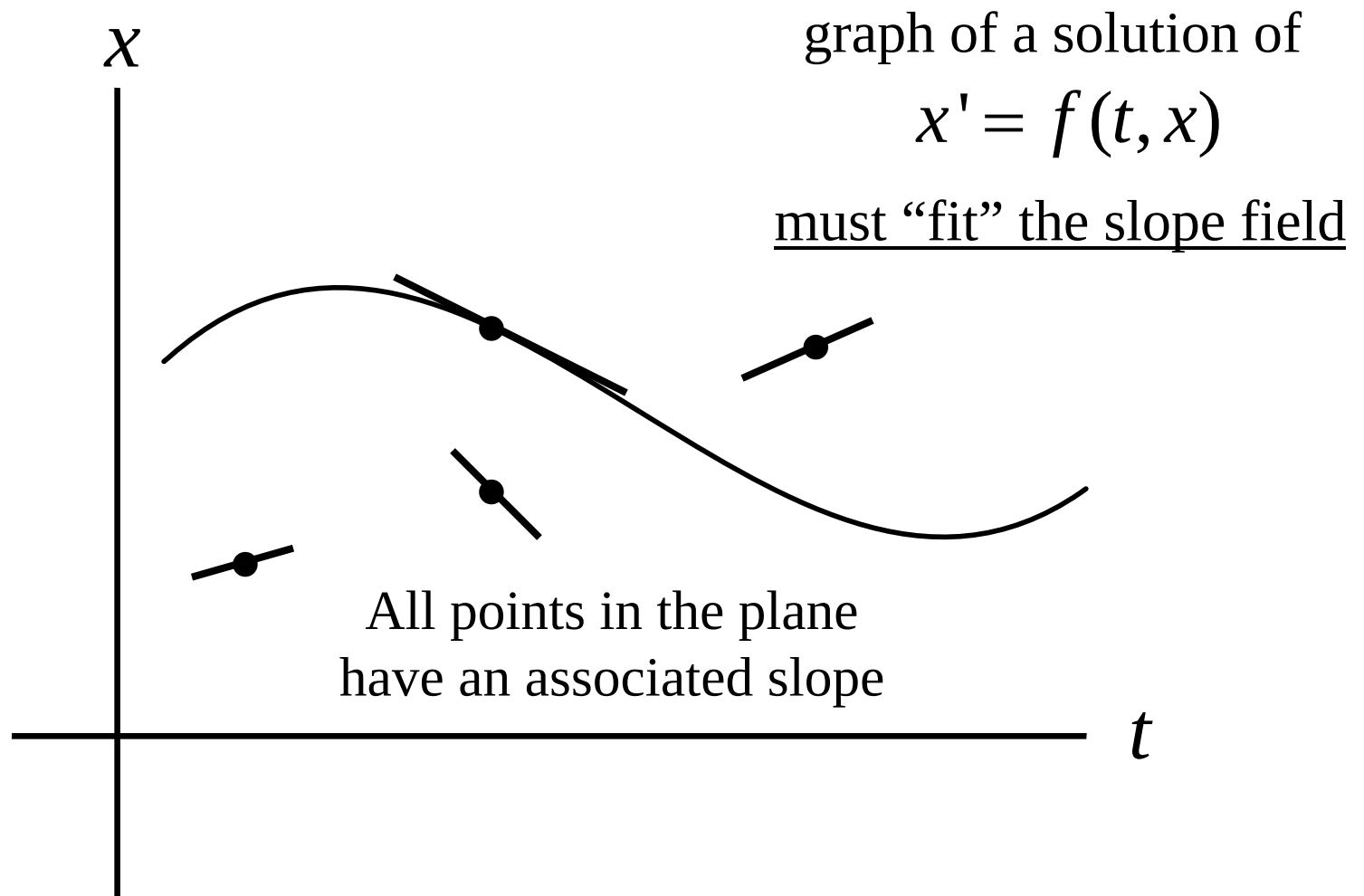
<http://math.arizona.edu/~cushing/250b.html>

The first theme of Chapter 1 concerns the existence of solutions to initial value problems.

