

MATH 250a

Fall Semester 2007

Section 2 (J. M. Cushing)

Thursday, November 1

<http://math.arizona.edu/~cushing/250a.html>

Example (continued)

Assume 95% of an antibiotic
is used or otherwise eliminated between treatments.

y_n = amount (mg) antibiotic in patient at time $n = 0, 1, 2, 3, \dots$

y_{n+1} = (residual left from treatment at time n) + (new dose)

$$y_{n+1} = 0.05y_n + d$$

Question:

What dosages d will in the long run keep y_n between 250 - 300 mg?

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

the solution formula
is a finite geometric series

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \lim_{n \rightarrow +\infty} \left(d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d \right) = ?$$

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \lim_{n \rightarrow +\infty} (d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d) = ?$$



This limit is called an *infinite series*.

Definition

An infinite series $\sum_{i=0}^{\infty} a_i$ *converges* if (and only if)

its sequence of partial sums $s_n = \sum_{i=0}^{n-1} a_i$ converges.

Definition

An infinite series $\sum_{i=0}^{\infty} a_i$ *converges* if (and only if)

its sequence of partial sums $s_n = \sum_{i=0}^{n-1} a_i$ converges.

An infinite series $\sum_{i=0}^{\infty} a_i$ *diverges* if (and only if)

its sequence of partial sums $s_n = \sum_{i=0}^{n-1} a_i$ diverges.

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \sum_{i=0}^{\infty} 0.05^i d = ?$$



- Does this infinite series converge ?
- If so, to what ?

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \sum_{i=0}^{\infty} 0.05^i d = ?$$

By definition:

$$\sum_{i=0}^{\infty} 0.05^i d = \lim_{n \rightarrow +\infty} \sum_{i=0}^{n-1} 0.05^i d$$

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \sum_{i=0}^{\infty} 0.05^i d = ?$$

By definition:

$$\sum_{i=0}^{\infty} 0.05^i d = \lim_{n \rightarrow +\infty} \sum_{i=0}^{n-1} 0.05^i d$$



A finite geometric series

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \sum_{i=0}^{\infty} 0.05^i d = ?$$

By definition:

$$\sum_{i=0}^{\infty} 0.05^i d = \lim_{n \rightarrow +\infty} \sum_{i=0}^{n-1} 0.05^i d$$

Recall:

$$\sum_{i=0}^{n-1} ar^i = a \frac{1-r^n}{1-r}$$

A finite geometric series



Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \sum_{i=0}^{\infty} 0.05^i d = ?$$

By definition:

$$\sum_{i=0}^{\infty} 0.05^i d = \lim_{n \rightarrow +\infty} \sum_{i=0}^{n-1} 0.05^i d$$

$$= \lim_{n \rightarrow +\infty} d \frac{1 - 0.05^n}{1 - 0.05}$$

Recall:

$$\sum_{i=0}^{n-1} ar^i = a \frac{1 - r^n}{1 - r}$$

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \sum_{i=0}^{\infty} 0.05^i d = ?$$

By definition:

$$\begin{aligned} \sum_{i=0}^{\infty} 0.05^i d &= \lim_{n \rightarrow +\infty} \sum_{i=0}^{n-1} 0.05^i d \\ &= \lim_{n \rightarrow +\infty} d \frac{1 - 0.05^n}{1 - 0.05} \\ &= d \frac{1}{1 - 0.05} \end{aligned}$$

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \sum_{i=0}^{\infty} 0.05^i d = ?$$

By definition:

$$\begin{aligned} \sum_{i=0}^{\infty} 0.05^i d &= \lim_{n \rightarrow +\infty} \sum_{i=0}^{n-1} 0.05^i d \\ &= \lim_{n \rightarrow +\infty} d \frac{1 - 0.05^n}{1 - 0.05} \\ &= d \frac{1}{1 - 0.05} = \frac{20}{19} d \end{aligned}$$

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \sum_{i=0}^{\infty} 0.05^i d = \frac{20}{19}d$$

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \sum_{i=0}^{\infty} 0.05^i d = \frac{20}{19}d$$

Question:

What dosages d will in the long run keep y_n between 250 - 300 mg?

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \sum_{i=0}^{\infty} 0.05^i d = \frac{20}{19}d$$

Question:

What dosages d will in the long run keep y_n between 250 - 300 mg?

Answer:

$$250 \leq \frac{20}{19}d \leq 300$$

Example

$$y_{n+1} = 0.05y_n + d$$

$$y_n = d + 0.05d + 0.05^2d + \cdots + 0.05^{n-1}d$$

What happens in the long run?

$$\lim_{n \rightarrow +\infty} y_n = \sum_{i=0}^{\infty} 0.05^i d = \frac{20}{19}d$$

Question:

What dosages d will in the long run keep y_n between 250 - 300 mg?

Answer:

$$251.0 \text{ mg} \approx \frac{5019}{20} \leq d \leq \frac{6019}{20} \approx 301.0 \text{ mg}$$

There are two basic questions concerning an infinite series:

- Does it converge ?

There are two basic questions concerning an infinite series:

- Does it converge ?
- If so, to what ?

There are two basic questions concerning an infinite series:

- Does it converge ?
- If so, to what ?

For geometric series, we know the answer to both questions :

$$\sum_{i=0}^{\infty} ar^i \text{ converges to } \frac{a}{1-r} \text{ if } |r| < 1$$

$$\sum_{i=0}^{\infty} ar^i \text{ diverges if } |r| \geq 1$$

There are two basic questions concerning an infinite series:

- Does it converge ?
- If so, to what ?

But not all infinite series are geometric series.

There are two basic questions concerning an infinite series:

- Does it converge ?
- If so, to what ?

But not all infinite series are geometric series.

First question is studied in Chapter 9

Second question is in general much more difficult.

Some relevant methods of calculation are studied in Chapter 10.

Convergence Tests for Series

(Chapter 9.3)

Convergence Tests for Series (Chapter 9.3)

A useful geometric interpretation of a series

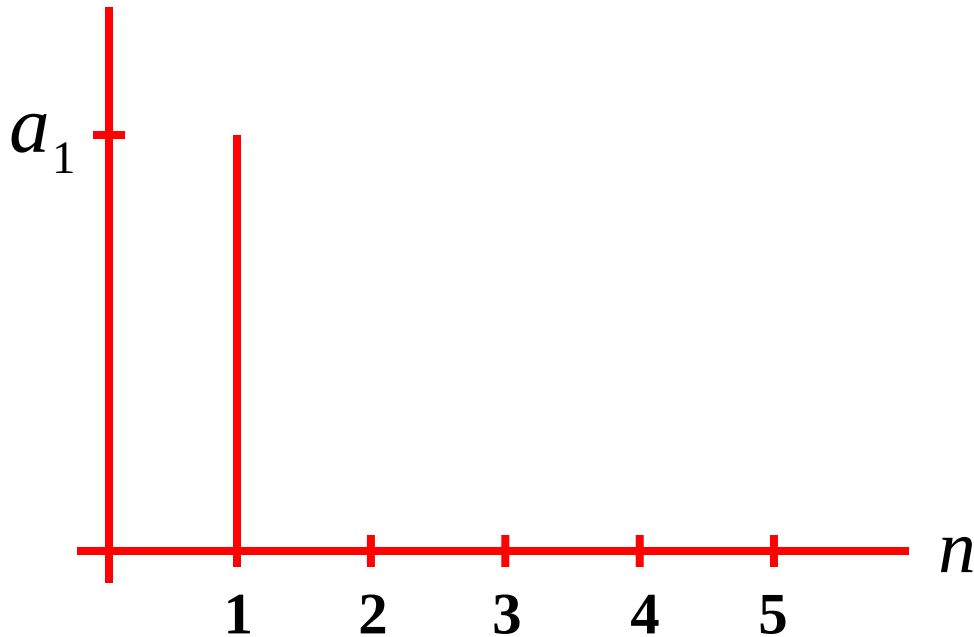
$$\sum_{n=1}^{\infty} a_n, \quad a_n \geq 0$$

Convergence Tests for Series

(Chapter 9.3)

A useful geometric interpretation of a series

$$\sum_{n=1}^{\infty} a_n, \quad a_n \geq 0$$

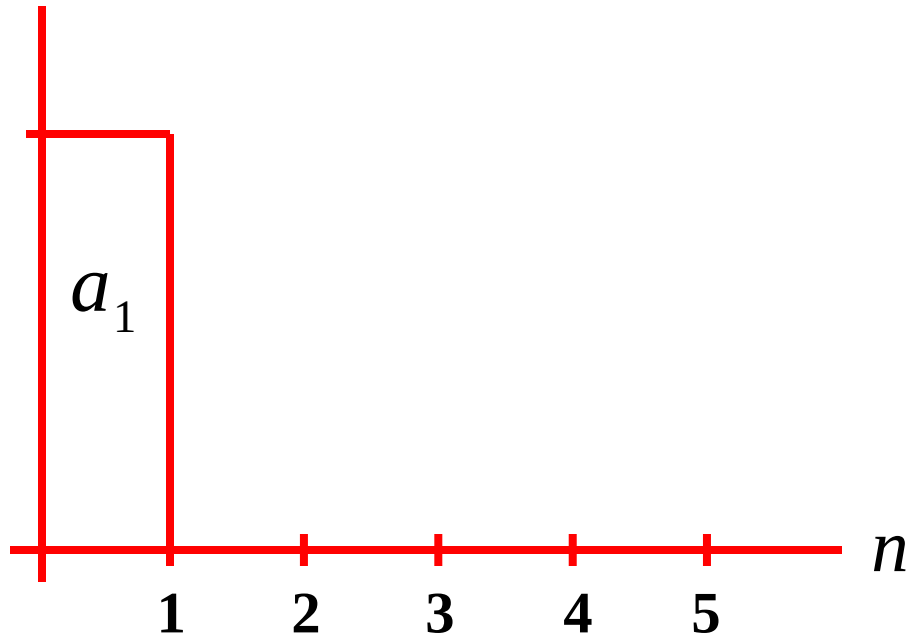


Convergence Tests for Series

(Chapter 9.3)

