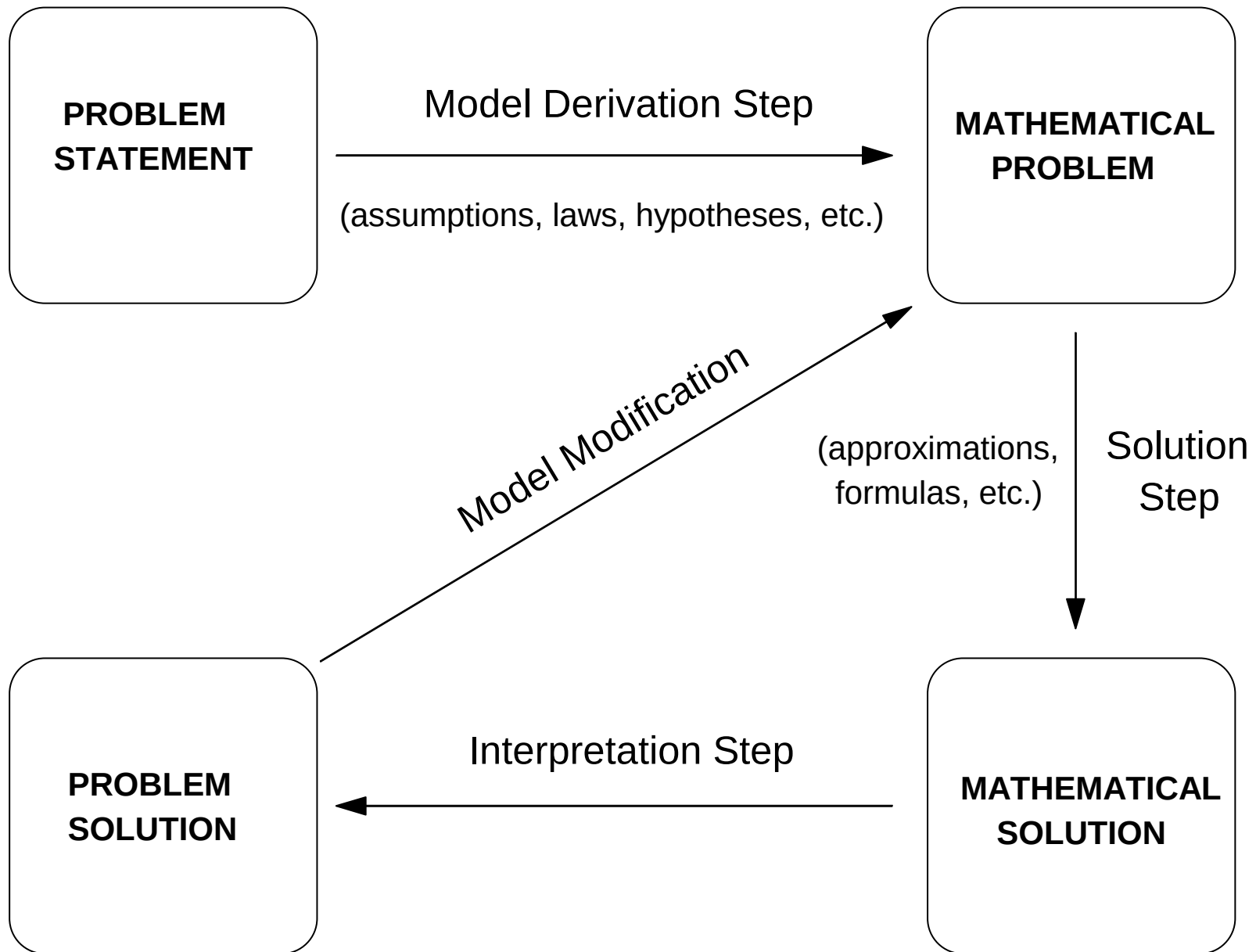


MATH 250b – Section 2

Spring Semester 2008

Thursday, 17-Jan-08

<http://math.arizona.edu/~cushing/250b.html>



1. An object, dropped from a high location, falls vertically with a constant acceleration of 9.8 meters/sec^2 .

How far will it fall in 1 second ? 5 seconds ? 1/2 minute ?

