

MATH 250a

Fall Semester 2007

Section 2 (J. M. Cushing)

Thursday, October 11

<http://math.arizona.edu/~cushing/250a.html>

$$\int_1^{+\infty} \frac{1}{x^p} dx = \begin{cases} \text{diverges if } 0 < p \leq 1 \\ \text{converges to } \frac{1}{p-1} \text{ if } 1 < p \end{cases}$$

$$\int_0^1 \frac{1}{x^p} dx = \begin{cases} \text{diverges if } 1 \leq p \\ \text{converges to } \frac{1}{1-p} \text{ if } 0 < p < 1 \end{cases}$$

$$\text{For } a > 0, \quad \int_0^{\infty} e^{-ax} dx = \text{converges to } \frac{1}{a}$$

Comparison Tests for Improper Integrals

$$0 \leq f(x) \leq g(x) \text{ and } \int_a^b g(x) dx \text{ converges} \Rightarrow \int_a^b f(x) dx \text{ converges}$$

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$$0 \leq g(x) \leq f(x)$$

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Area under $f(x)$ contains
area under $g(x)$.

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Area under $f(x)$ contains
area under $g(x)$.

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x}} dx \quad \text{converge or diverge?}$$

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$$\frac{1+e^{-x}}{\sqrt{x}} = f(x)$$

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x}} dx \quad \text{converge or diverge?}$$

$$\frac{1}{\sqrt{x}} \leq \frac{1+e^{-x}}{\sqrt{x}} = f(x)$$

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x}} dx \quad \text{converge or diverge?}$$

$$0 \leq g(x) = \frac{1}{\sqrt{x}} \leq \frac{1+e^{-x}}{\sqrt{x}} = f(x)$$

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$$0 \leq g(x) = \frac{1}{\sqrt{x}} \leq \frac{1+e^{-x}}{\sqrt{x}} = f(x)$$

$$\int_1^{\infty} \frac{1}{\sqrt{x}} dx \text{ diverges}$$

$$p = 1/2 < 1$$

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x}} dx \quad \text{diverges}$$

$$0 \leq g(x) = \frac{1}{\sqrt{x}} \leq \frac{1+e^{-x}}{\sqrt{x}} = f(x)$$

$$\int_1^{\infty} \frac{1}{\sqrt{x}} dx \quad \text{diverges}$$

Comparison Test implies

Example

$$\int_0^{\infty} e^{-x^2} dx$$

Example

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For $x \geq 1$: $x^2 \geq x$

Example

$$\int_0^{\infty} e^{-x^2} dx$$

For $x \geq 1$: $x^2 \geq x \Rightarrow -x^2 \leq -x$

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$$\int_0^{\infty} e^{-x^2} dx$$

$$\text{For } x \geq 1: \quad x^2 \geq x \Rightarrow -x^2 \leq -x \Rightarrow e^{-x^2} \leq e^{-x}$$

Example

$$\int_0^{\infty} e^{-x^2} dx = \int_0^1 e^{-x^2} dx + \int_1^{\infty} e^{-x^2} dx$$

For $x \geq 1$: $x^2 \geq x \Rightarrow -x^2 \leq -x \Rightarrow e^{-x^2} \leq e^{-x}$

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Proper

Improper

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$$0 \leq f(x) = e^{-x^2} \leq e^{-x} = g(x)$$

Example

$$\int_0^{\infty} e^{-x^2} dx = \int_0^1 e^{-x^2} dx + \int_1^{\infty} e^{-x^2} dx$$

$$\text{For } x \geq 1: \quad x^2 \geq x \Rightarrow -x^2 \leq -x \Rightarrow e^{-x^2} \leq e^{-x}$$

$$0 \leq f(x) = e^{-x^2} \leq e^{-x} = g(x)$$

$$\int_1^{\infty} g(x) dx = \int_1^{\infty} e^{-x} dx \quad \text{converges}$$

Example

$$\int_0^{\infty} e^{-x^2} dx = \int_0^1 e^{-x^2} dx + \int_1^{\infty} e^{-x^2} dx$$

For $x \geq 1$: $x^2 \geq x \Rightarrow -x^2 \leq -x \Rightarrow e^{-x^2} \leq e^{-x}$

$$0 \leq f(x) = e^{-x^2} \leq e^{-x} = g(x)$$

$$\int_1^{\infty} g(x) dx = \int_1^{\infty} e^{-x} dx \quad \text{converges}$$

Comparison Test implies convergence

Example

$$\int_0^{\infty} e^{-x^2} dx \quad \text{converges}$$

$$\text{For } x \geq 1: \quad x^2 \geq x \Rightarrow -x^2 \leq -x \Rightarrow e^{-x^2} \leq e^{-x}$$

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$$\int_1^{\infty} g(x) dx = \int_1^{\infty} e^{-x} dx \quad \text{converges}$$

A problem: which test to attempt?