A useful geometric interpretation of a series

$$\sum_{n=1}^{\infty} a_n, \quad a_n \geq 0$$

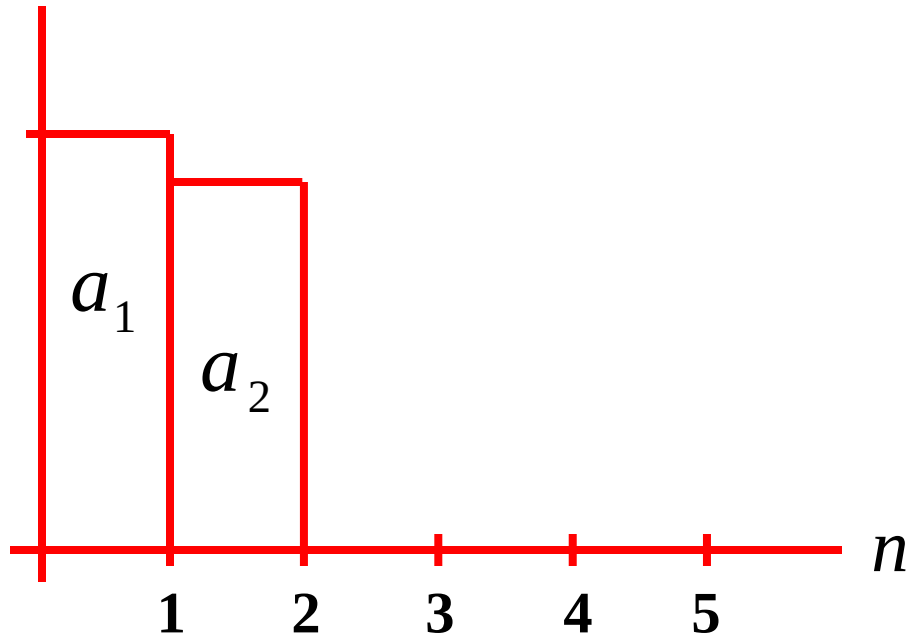


Convergence Tests for Series

(Chapter 9.3)

A useful geometric interpretation of a series

$$\sum_{n=1}^{\infty} a_n, \quad a_n \geq 0$$

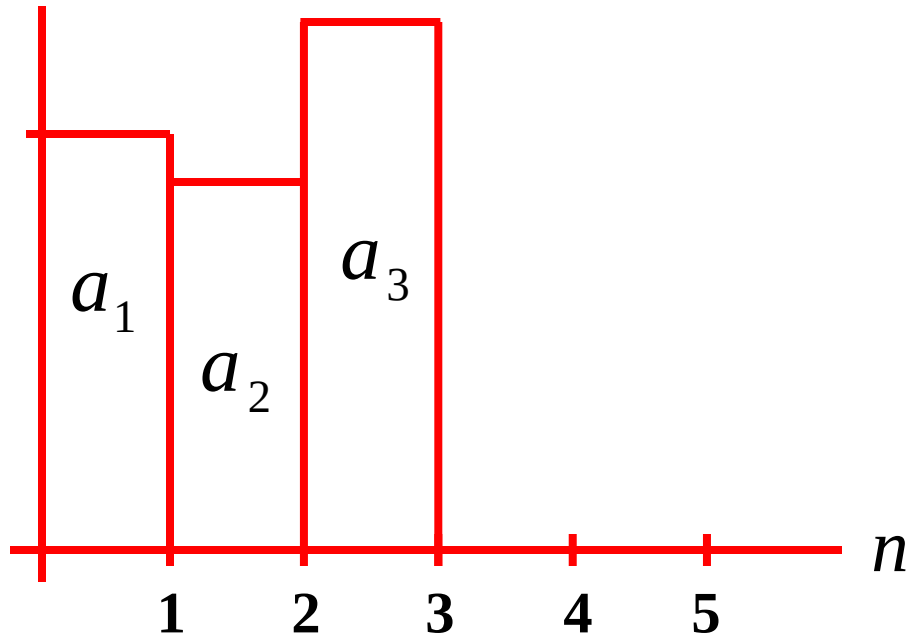


Convergence Tests for Series

(Chapter 9.3)

A useful geometric interpretation of a series

$$\sum_{n=1}^{\infty} a_n, \quad a_n \geq 0$$

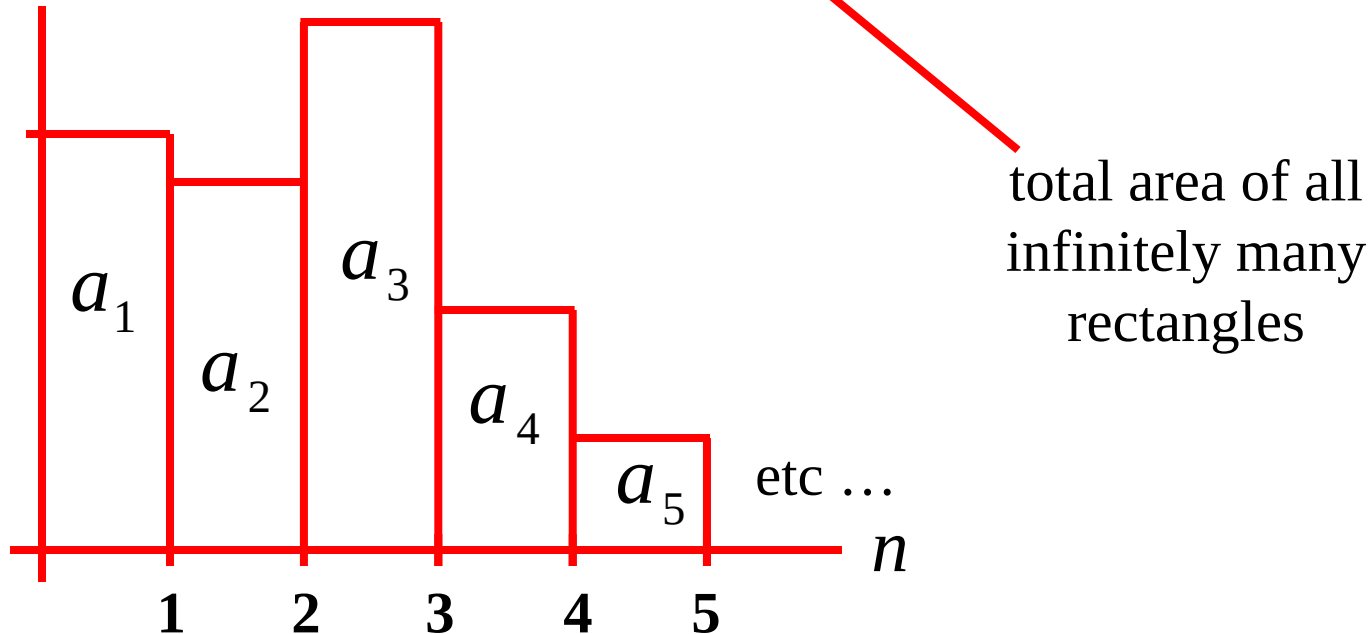


Convergence Tests for Series

(Chapter 9.3)

A useful geometric interpretation of a series

$$\sum_{n=1}^{\infty} a_n, \quad a_n \geq 0$$

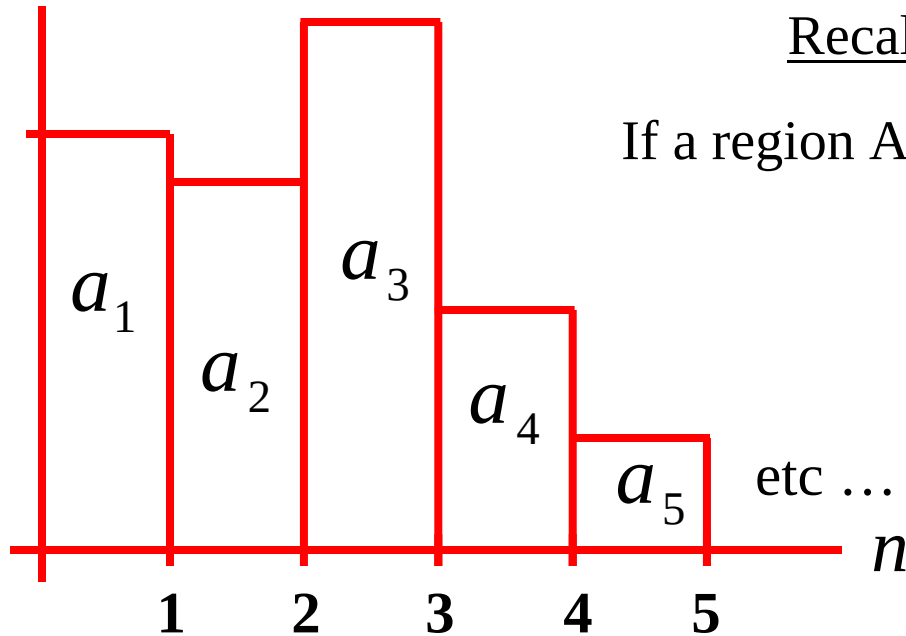


Convergence Tests for Series

(Chapter 9.3)