The second theme of Chapter 1 deals with

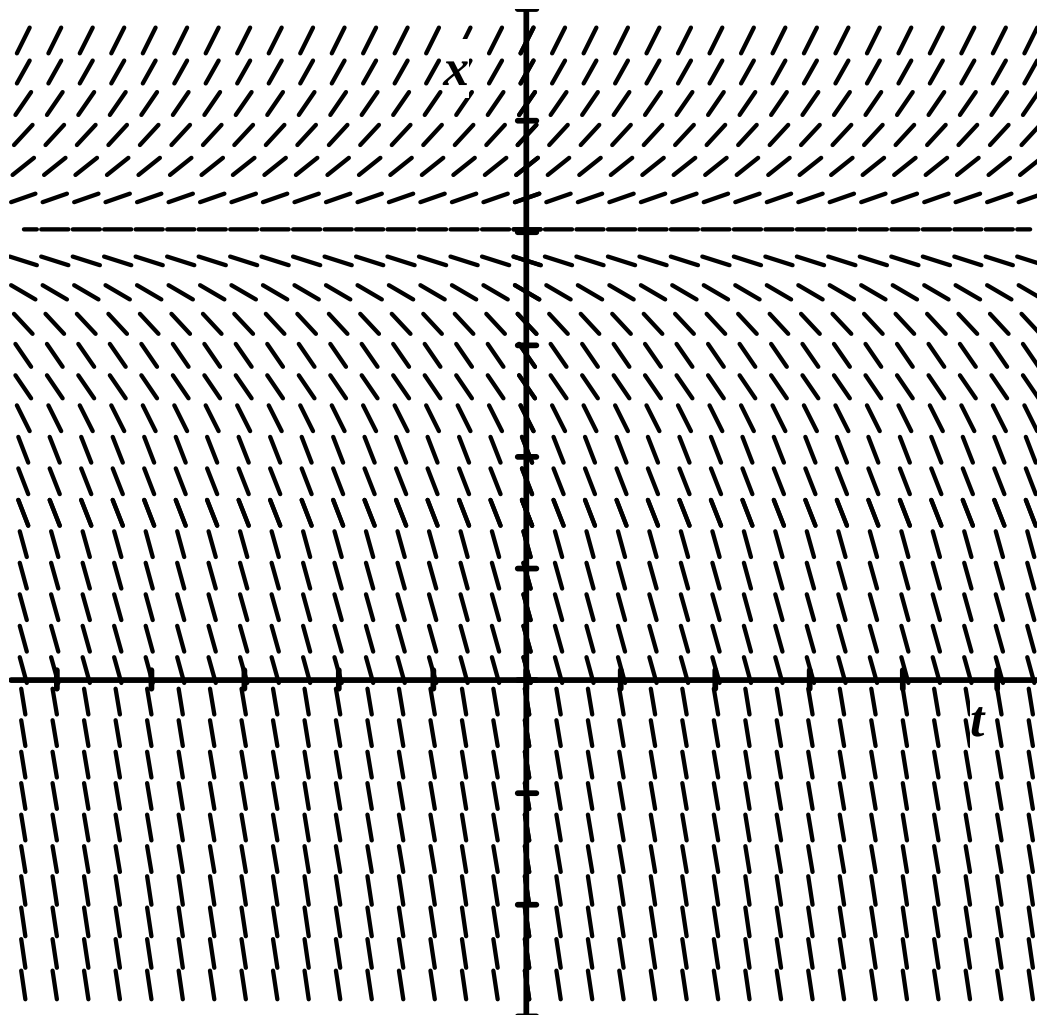
- some basic graphic & numerical approximations to solutions.
 - slope fields
 - numerical approximations
 - based on approximation methods for integrals
 - (from calculus)