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Example

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If you think this *converges*, you would look for a *larger* test function $g(x)$.

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1 + e^{-x}}{\sqrt{x+1}} dx$$

If you think this *converges*, you would look for a *larger* test function $g(x)$.

If you think this *diverges*, you would look for a *smaller* test function $g(x)$.

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1 + e^{-x}}{\sqrt{x+1}} dx$$

If you think this *converges*, you would look for a *larger* test function $g(x)$.

If you think this *diverges*, you would look for a *smaller* test function $g(x)$.

To decide, we need to make a reasonable conjecture.

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

How does $\frac{1+e^{-x}}{\sqrt{x+1}}$ behave for large x ?

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For large x : $\frac{1+e^{-x}}{\sqrt{x+1}}$

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very small

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Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

How does $\frac{1+e^{-x}}{\sqrt{x+1}}$ behave for large x ?

For large x : $\frac{1+\boxed{e^{-x}}}{\sqrt{x+1}} \approx \frac{1}{\sqrt{x+1}}$

very small

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

How does $\frac{1+e^{-x}}{\sqrt{x+1}}$ behave for large x ?

For large x : $\frac{1+e^{-x}}{\sqrt{x+1}} \approx \frac{1}{\sqrt{x+1}} =$

negligible

A problem: which test to attempt?

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How does $\frac{1+e^{-x}}{\sqrt{x+1}}$ behave for large x ?

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A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

How does $\frac{1+e^{-x}}{\sqrt{x+1}}$ behave for large x ?

$$\text{For large } x: \frac{1+e^{-x}}{\sqrt{x+1}} \approx \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}}$$

has a *divergent*
integral

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

This suggests (be *does not prove*) divergence.

For large x : $\frac{1+e^{-x}}{\sqrt{x+1}} \approx \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}}$

has a *divergent*
integral

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

This suggests (be *does not prove*) divergence.
Therefore, we search for a *smaller* test function.

For large x : $\frac{1+e^{-x}}{\sqrt{x+1}} \approx \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}}$

has a *divergent*
integral

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

$$\frac{1+e^{-x}}{\sqrt{x+1}}$$

How to make this
smaller for large x ?

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

$$\frac{1+e^{-x}}{\sqrt{x+1}} \geq \frac{1}{\sqrt{x+1}}$$

How to make this
smaller for large x ?

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

$$\frac{1+e^{-x}}{\sqrt{x+1}} \geq \frac{1}{\sqrt{x+x}}$$

How to make this
smaller for large x ?

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

$$\frac{1+e^{-x}}{\sqrt{x+1}} \geq \frac{1}{\sqrt{x+x}} = \frac{1}{\sqrt{2x}}$$

How to make this
smaller for large x ?

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

$$\frac{1+e^{-x}}{\sqrt{x+1}} \geq \frac{1}{\sqrt{x+x}} = \frac{1}{\sqrt{2x}} = \frac{1}{\sqrt{2}} \frac{1}{x^{1/2}}$$

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$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx$$

$$\frac{1+e^{-x}}{\sqrt{x+1}} \geq \frac{1}{\sqrt{x+x}} = \frac{1}{\sqrt{2x}} = \frac{1}{\sqrt{2}} \frac{1}{x^{1/2}} = g(x) \geq 0$$

$g(x)$ has a *divergent* integral,

because $p = 1/2 < 1$

A problem: which test to attempt?

Example

$$\int_1^{\infty} \frac{1+e^{-x}}{\sqrt{x+1}} dx \quad \text{divergent}$$

$$\frac{1+e^{-x}}{\sqrt{x+1}} \geq \frac{1}{\sqrt{x+x}} = \frac{1}{\sqrt{2x}} = \frac{1}{\sqrt{2}} \frac{1}{x^{1/2}} = g(x) \geq 0$$

Comparison Test implies

A problem: which test to attempt?