A useful geometric interpretation of a series

$$\sum_{n=1}^{\infty} a_n, \quad a_n \geq 0$$



Recall related areas principle:

If a region A is contained in a region B, then

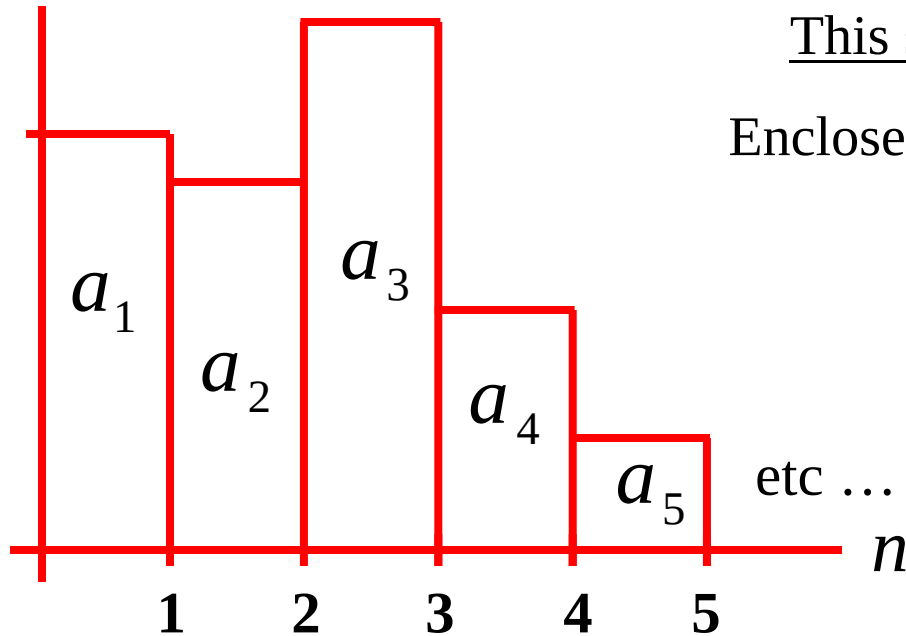
- B has finite area implies A does too
- A has infinite area implies B does too

Convergence Tests for Series

(Chapter 9.3)

A useful geometric interpretation of a series

$$\sum_{n=1}^{\infty} a_n, \quad a_n \geq 0$$



This suggests we might be able to

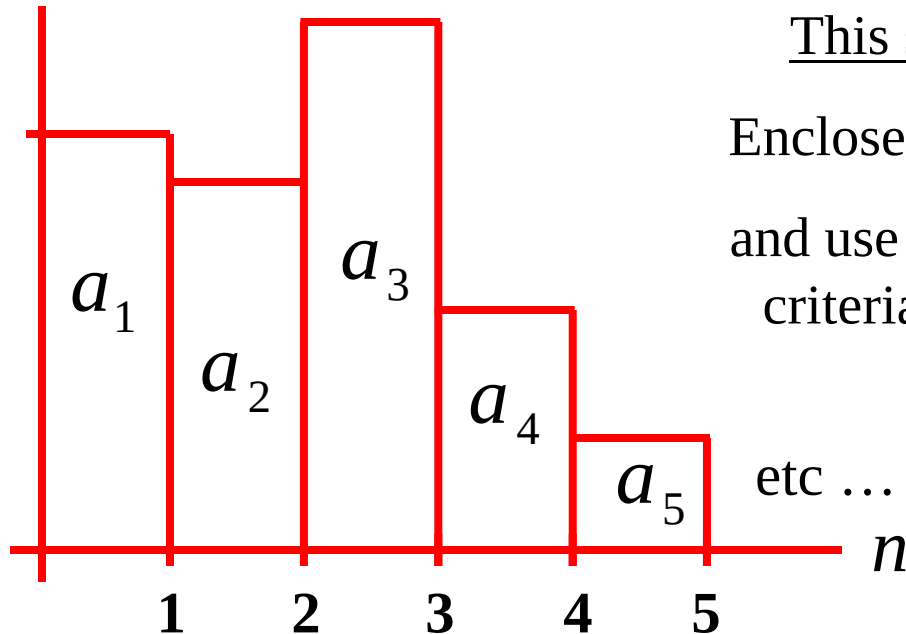
Enclose $\sum_{n=1}^{\infty} a_n$ in an area under a curve

Convergence Tests for Series

(Chapter 9.3)

A useful geometric interpretation of a series

$$\sum_{n=1}^{\infty} a_n, \quad a_n \geq 0$$



This suggests we might be able to

Enclose $\sum_{n=1}^{\infty} a_n$ in an area under a curve
and use *improper integral* convergence
criteria to prove *series* convergence.