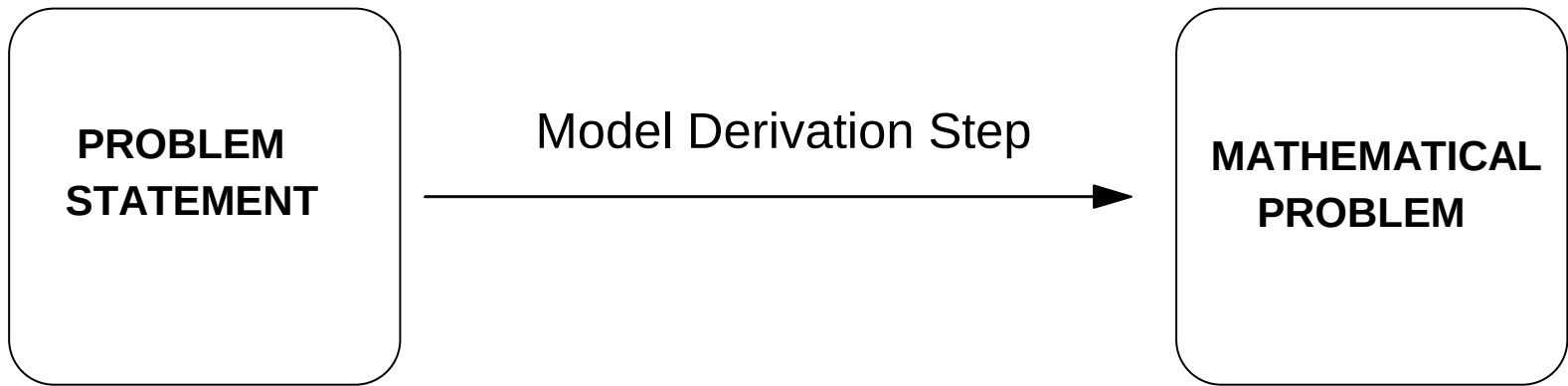
1. An object, dropped from a high location, falls vertically with a constant acceleration of 9.8 meters/sec^2 .

How far will it fall in 1 second ? 5 seconds ? 1/2 minute ?

How are you going to approach this problem ?

What steps will you take to arrive at an answer ?

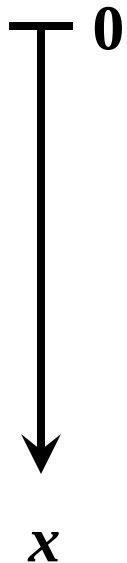
What's your first step ?