Example

$$\int_0^1 \frac{1 + e^{-x}}{\sqrt{x^2 + x}} dx$$

A problem: which test to attempt?

Example

$$\int_0^1 \frac{1+e^{-x}}{\sqrt{x^2+x}} dx$$

How does $\frac{1+e^{-x}}{\sqrt{x^2+x}}$ behave for x *small* (an positive)?

A problem: which test to attempt?

Example

$$\int_0^1 \frac{1+e^{-x}}{\sqrt{x^2+x}} dx$$

For x small (an positive): $\frac{1+e^{-x}}{\sqrt{x^2+x}} \approx \frac{1+1}{\sqrt{x^2+x}}$

A problem: which test to attempt?

Example

$$\int_0^1 \frac{1+e^{-x}}{\sqrt{x^2+x}} dx$$

For x small (an positive): $\frac{1+e^{-x}}{\sqrt{x^2+x}} \approx \frac{2}{\sqrt{x^2+x}}$

negligible

A problem: which test to attempt?

Example

$$\int_0^1 \frac{1 + e^{-x}}{\sqrt{x^2 + x}} dx$$

For x small (an positive): $\frac{1 + e^{-x}}{\sqrt{x^2 + x}} \approx \frac{2}{\sqrt{x}}$

negligible

A problem: which test to attempt?

Example

$$\int_0^1 \frac{1+e^{-x}}{\sqrt{x^2+x}} dx$$

For x small (an positive): $\frac{1+e^{-x}}{\sqrt{x^2+x}} \approx 2 \frac{1}{\sqrt{x}}$



has a convergent integral
because $p = 1/2 < 1$.

A problem: which test to attempt?

Example

$$\int_0^1 \frac{1 + e^{-x}}{\sqrt{x^2 + x}} dx$$

We conjecture *convergence*
and therefore look for a *larger* test function

A problem: which test to attempt?

Example

$$\int_0^1 \frac{1+e^{-x}}{\sqrt{x^2+x}} dx$$

$$0 \leq f(x) = \frac{1+e^{-x}}{\sqrt{x^2+x}}$$

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Example

$$\int_0^1 \frac{1+e^{-x}}{\sqrt{x^2+x}} dx$$

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Has a *convergent* integral.

A problem: which test to attempt?

Example

$$\int_0^1 \frac{1+e^{-x}}{\sqrt{x^2+x}} dx \quad \text{convergent}$$

$$0 \leq f(x) = \frac{1+e^{-x}}{\sqrt{x^2+x}} \leq \frac{1+1}{\sqrt{x}} = 2 \frac{1}{\sqrt{x}} = g(x)$$

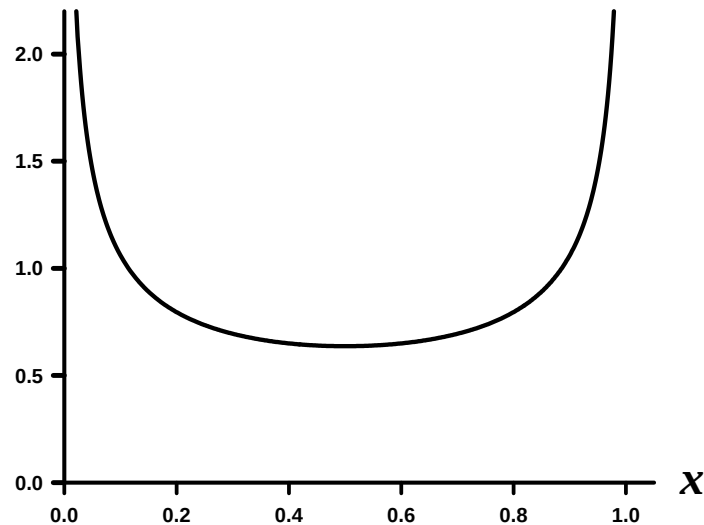
Has a *convergent* integral.