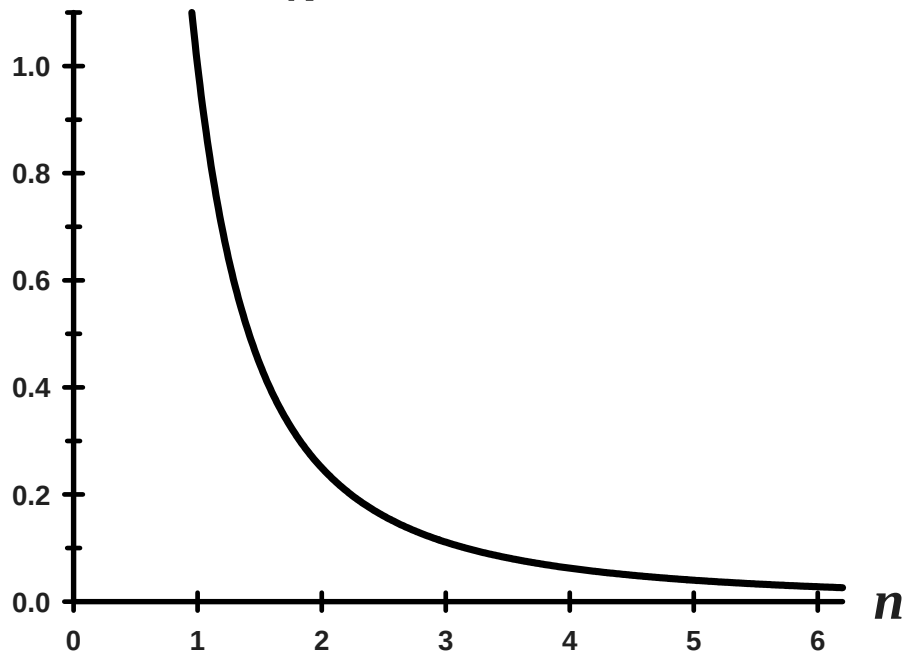
Example

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$$

Example

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

$$f(x) = \frac{1}{x^2}$$

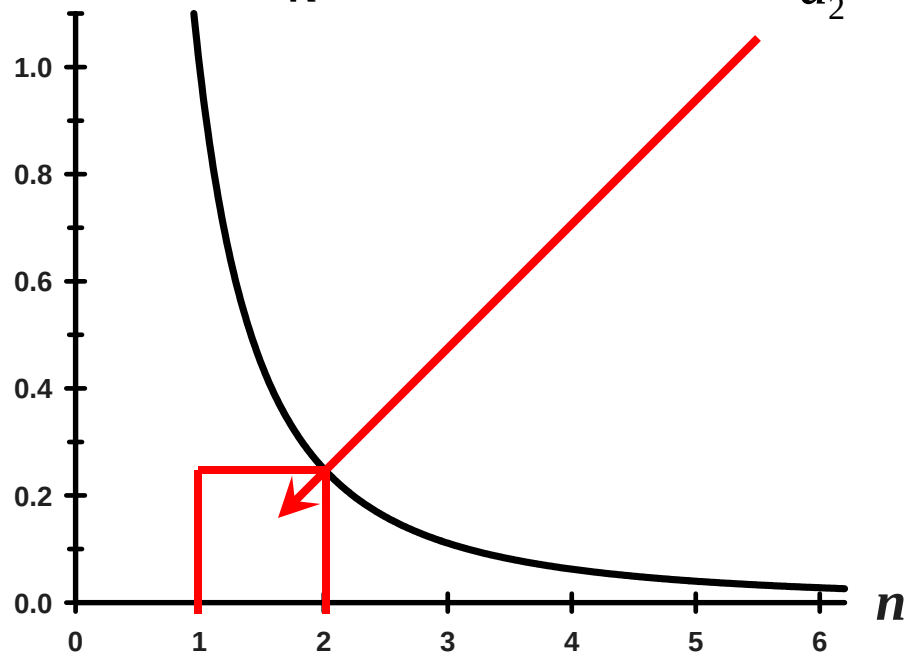


Example

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

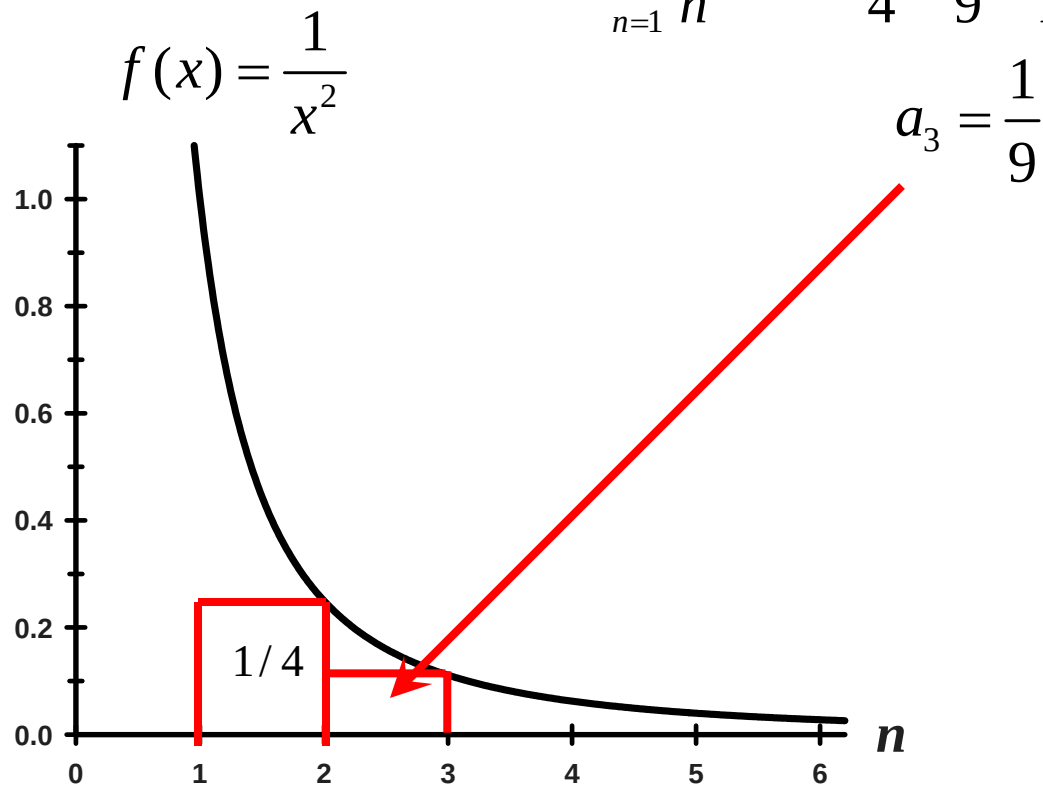
$$f(x) = \frac{1}{x^2}$$

$$a_2 = \frac{1}{4}$$



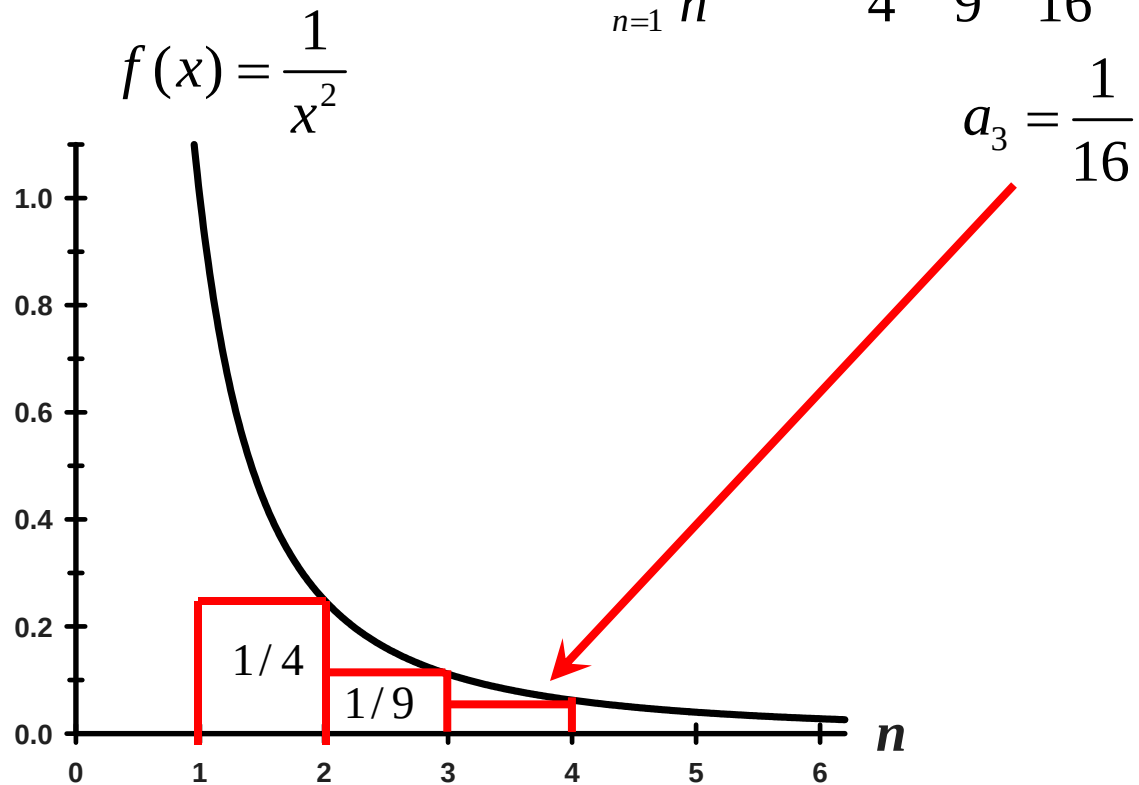
Example

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$



Example

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$



Example

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

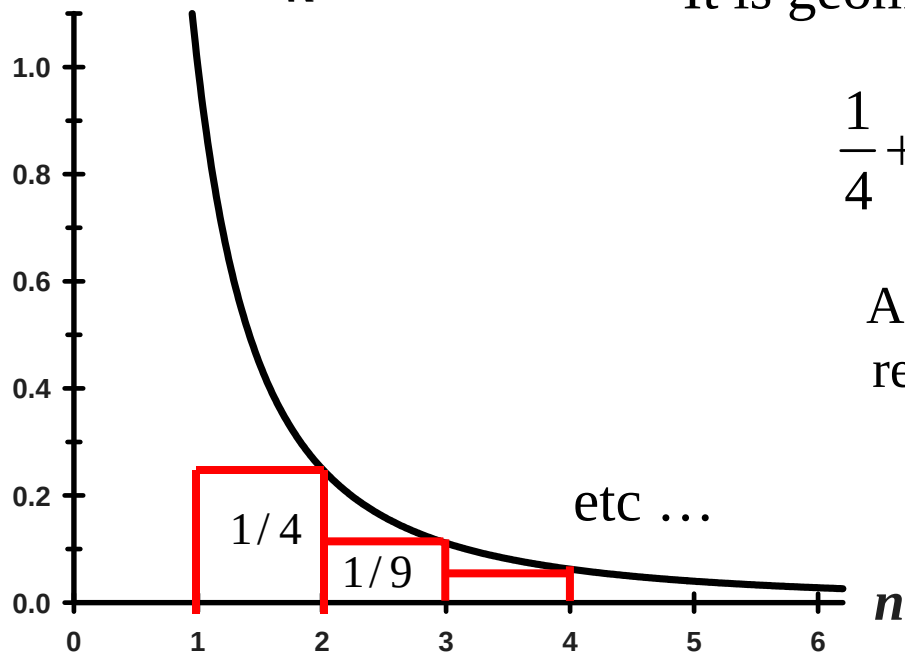
$$f(x) = \frac{1}{x^2}$$

It is geometrically clear that

$$\frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots \leq \int_1^{\infty} \frac{1}{x^2} dx$$

Area of all
rectangles

Area under
curve



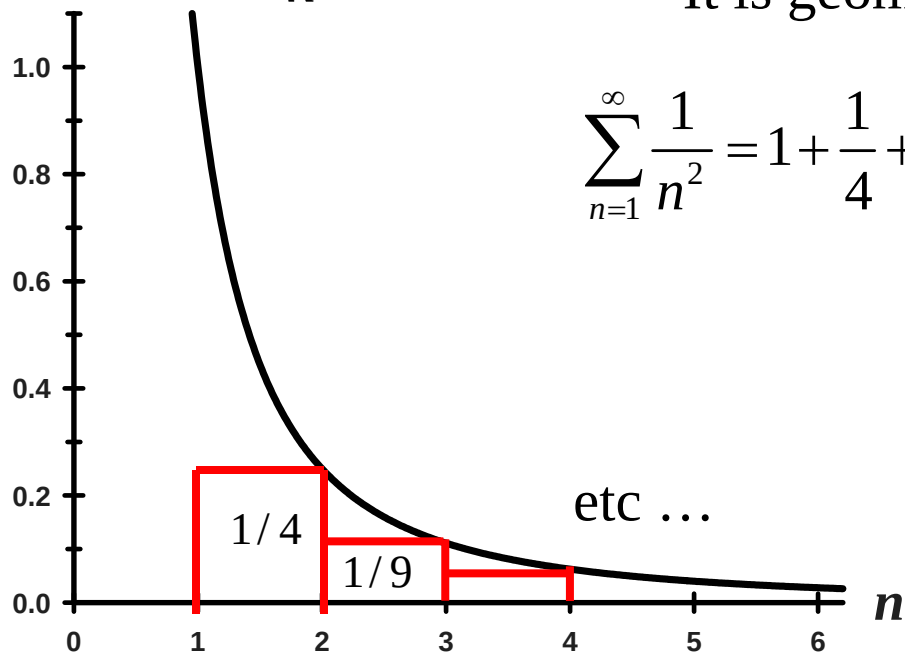
Example

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

$$f(x) = \frac{1}{x^2}$$

It is geometrically clear that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots \leq 1 + \int_1^{\infty} \frac{1}{x^2} dx$$