1.2.1 Slope Fields



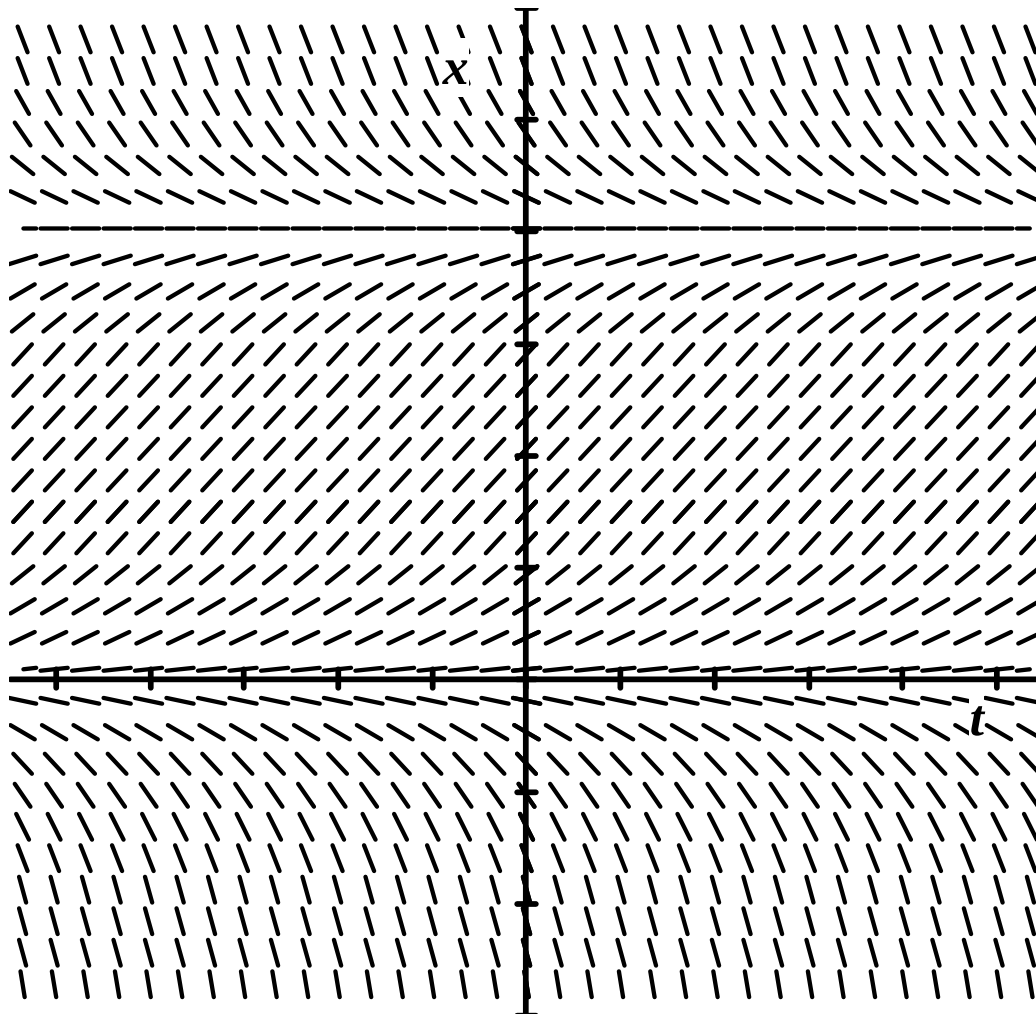
Grid Method: Examples

$$x' = x - 1$$



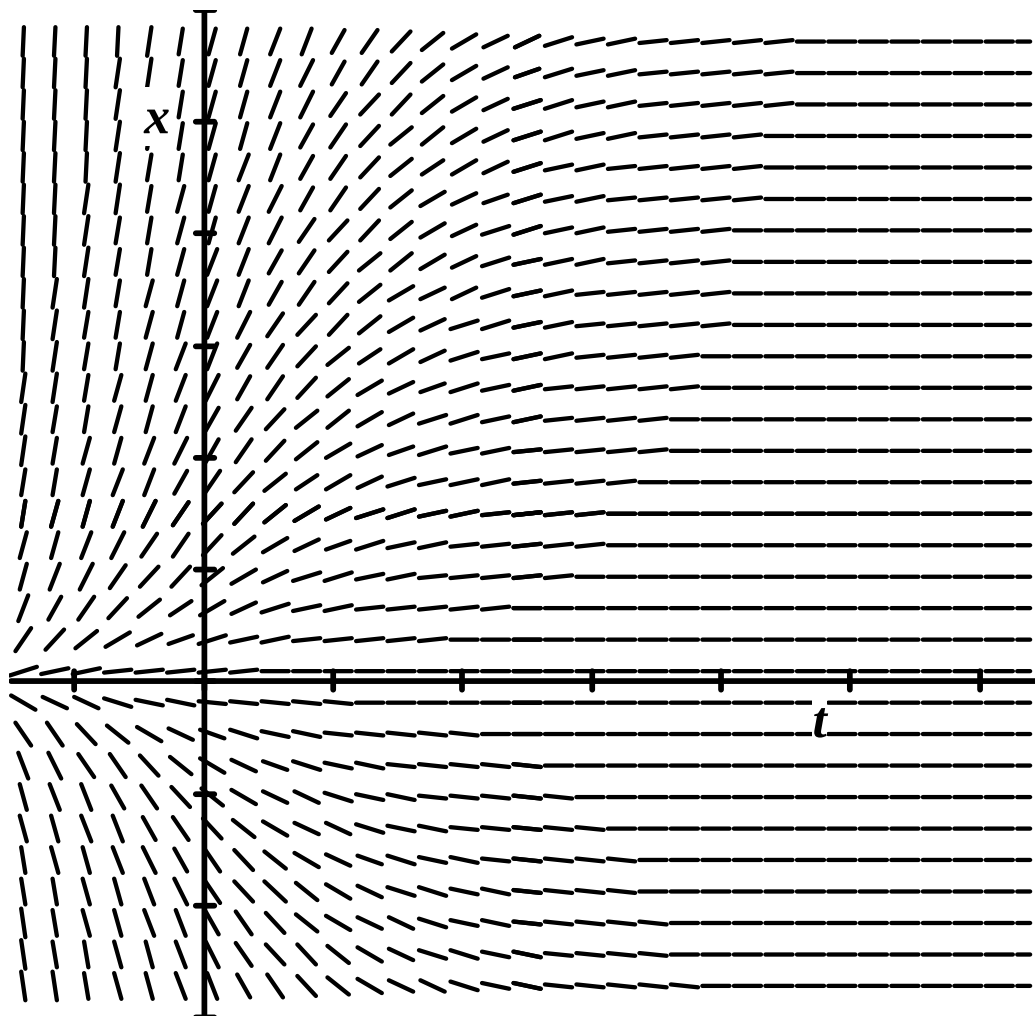
Grid Method: Examples

$$x' = (1-x)x$$



Grid Method: Examples

$$x' = e^{-t} x$$



Slope Fields

We want a systematic way to construct a sketch of the slope field

Two most common procedures are:

Grid method

Choose a grid of (t,x) points and plot line segments
with appropriate slopes at each (use a computer)

Isocline method

Make a selection of slopes and for each plot the points
 (t,x) associated with that slope

Isocline Method: Examples

$$x' = x^2 + t^2$$

Isocline equation: $x^2 + t^2 = m$

$m > 0 \Rightarrow$ isoclines are circles (radius $\sqrt{m}$)

$m < 0 \Rightarrow$ no isoclines

$m = 0 \Rightarrow$ isocline is a point (the origin)