Comparison Test implies

Example from Probability Theory

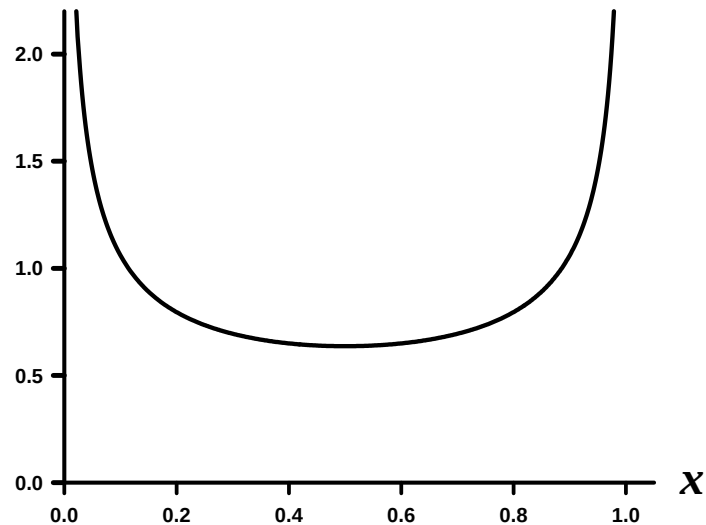
$$f(x) = \frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}}$$

This is an example of a *beta probability distribution* for a random variable x varying on the finite interval $0 < x < 1$



Example from Probability Theory

$$\int_a^b \frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}} dx = \begin{cases} \text{probability random variable } x \\ \text{lies between } a \text{ and } b \end{cases}$$



Example from Probability Theory

$$\int_0^{1/2} \frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}} dx = \begin{cases} \text{probability random variable } x \\ \text{lies between } 0 \text{ and } 1/2 \end{cases}$$

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improper

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Example from Probability Theory

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convergent or divergent ?

For x small: $\frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}} = \frac{1}{\pi} \frac{1}{\sqrt{x-x^2}} \approx \frac{1}{\pi} \frac{1}{\sqrt{x}}$

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has convergent integral

Example from Probability Theory

$$\int_0^{1/2} \frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}} dx$$

We conjecture convergence.

To use Comparison Test we look for *larger* test function.

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To use Comparison Test we look for *larger* test function.

For x between 0 and $1/2$

$$0 \leq f(x) = \frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}}$$

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has a convergent integral,
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Example from Probability Theory

$$\int_0^{1/2} \frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}} dx \quad \text{converges}$$

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The Comparison Test implies

Example from Probability Theory

An exercise: show that

$$\int_0^{1/2} \frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}} dx = \frac{1}{2}$$

Applications of Definite Integrals

Objects in Motion & the Linear Friction Law

Applications of Definite Integrals

Objects in Motion & the Linear Friction Law



Applications of Definite Integrals

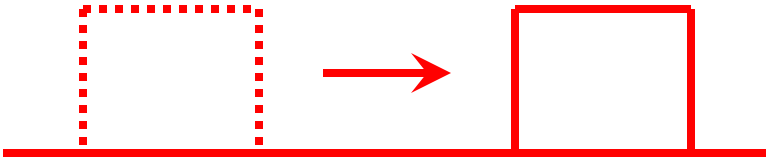
Objects in Motion & the Linear Friction Law