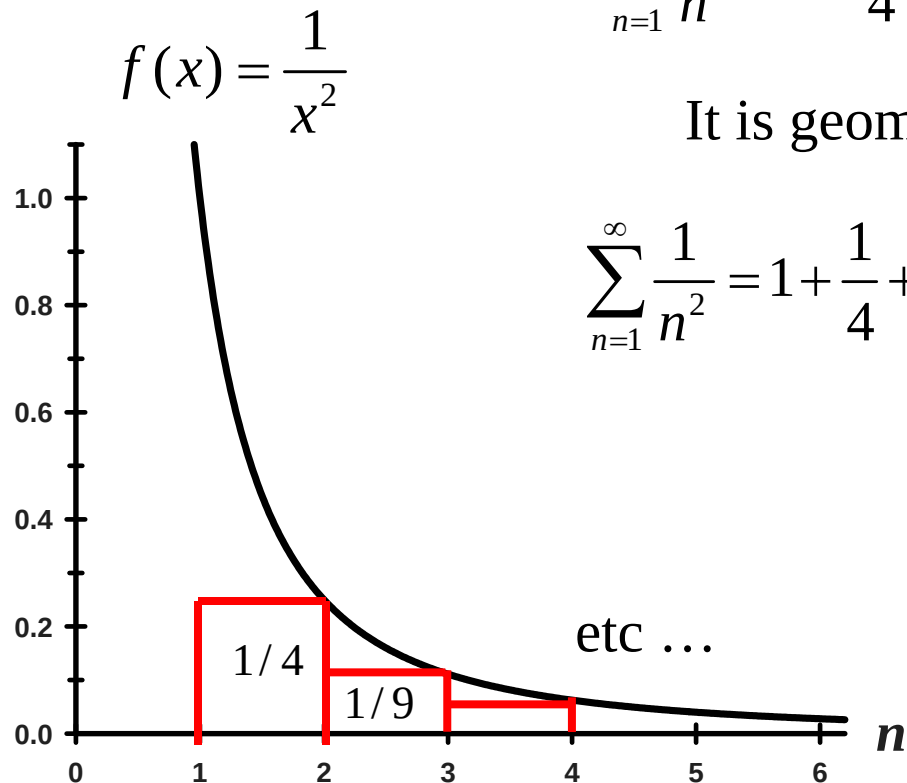
Example

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

It is geometrically clear that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots \leq 1 + \int_1^{\infty} \frac{1}{x^2} dx$$

From our study of improper
integrals we know this integral
converges.



Example

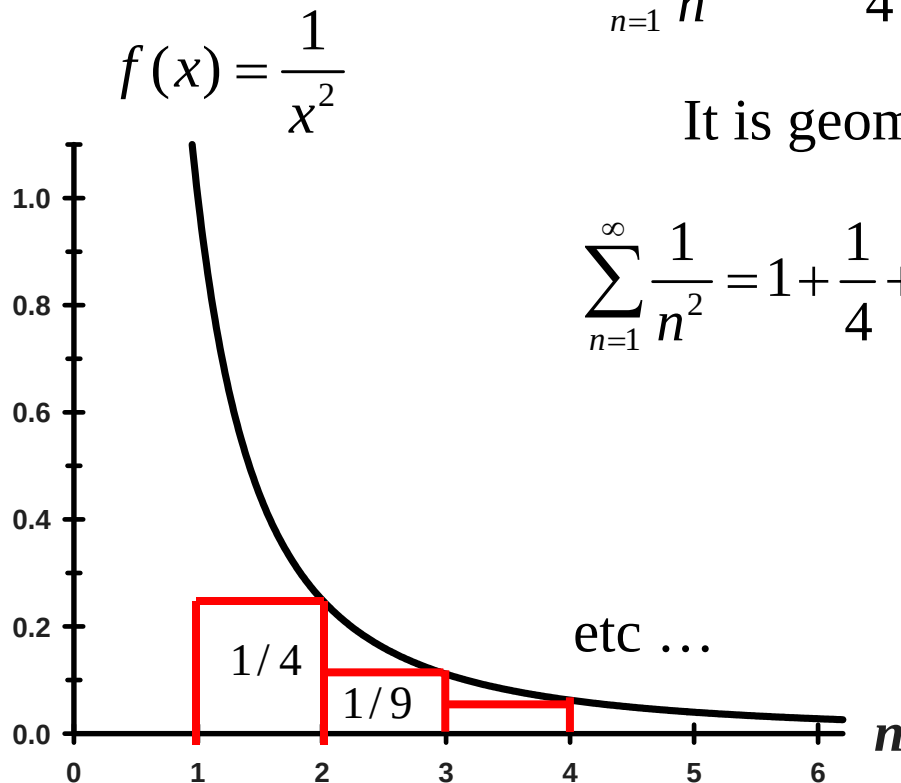
$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots$$

It is geometrically clear that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots \leq 1 + \int_1^{\infty} \frac{1}{x^2} dx$$

From our study of improper integrals we know this integral converges.

We conclude *the series converges*.

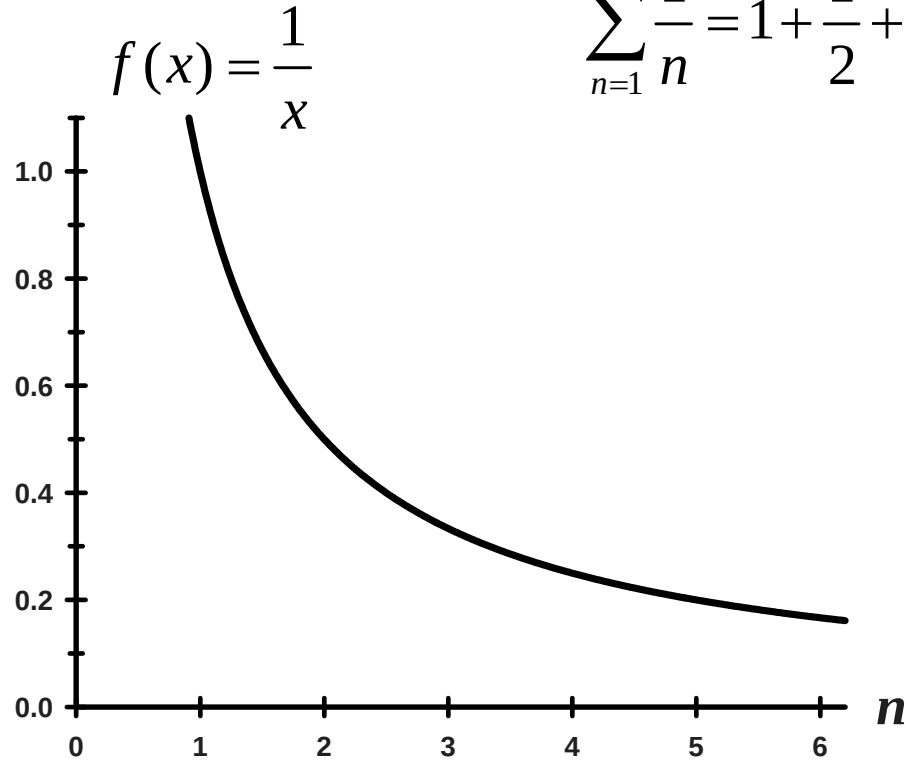


Example
(the Harmonic Series)

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

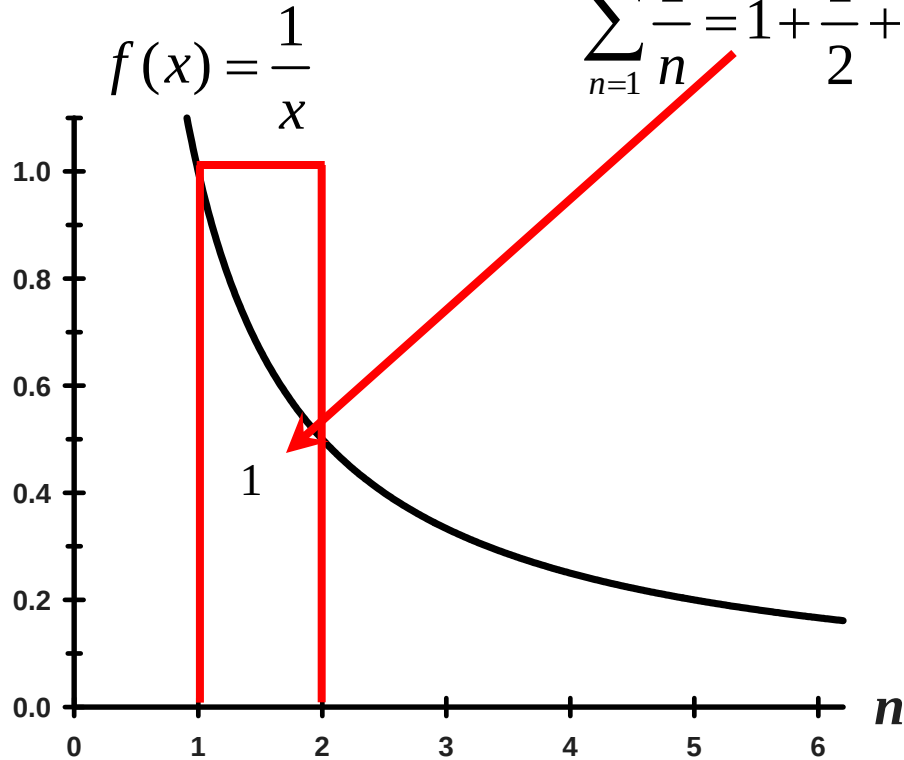
Example
(the Harmonic Series)