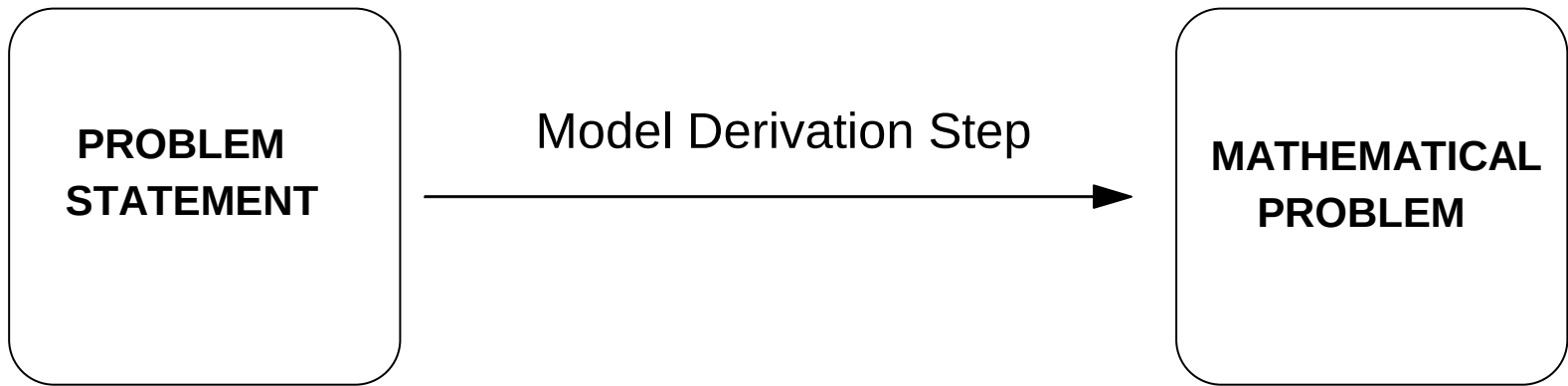


Identification of quantities, variables, etc. & their symbolization

x = distance (meters) from drop point, t = time (seconds)

a = acceleration (m/s^2)

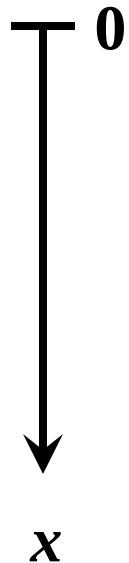




Identification of quantities, variables, etc. & their symbolization

x = distance (meters) from drop point, t = time (seconds)

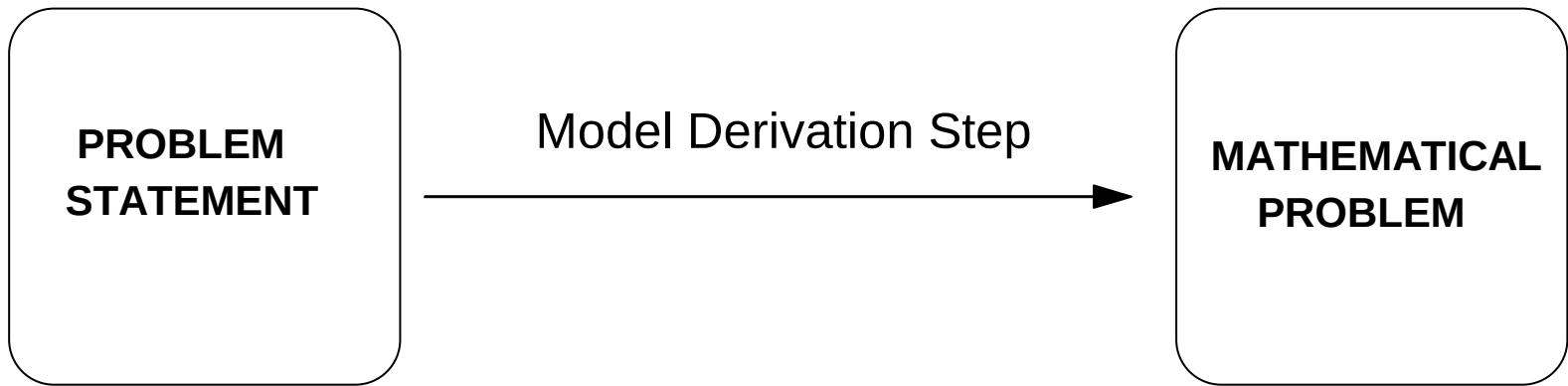
a = acceleration (m/s^2)



Classify and relate quantities & variables

$x = x(t)$ = dependent variable, t = independent variable

$a = x''(t)$, a relevant "parameter" = 9.8 (*meters / sec²*)



Identification of quantities, variables, etc. & their symbolization

x = distance (meters) from drop point, t = time (seconds)

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Classify and relate quantities & variables

$x = x(t)$ = dependent variable, t = independent variable

$a = x''(t)$, a relevant "parameter" = 9.8 (*meters / sec²*)