Push or hit
to set into motion

Applications of Definite Integrals

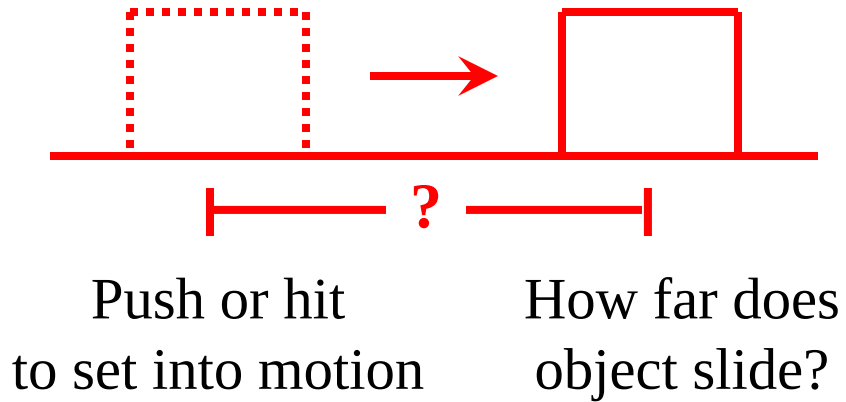
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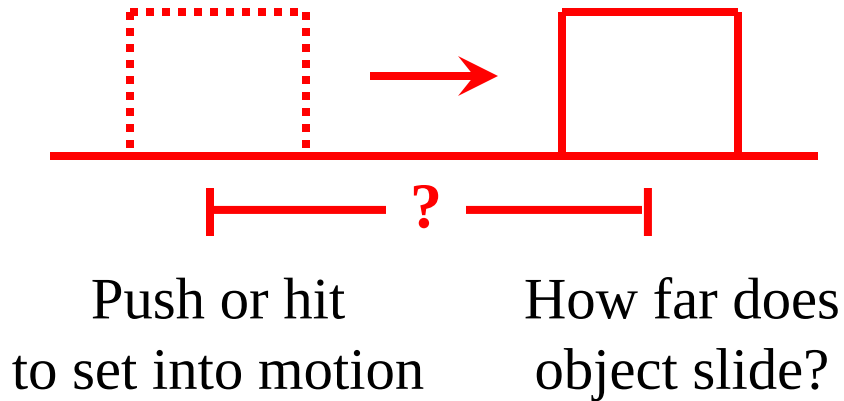
Applications of Definite Integrals

Objects in Motion & the Linear Friction Law



Applications of Definite Integrals

Objects in Motion & the Linear Friction Law



How does distance depend on

Weight of the object?
Magnitude of initial push?
Properties of the surface?

Applications of Definite Integrals

Objects in Motion & the Linear Friction Law

Linear Friction Law

(force of friction is proportional to object's velocity)

Applications of Definite Integrals

Objects in Motion & the Linear Friction Law

Linear Friction Law

(force of friction is proportional to object's velocity)

implies

$$v(t) = v_0 e^{-ct/m}$$

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Objects in Motion & the Linear Friction Law

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object's velocity
at time t



Applications of Definite Integrals

Objects in Motion & the Linear Friction Law

Linear Friction Law

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object's velocity
at time t

object's initial
velocity
(at time $t = 0$)

Applications of Definite Integrals

Objects in Motion & the Linear Friction Law

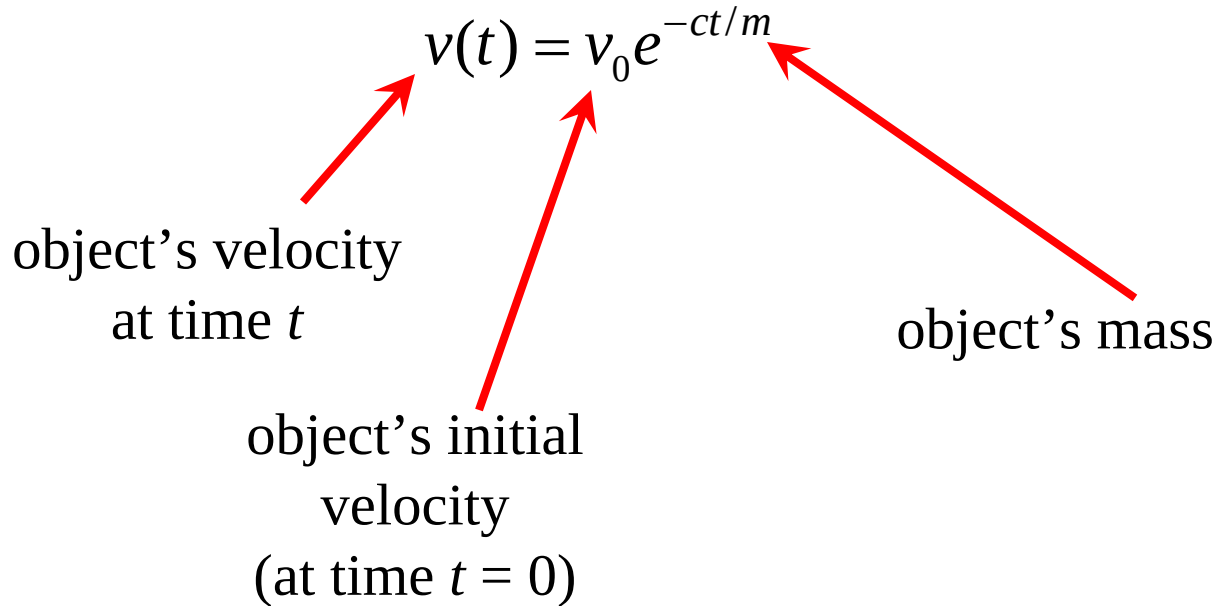
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object's velocity
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object's initial
velocity
(at time $t = 0$)