Isocline Method: Examples

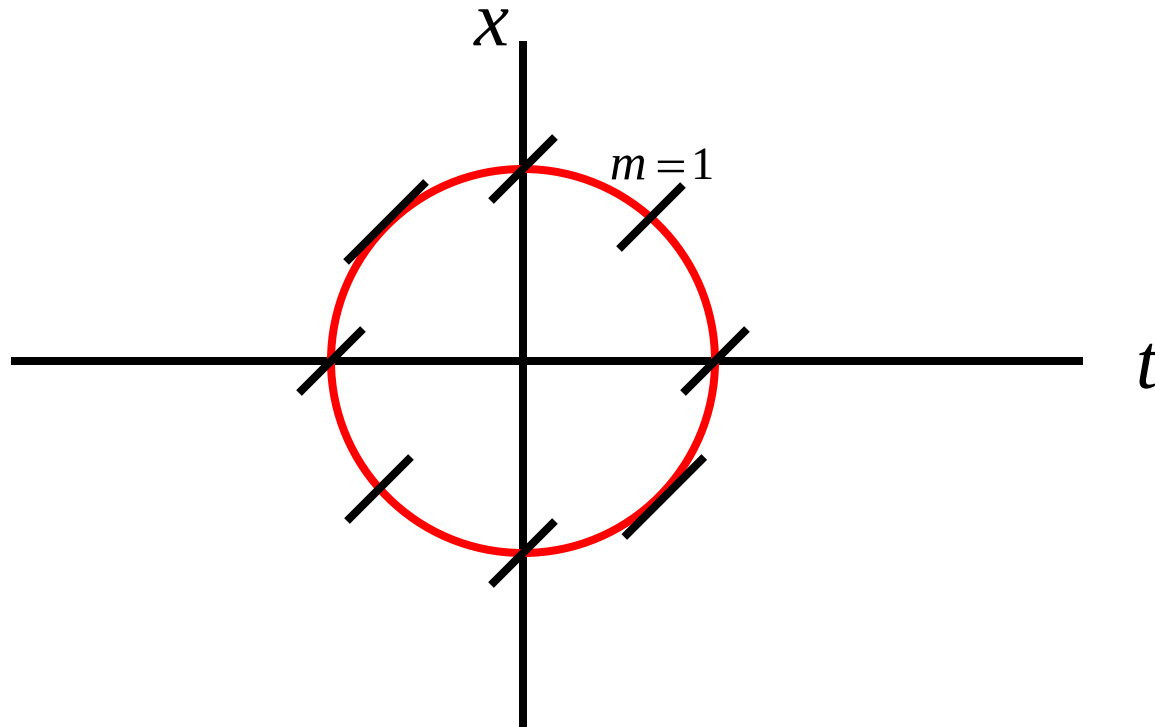
$$x' = x^2 + t^2$$

Isocline equation: $x^2 + t^2 = m$

$m > 0 \Rightarrow$ isoclines are circles (radius $\sqrt{m}$)

$m < 0 \Rightarrow$ no isoclines

$m = 0 \Rightarrow$ isocline is a point (the origin)