Derive equation for "state variable"

$x''(t) = 9.8$ (directly from problem statement)

$x(0) = 0$ (our choice of reference point)

$x'(0) = 0$ (mathematical description of "dropped")

0
↓
 x

**PROBLEM
STATEMENT**

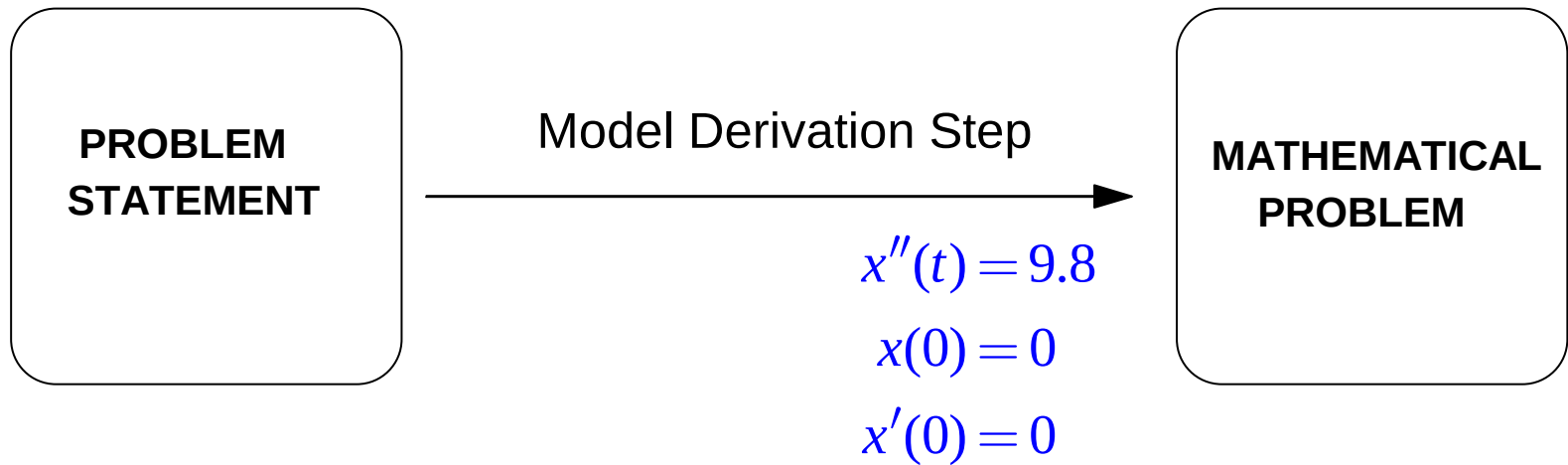
Model Derivation Step

**MATHEMATICAL
PROBLEM**

$$x''(t) = 9.8$$

$$x(0) = 0$$

$$x'(0) = 0$$



Try this: re-do this step using velocity (instead of position) as the unknown (state) variable.

**PROBLEM
STATEMENT**

Model Derivation Step

**MATHEMATICAL
PROBLEM**

$$x''(t) = 9.8$$

$$x(0) = 0$$

$$x'(0) = 0$$

Find a function $x(t)$
that satisfies
the "model equations"

Solution
Step

**MATHEMATICAL
SOLUTION**

**PROBLEM
STATEMENT**

Model Derivation Step

$$x''(t) = 9.8$$

$$x(0) = 0$$

$$x'(0) = 0$$

**MATHEMATICAL
PROBLEM**

Solution
Step

$$x(t) = 4.9t^2$$

**MATHEMATICAL
SOLUTION**

**PROBLEM
STATEMENT**

Model Derivation Step

**MATHEMATICAL
PROBLEM**

$$x''(t) = 9.8$$

$$x(0) = 0$$

$$x'(0) = 0$$

$$x(1) = 4.9(1)^2 = 4.9 \frac{m}{\text{sec}^2} (1\text{sec})^2 = 4.9m$$

$$x(5) = 4.9(5)^2 = 122.5m$$

$$x(30) = 4.9(30)^2 = 4,410m$$

Solution
Step

$$x(t) = 4.9t^2$$

**MATHEMATICAL
SOLUTION**

Interpretation Step

**PROBLEM
SOLUTION**

In this problem, perhaps better
termed a “Utilization Step”