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$



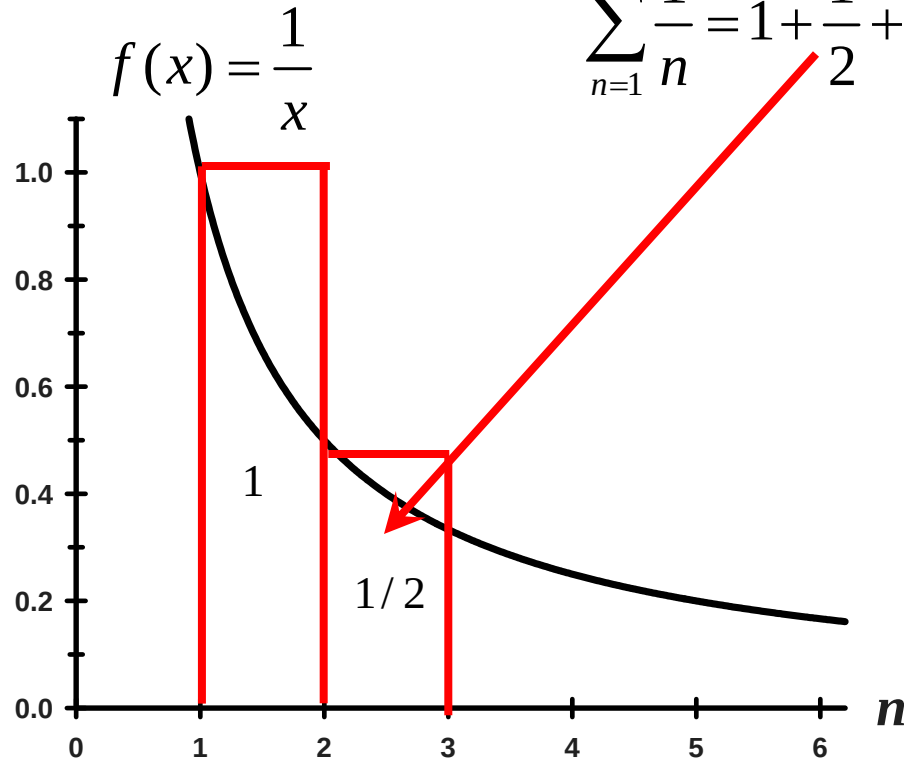
Example
(the Harmonic Series)

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$



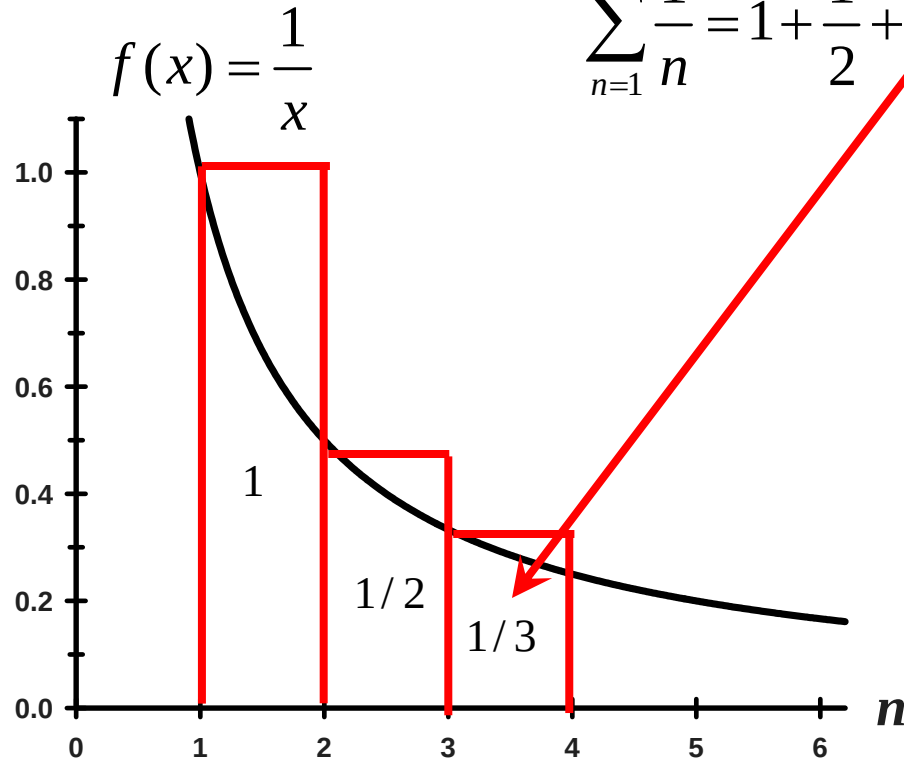
Example
(the Harmonic Series)

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$



Example
(the Harmonic Series)

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$



Example
(the Harmonic Series)

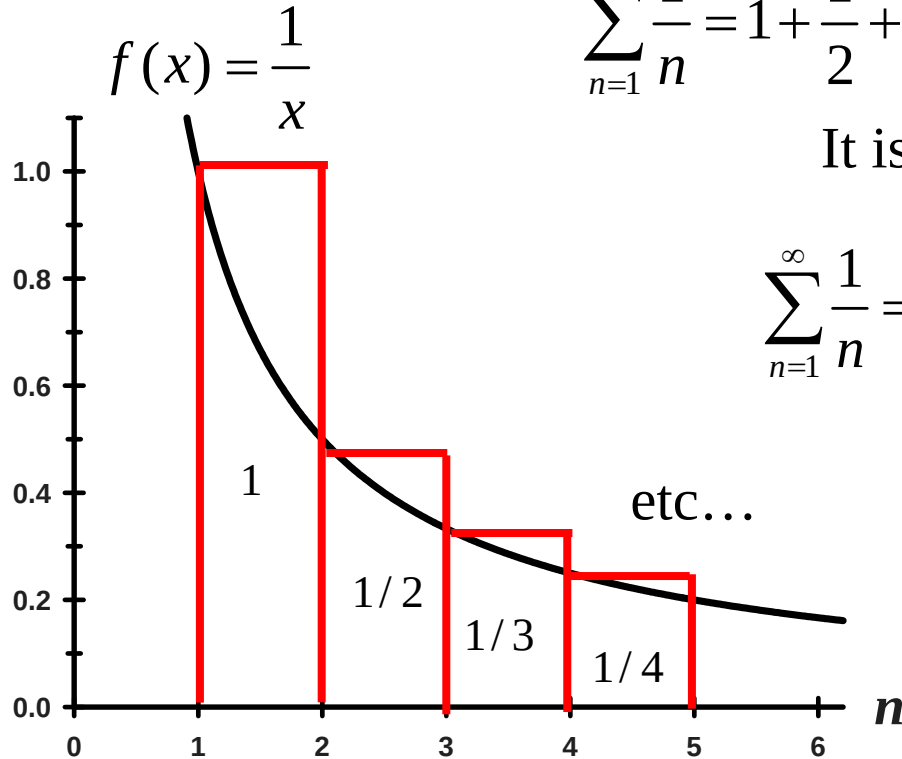
$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

It is geometrically clear that

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \geq \int_1^{\infty} \frac{1}{x} dx$$

Area of all
rectangles

Area under
curve



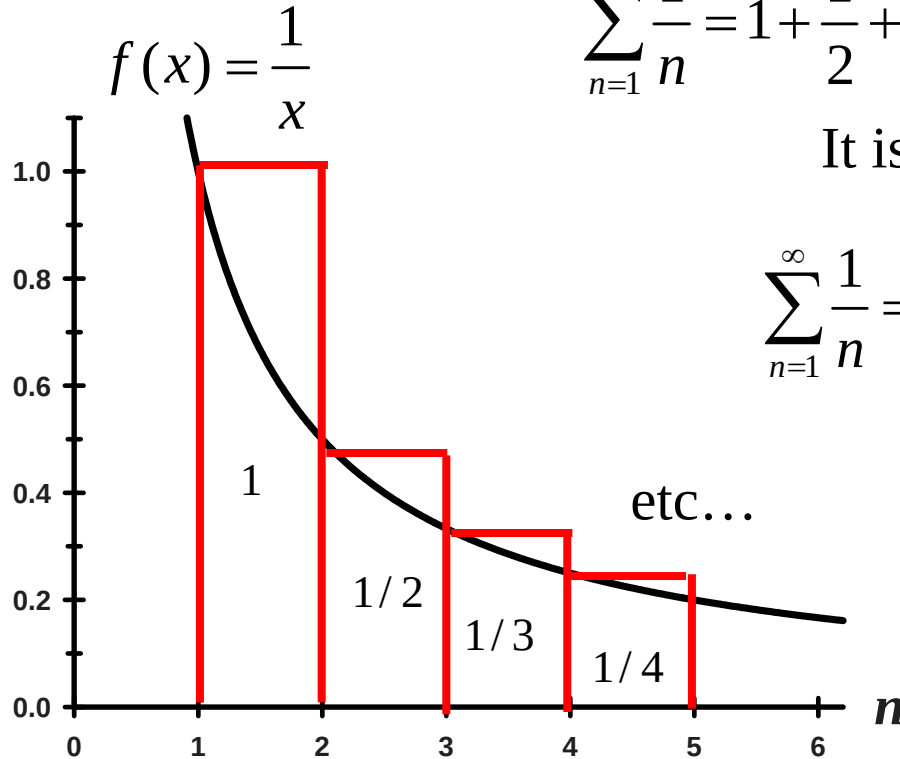
Example
(the Harmonic Series)

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

It is geometrically clear that

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \geq \int_1^{\infty} \frac{1}{x} dx$$

From our study of improper integrals we know this integral *diverges*.