Isocline Method: Examples

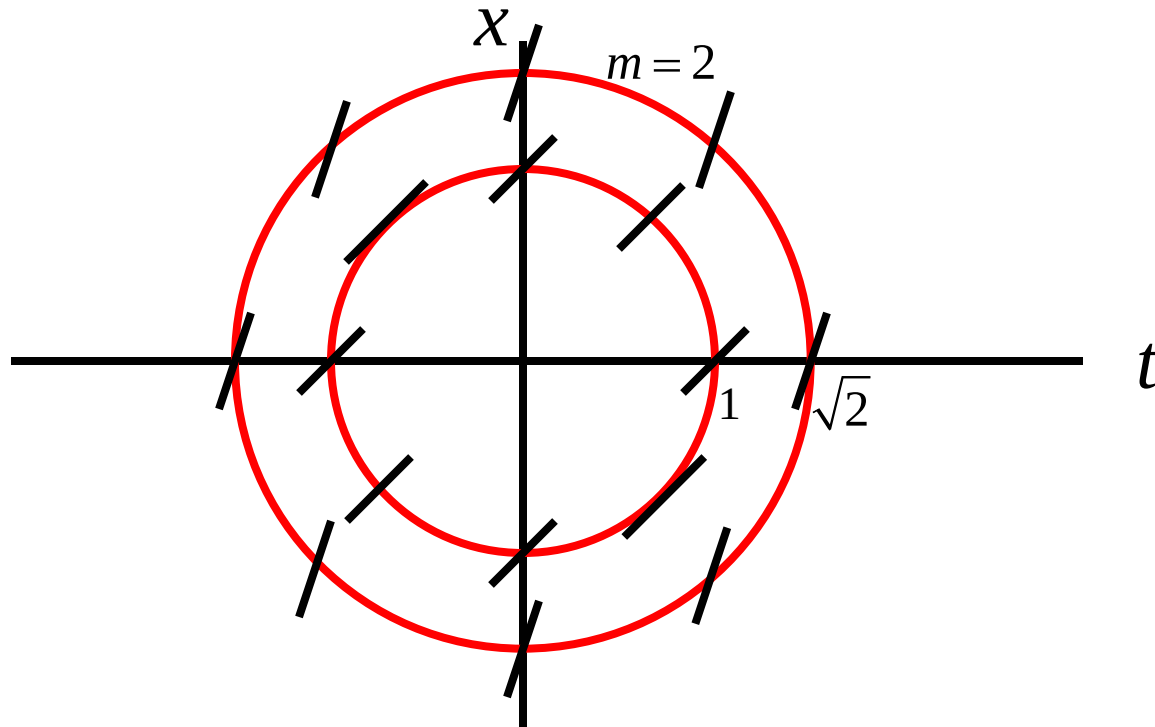
$$x' = x^2 + t^2$$

Isocline equation: $x^2 + t^2 = m$

$m > 0 \Rightarrow$ isoclines are circles (radius $\sqrt{m}$)

$m < 0 \Rightarrow$ no isoclines

$m = 0 \Rightarrow$ isocline is a point (the origin)



Isocline Method: Examples

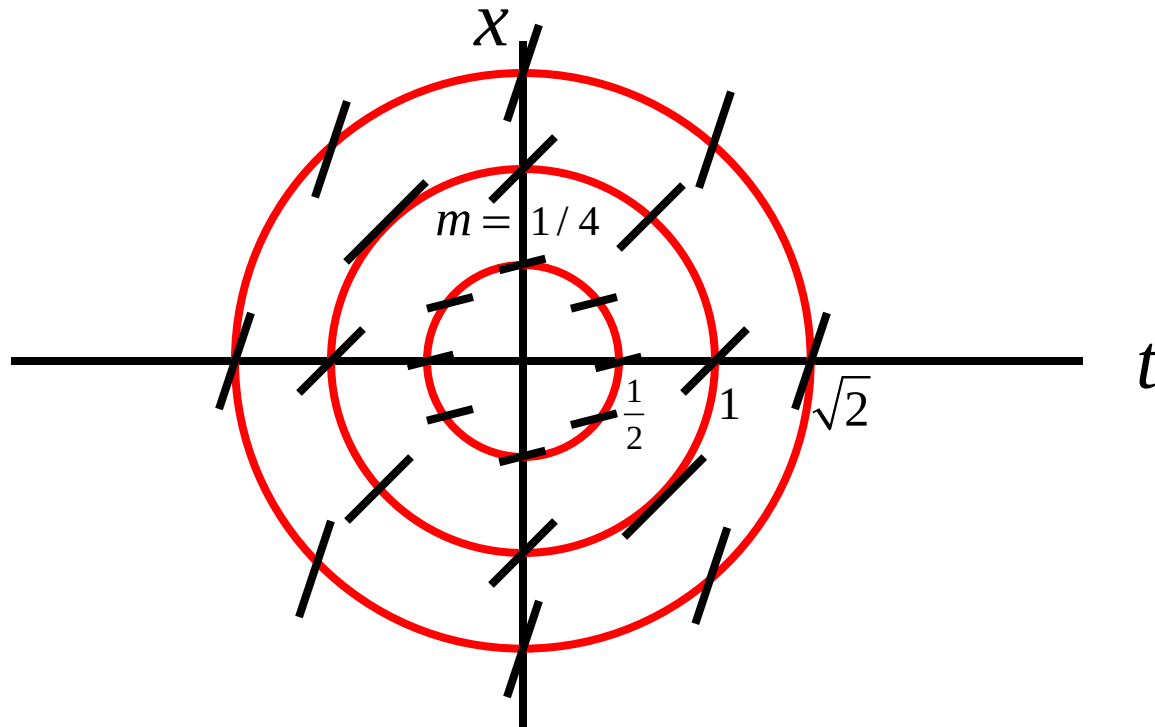
$$x' = x^2 + t^2$$

Isocline equation: $x^2 + t^2 = m$

$m > 0 \Rightarrow$ isoclines are circles (radius $\sqrt{m}$)