2. *An object, dropped from a high location, falls vertically with a constant acceleration of 9.8 meters/sec^2 .*

If the ground is 100 meters below, what is the object's velocity when it hits the ground ?

What's the same in this problem ?

What's different ?

**PROBLEM
STATEMENT**

Model Derivation Step

**MATHEMATICAL
PROBLEM**

$$x''(t) = 9.8$$

$$x(0) = 0$$

$$x'(0) = 0$$

Same: model equations.

**PROBLEM
STATEMENT**

Model Derivation Step

$$x''(t) = 9.8$$

$$x(0) = 0$$

$$x'(0) = 0$$

**MATHEMATICAL
PROBLEM**

Same: model equations.

Same: solution step.

Solution
Step

$$x(t) = 4.9t^2$$

**MATHEMATICAL
SOLUTION**

**PROBLEM
STATEMENT**

Model Derivation Step

**MATHEMATICAL
PROBLEM**

$$x''(t) = 9.8$$

$$x(0) = 0$$

$$x'(0) = 0$$

Different: the utilization step.

Solution
Step

$$\text{impact time: } 4.9t^2 = 100 \Rightarrow t_i = 4.518\text{sec}$$

$$\text{impact velocity} = x'(t_i) = 9.8t_i = 44.28 \text{ (m/sec)}$$

**PROBLEM
SOLUTION**

Interpretation Step

$x(t) = 4.9t^2$
**MATHEMATICAL
SOLUTION**

In this problem, perhaps better
termed a “Utilization Step”

Data from a lab experiment*

t (seconds)	x (meters)	x predicted (meters)
0.000	0.00	0.00
0.347	0.61	0.59
0.470	1.00	1.08
0.519	1.22	1.32
0.582	1.52	1.66
0.650	1.83	2.07
0.674	2.00	2.23
0.717	2.13	2.52
0.766	2.44	2.88
0.823	2.74	3.32
0.870	3.00	3.71
1.031	4.00	5.21
1.193	5.00	6.98
1.354	6.00	8.98
1.501	7.00	11.04
1.727	8.50	14.61
1.873	9.50	17.19