Example
(the Harmonic Series)

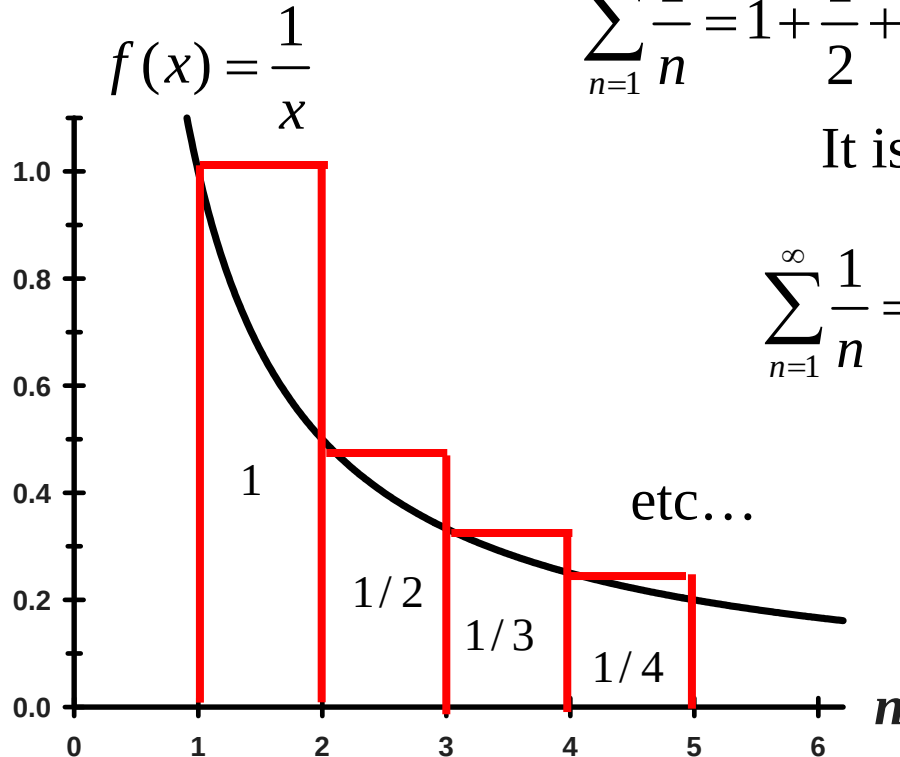
$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

It is geometrically clear that

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \geq \int_1^{\infty} \frac{1}{x} dx$$

From our study of improper
integrals we know this integral
diverges

We conclude *the series diverges*.



The Integral Test
(Theorem 9.3, text)

Suppose $a_n = f(n)$ where $f(x)$ is decreasing and positive for $x \geq c$.

$$\int_c^{\infty} f(x) dx \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\int_c^{\infty} f(x) dx \text{ diverges} \Rightarrow \sum a_n \text{ diverges}$$

The Integral Test (Theorem 9.3, text)

Suppose $a_n = f(n)$ where $f(x)$ is decreasing and positive for $x \geq c$.

$$\int_c^{\infty} f(x) dx \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\int_c^{\infty} f(x) dx \text{ diverges} \Rightarrow \sum a_n \text{ diverges}$$

Example

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad (p\text{-series})$$

The Integral Test (Theorem 9.3, text)

Suppose $a_n = f(n)$ where $f(x)$ is decreasing and positive for $x \geq c$.

$$\int_c^{\infty} f(x) dx \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\int_c^{\infty} f(x) dx \text{ diverges} \Rightarrow \sum a_n \text{ diverges}$$

Example

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad (p\text{-series}) \quad \text{Take } f(x) = \frac{1}{x^p}.$$

The Integral Test (Theorem 9.3, text)

Suppose $a_n = f(n)$ where $f(x)$ is decreasing and positive for $x \geq c$.

$$\int_c^{\infty} f(x) dx \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\int_c^{\infty} f(x) dx \text{ diverges} \Rightarrow \sum a_n \text{ diverges}$$

Example

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad (p\text{-series}) \quad \text{Take } f(x) = \frac{1}{x^p}.$$

$$\text{Recall: } \int_1^{\infty} \frac{1}{x^p} dx \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

The Integral Test (Theorem 9.3, text)

Suppose $a_n = f(n)$ where $f(x)$ is decreasing and positive for $x \geq c$.

$$\int_c^{\infty} f(x) dx \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\int_c^{\infty} f(x) dx \text{ diverges} \Rightarrow \sum a_n \text{ diverges}$$

Example

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad (p\text{-series}) \quad \text{Take } f(x) = \frac{1}{x^p}.$$

$$\int_1^{\infty} \frac{1}{x^p} dx \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p -Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n \text{ converges} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1.
$$\sum a_n \text{ converges} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0$$

Warning ... this does not say

$$\lim_{n \rightarrow \infty} a_n = 0 \Rightarrow \sum a_n \text{ converges}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n \text{ converges} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

Warning ... this does not say

This is a false
assertion. $\lim_{n \rightarrow \infty} a_n = 0 \Rightarrow \sum a_n \text{ converges}$

As this example shows : $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$ but $\sum_{n=1}^{\infty} \frac{1}{n}$ *diverges !*

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$



A necessary, but *not* sufficient
condition for convergence

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n \text{ converges} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

$$\lim_{n \rightarrow +\infty} \frac{n}{n+1} = ?$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n \text{ converges} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

$$\lim_{n \rightarrow +\infty} \frac{n}{n+1} = ?$$

$$\frac{n}{n+1} = \frac{1}{1+n^{-1}}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1.
$$\sum a_n \text{ converges} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0$$

$$\lim_{n \rightarrow +\infty} \frac{n}{n+1} = 1 \neq 0$$

$$\frac{n}{n+1} = \frac{1}{1+n^{-1}}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1.
$$\sum a_n \text{ converges} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0$$

$$\lim_{n \rightarrow +\infty} \frac{n}{n+1} = 1 \neq 0 \Rightarrow \sum_{n=1}^{\infty} \frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots \text{ diverges}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n \text{ converges} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

$$\lim_{n \rightarrow +\infty} \frac{n}{n+1} = 1 \neq 0 \Rightarrow \sum_{n=1}^{\infty} \frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots \text{ diverges}$$

$$\lim_{n \rightarrow +\infty} \frac{n}{n^2 + 1} = ?$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

$$\lim_{n \rightarrow +\infty} \frac{n}{n+1} = 1 \neq 0 \Rightarrow \sum_{n=1}^{\infty} \frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots \text{ diverges}$$

$$\lim_{n \rightarrow +\infty} \frac{n}{n^2 + 1} = ?$$

$$\frac{n}{n^2 + 1} = \frac{n^{-1}}{1 + n^{-2}}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n \text{ converges} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

$$\lim_{n \rightarrow +\infty} \frac{n}{n+1} = 1 \neq 0 \Rightarrow \sum_{n=1}^{\infty} \frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots \text{ diverges}$$

$$\lim_{n \rightarrow +\infty} \frac{n}{n^2 + 1} = 0$$

$$\frac{n}{n^2 + 1} = \frac{n^{-1}}{1 + n^{-2}}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

$$\lim_{n \rightarrow +\infty} \frac{n}{n+1} = 1 \neq 0 \Rightarrow \sum_{n=1}^{\infty} \frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots \text{ diverges}$$

$$\lim_{n \rightarrow +\infty} \frac{n}{n^2 + 1} = 0 \text{ does not imply } \sum_{n=1}^{\infty} \frac{n}{n^2 + 1} \text{ converges}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1.
$$\sum a_n \text{ converges} \Rightarrow \lim_{n \rightarrow \infty} a_n = 0$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$
2. $\sum a_n, \sum b_n$ converge $\Rightarrow \sum (a_n + b_n), \sum ka_n$ converge

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

2. $\sum a_n, \sum b_n$ converge $\Rightarrow \sum (a_n + b_n), \sum ka_n$ converge

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^3}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

2. $\sum a_n, \sum b_n$ converge $\Rightarrow \sum (a_n + b_n), \sum ka_n$ converge

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^3} = \sum_{n=1}^{\infty} \left(2\frac{1}{n^2} + \frac{1}{n^3} \right)$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

2. $\sum a_n, \sum b_n$ converge $\Rightarrow \sum (a_n + b_n), \sum ka_n$ converge

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^3} = \sum_{n=1}^{\infty} \left(2\frac{1}{n^2} + \frac{1}{n^3} \right)$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2}, \sum_{n=1}^{\infty} \frac{1}{n^3} \text{ converge}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