$m < 0 \Rightarrow$ no isoclines

$m = 0 \Rightarrow$ isocline is a point (the origin)



Isocline Method: Examples

$$x' = x^2 + t^2$$

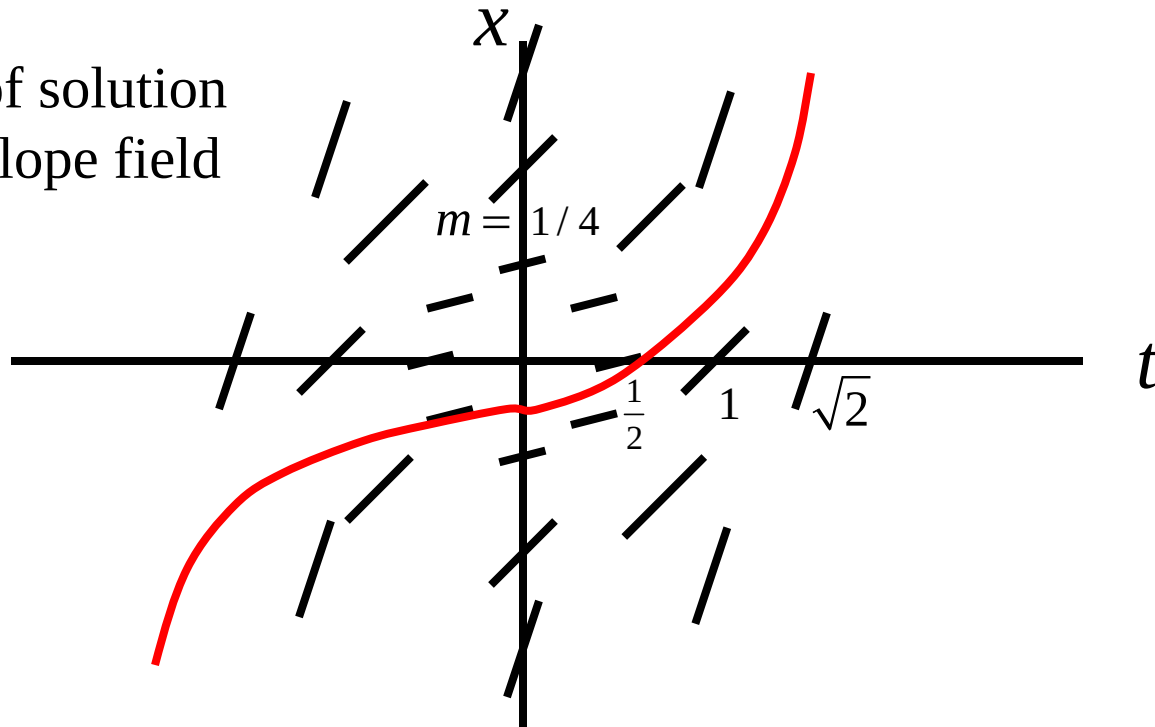
Isocline equation: $x^2 + t^2 = m$

$m > 0 \Rightarrow$ isoclines are circles (radius $\sqrt{m}$)

$m < 0 \Rightarrow$ no isoclines

$m = 0 \Rightarrow$ isocline is a point (the origin)

Note: graph of solution
must fit into slope field



1.2.2 Numerical Approximations

$$x' = f(t, x)$$

$$x(t_0) = x_0$$

How can we obtain numerical approximations to $x(t)$ for $t > t_0$?

Examples

$$x' = x$$

$$x(0) = 1$$

$$x(1) \approx ??$$

$$x' = x^2 + t^2$$

$$x(0) = 0$$

$$x(1) \approx ??$$

Use the Left Hand Rectangle Rule for integrals

$$\int_a^b f(t, x(t)) dt \approx (b - a) f(a, x(a))$$