model

$$x''(t) = 9.8$$

$$x(0) = 0$$

$$x'(0) = 0$$

prediction

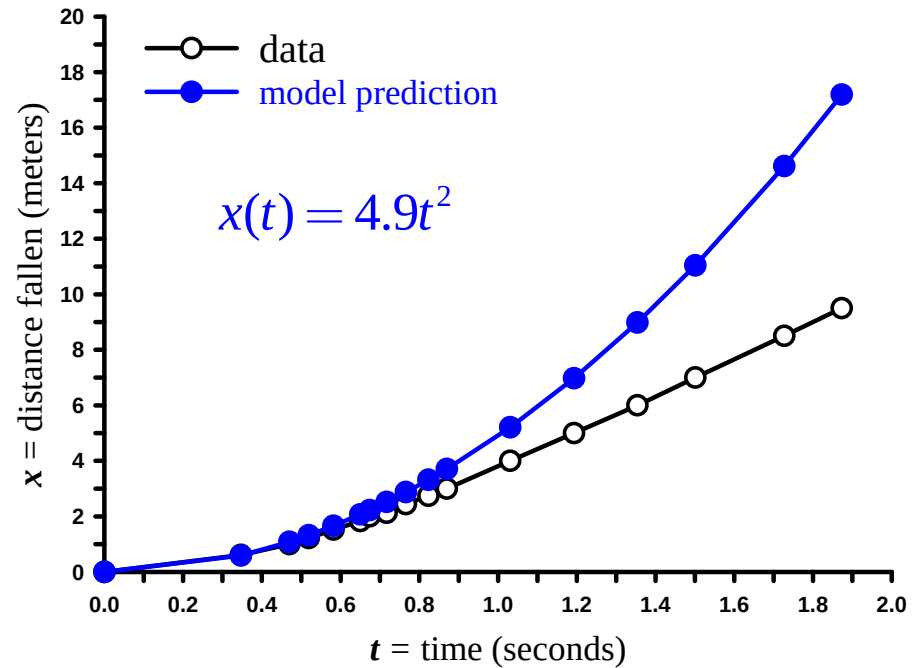
$$x(t) = 4.9t^2$$



* M. Peastrel, R. Lynch and A. Armenti, Jr., 1993, *The Physics of Sports*
(A. Armenti, Jr., editor), American Institute of Physics, New York, pp. 301-303

Data from a lab experiment*

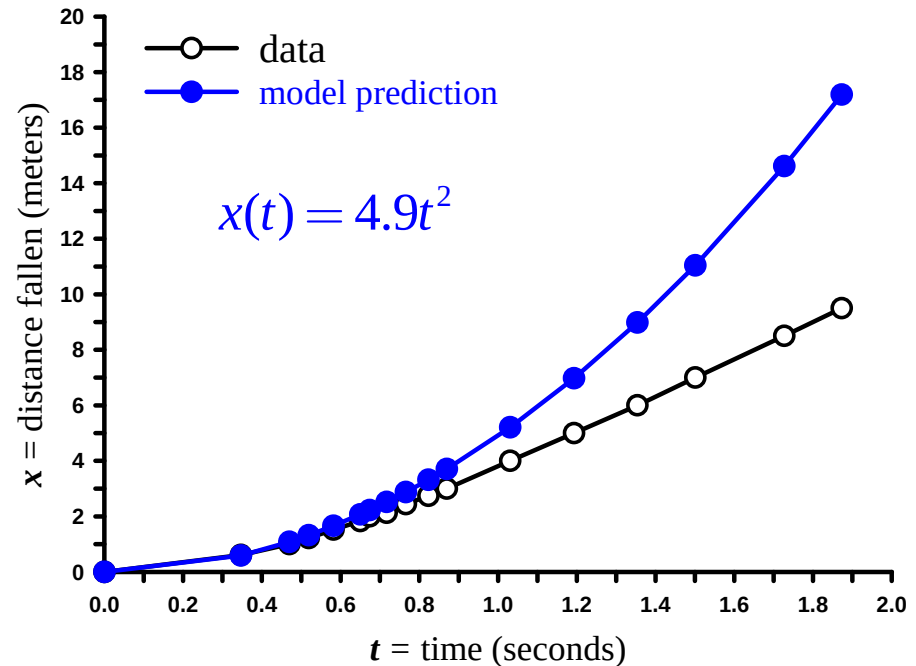
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0.347	0.61	0.59	-3.28
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1.354	6.00	8.98	49.72
1.501	7.00	11.04	57.71
1.727	8.50	14.61	71.93
1.873	9.50	17.19	80.95