2. $\sum a_n, \sum b_n$ converge $\Rightarrow \sum (a_n + b_n), \sum ka_n$ converge

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^3} = \sum_{n=1}^{\infty} \left(2 \frac{1}{n^2} + \frac{1}{n^3} \right)$$

$$\sum_{n=1}^{\infty} 2 \frac{1}{n^2}, \sum_{n=1}^{\infty} \frac{1}{n^3} \text{ converge}$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

2. $\sum a_n, \sum b_n$ converge $\Rightarrow \sum (a_n + b_n), \sum ka_n$ converge

$$\sum_{n=1}^{\infty} \frac{2n+1}{n^3} = \sum_{n=1}^{\infty} \left(2\frac{1}{n^2} + \frac{1}{n^3} \right) \text{ converges}$$

because $\sum_{n=1}^{\infty} 2\frac{1}{n^2}, \sum_{n=1}^{\infty} \frac{1}{n^3}$ converge

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$
2. $\sum a_n, \sum b_n$ converge $\Rightarrow \sum (a_n + b_n), \sum ka_n$ converge
3. Changing a *finite* number of terms in a series does not affect its convergence or divergence.

$$-1 + 0.5^3 - \pi + 3 + 0.5^4 + 0.5^5 + 0.5^6 + \dots$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$
2. $\sum a_n, \sum b_n$ converge $\Rightarrow \sum (a_n + b_n), \sum ka_n$ converge
3. Changing a *finite* number of terms in a series does not affect its convergence or divergence.

$$0.5^3 + 0.5^4 + 0.5^5 + 0.5^6 + \dots$$

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

2. $\sum a_n, \sum b_n$ converge $\Rightarrow \sum (a_n + b_n), \sum ka_n$ converge

3. Changing a *finite* number of terms in a series does not affect its convergence or divergence.

$$1 + 0.5 + 0.5^2 + 0.5^3 + 0.5^4 + 0.5^5 + 0.5^6 + \dots$$

convergent (a geometric series)

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

2. $\sum a_n, \sum b_n$ converge $\Rightarrow \sum (a_n + b_n), \sum ka_n$ converge

3. Changing a *finite* number of terms in a series does not affect its convergence or divergence.

$$1 + 0.5 + 0.5^2 + 0.5^3 + 0.5^4 + 0.5^5 + 0.5^6 + \dots$$

Changed only a finite number of terms, so original series converges

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

Some Basic Facts To Help with Other Series

1. $\sum a_n$ converges $\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$
2. $\sum a_n, \sum b_n$ converge $\Rightarrow \sum (a_n + b_n), \sum ka_n$ converge
3. Changing a *finite* number of terms in a series does not affect its convergence or divergence.

$$-1 + 0.5^3 - \pi + 3 + 0.5^4 + 0.5^5 + 0.5^6 + \dots$$

converges

Some Basic Series

Geometric Series

$$\sum_{n=1}^{\infty} ar^n \begin{cases} \text{converges if } |r| < 1 \\ \text{diverges if } |r| \geq 1 \end{cases}$$

p-Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } p \leq 1 \end{cases}$$

We can also use these to analyze other series, by *comparison methods*.

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + n^2 + n + 1}}$$

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + n^2 + n + 1}}$$

$$a_n = \frac{1}{\sqrt{n^3 + n^2 + n + 1}} :$$

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + n^2 + n + 1}}$$

$$a_n = \frac{1}{\sqrt{n^3 + n^2 + n + 1}} :$$

behaves like $\frac{1}{\sqrt{n^3}} = \frac{1}{n^{3/2}} = b_n$ for large n and $\sum b_n$ converges

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + n^2 + n + 1}}$$

$$a_n = \frac{1}{\sqrt{n^3 + n^2 + n + 1}} :$$

behaves like $\frac{1}{\sqrt{n^3}} = \frac{1}{n^{3/2}} = b_n$ for large n and $\sum b_n$ converges

so we look for an *upper* bound in order to apply test

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + n^2 + n + 1}}$$

$$a_n = \frac{1}{\sqrt{n^3 + n^2 + n + 1}} \leq \frac{1}{\sqrt{n^3}}$$

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + n^2 + n + 1}}$$

$$a_n = \frac{1}{\sqrt{n^3 + n^2 + n + 1}} \leq \frac{1}{\sqrt{n^3}} = \frac{1}{n^{3/2}} = b_n$$

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + n^2 + n + 1}}$$

$$a_n = \frac{1}{\sqrt{n^3 + n^2 + n + 1}} \leq \frac{1}{\sqrt{n^3}} = \frac{1}{n^{3/2}} = b_n$$

A p -series
with $p = 3/2 > 1$.

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + n^2 + n + 1}} \quad \text{converges}$$

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

$$b_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

$$b_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

behaves like $\frac{n}{\sqrt{n^3}} = \frac{1}{n^{1/2}} = b_n$ for large n and $\sum b_n$ diverges

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

$$b_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

behaves like $\frac{n}{\sqrt{n^3}} = \frac{1}{n^{1/2}} = b_n$ for large n and $\sum b_n$ diverges

so we look for an *lower* bound in order to apply test

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

$$b_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \geq \frac{n}{\sqrt{n^3 + n^3 + n^3 + n^3}} :$$

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

$$b_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \geq \frac{n}{\sqrt{n^3 + n^3 + n^3 + n^3}} = \frac{1}{2} \frac{n}{n^{3/2}}$$

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

$$b_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \geq \frac{n}{\sqrt{n^3 + n^3 + n^3 + n^3}} = \frac{1}{2} \frac{n}{n^{3/2}} = \frac{1}{2} \frac{1}{n^{1/2}} = a_n$$

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

$$b_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \geq \frac{n}{\sqrt{n^3 + n^3 + n^3 + n^3}} = \frac{1}{2} \frac{n}{n^{3/2}} = \frac{1}{2} \frac{1}{n^{1/2}} = a_n$$

A p -series
with $p = 1/2 < 1$.

The Comparison Test (Theorem 9.4, text)

Suppose $0 \leq a_n \leq b_n$ for all n .

$$\sum b_n \text{ converges} \Rightarrow \sum a_n \text{ converges}$$

$$\sum a_n \text{ diverges} \Rightarrow \sum b_n \text{ diverges}$$

(A direct analog of the comparison test for improper integrals.)

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \quad \text{diverges}$$

Limit Comparison Test (Theorem 9.5, text)

Suppose $a_n, b_n > 0$ for all n and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$.

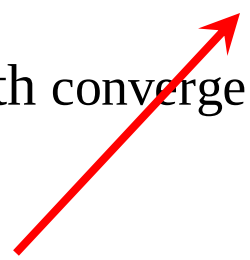
Then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

Limit Comparison Test (Theorem 9.5, text)

Suppose $a_n, b_n > 0$ for all n and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$.

Then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

a_n behaves like cb_n



Limit Comparison Test (Theorem 9.5, text)

Suppose $a_n, b_n > 0$ for all n and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$.

Then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

For large n :
$$a_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \approx \frac{n}{\sqrt{n^3}} = \frac{1}{n^{1/2}} = b_n$$

a divergent p-series

Limit Comparison Test (Theorem 9.5, text)

Suppose $a_n, b_n > 0$ for all n and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$.

Then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

For large n : $a_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \approx \frac{n}{\sqrt{n^3}} = \frac{1}{n^{1/2}} = b_n$

$$\frac{a_n}{b_n} = \frac{\frac{n}{\sqrt{n^3 + n^2 + n + 1}}}{\frac{n}{\sqrt{n^3}}}$$

Limit Comparison Test (Theorem 9.5, text)

Suppose $a_n, b_n > 0$ for all n and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$.

Then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

For large n : $a_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \approx \frac{n}{\sqrt{n^3}} = \frac{1}{n^{1/2}} = b_n$

$$\frac{a_n}{b_n} = \frac{\frac{n}{\sqrt{n^3 + n^2 + n + 1}}}{\frac{n}{\sqrt{n^3}}} = \sqrt{\frac{n^3}{n^3 + n^2 + n + 1}}$$

Limit Comparison Test (Theorem 9.5, text)

Suppose $a_n, b_n > 0$ for all n and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$.

Then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

For large n : $a_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \approx \frac{n}{\sqrt{n^3}} = \frac{1}{n^{1/2}} = b_n$

$$\frac{a_n}{b_n} = \frac{\frac{n}{\sqrt{n^3 + n^2 + n + 1}}}{\frac{n}{\sqrt{n^3}}} = \sqrt{\frac{n^3}{n^3 + n^2 + n + 1}} = \sqrt{\frac{1}{1 + n^{-1} + n^{-2} + n^{-3}}}$$

Limit Comparison Test (Theorem 9.5, text)

Suppose $a_n, b_n > 0$ for all n and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$.

Then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}}$$

For large n :
$$a_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \approx \frac{n}{\sqrt{n^3}} = \frac{1}{n^{1/2}} = b_n$$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 1 = c > 0$$

Limit Comparison Test (Theorem 9.5, text)

Suppose $a_n, b_n > 0$ for all n and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$.

Then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

Example

$$\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \text{ diverges}$$

For large n : $a_n = \frac{n}{\sqrt{n^3 + n^2 + n + 1}} \approx \frac{n}{\sqrt{n^3}} = \frac{1}{n^{1/2}} = b_n$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 1 = c > 0$$