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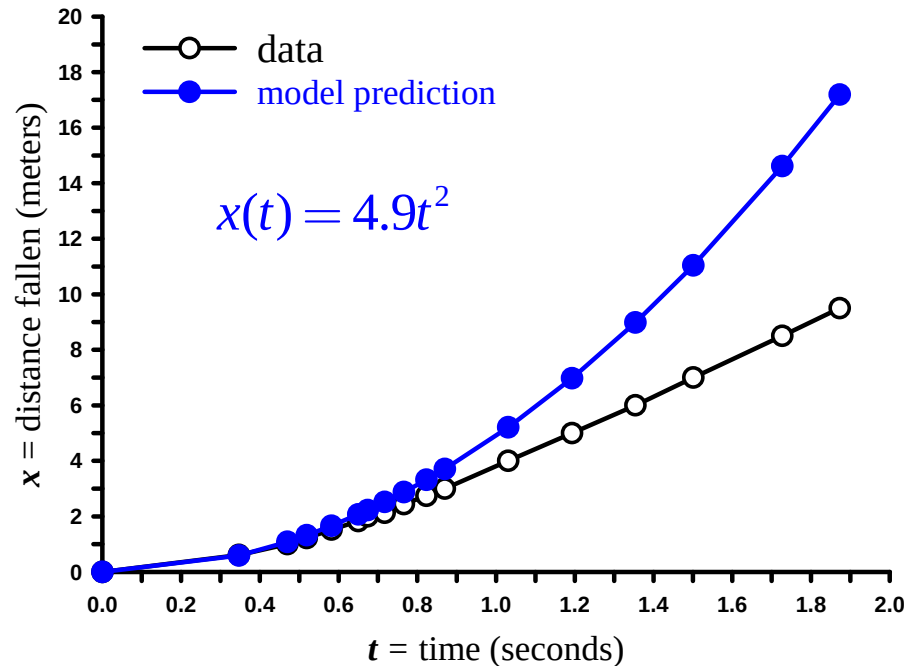
Problem

How to explain the discrepancy
between model & data?

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Some possibilities

Mathematical errors

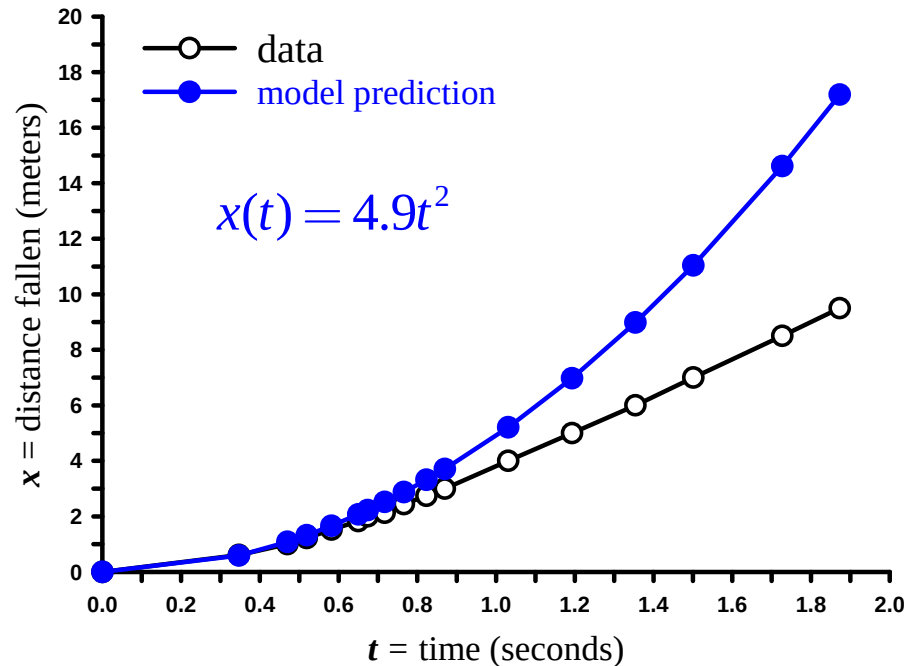
Experimental errors

Model is “wrong” - overlooked/ignored
important mechanism(s)

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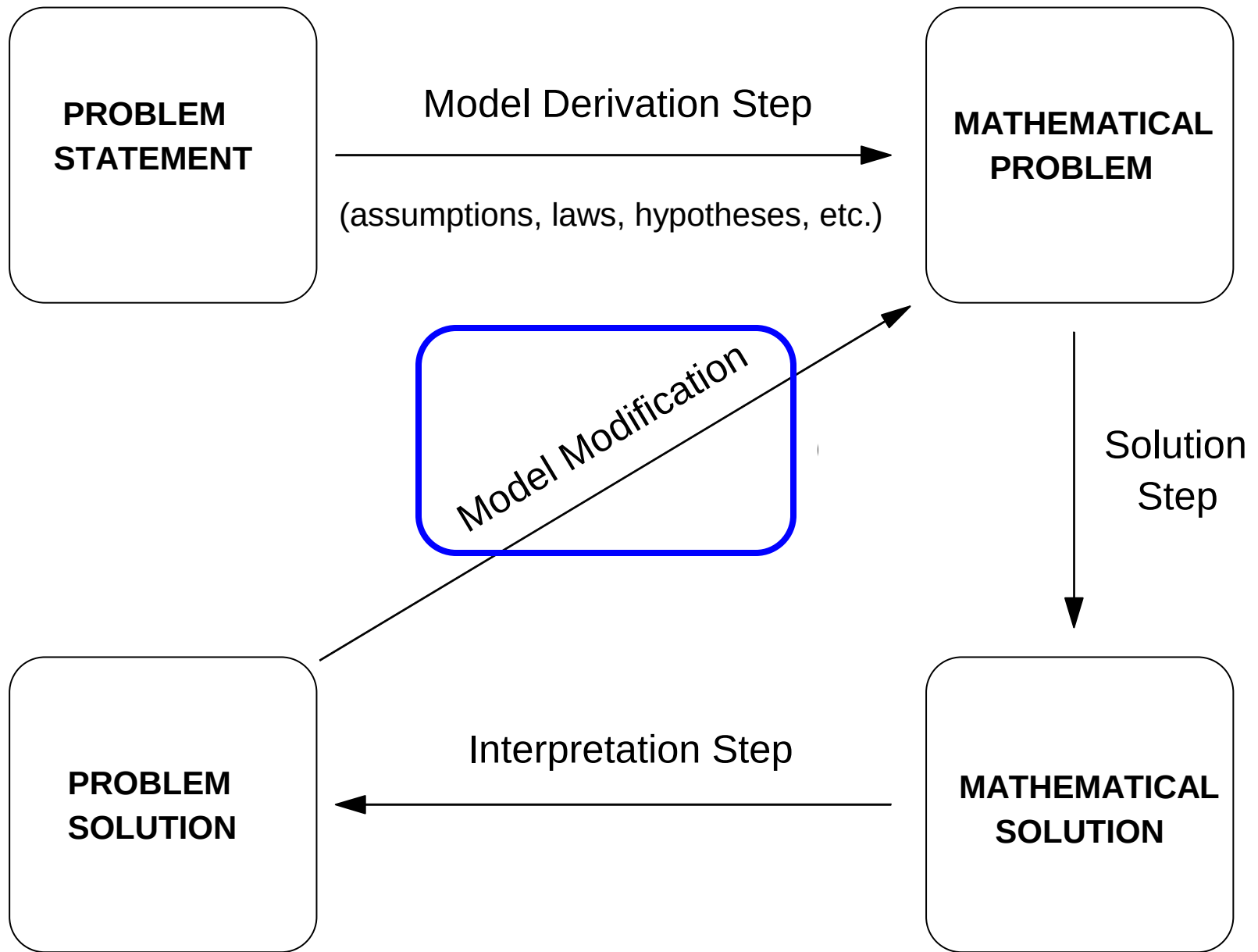


Turns out the experiment was done with a *badminton shuttlecock*.

By design, shuttlecocks are subject to considerable *air resistance/friction*.

We need to *modify our model* so as to take friction into account.

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For discussion next class

Re-do problems using velocity as unknown (state) variable.

Problems from text : Page 16: #1, #3, #9