object's mass

Applications of Definite Integrals

Objects in Motion & the Linear Friction Law

Linear Friction Law

(force of friction is proportional to object's velocity)

implies

$$v(t) = v_0 e^{-ct/m}$$

object's velocity
at time t

object's initial
velocity
(at time $t = 0$)

coefficient of
friction

object's mass

Applications of Definite Integrals

Objects in Motion & the Linear Friction Law

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Applications of Definite Integrals

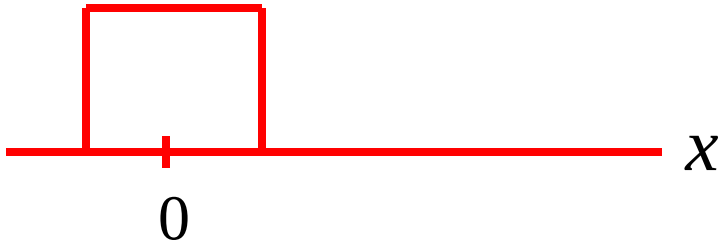
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Applications of Definite Integrals

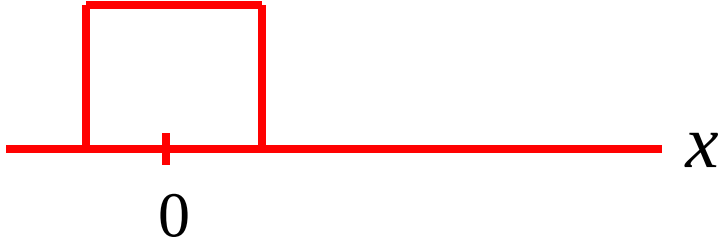
Objects in Motion & the Linear Friction Law

Linear Friction Law

(force of friction is proportional to object's velocity)

implies

$$v(t) = v_0 e^{-ct/m}$$



$x(t)$ = distance traveled at time t

Applications of Definite Integrals

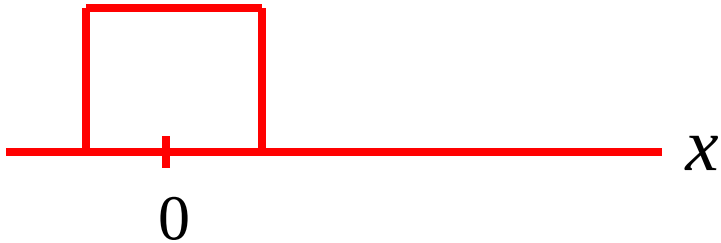
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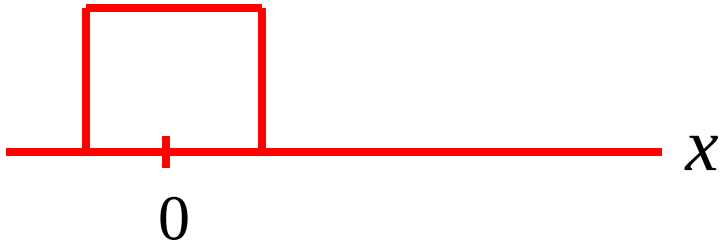
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Fundamental Theorem

$$\int_0^T v(t) dt = \int_0^T \frac{dx(t)}{dt} dt = x(T) - x(0)$$



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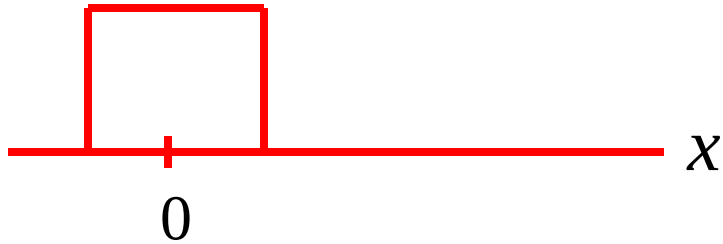
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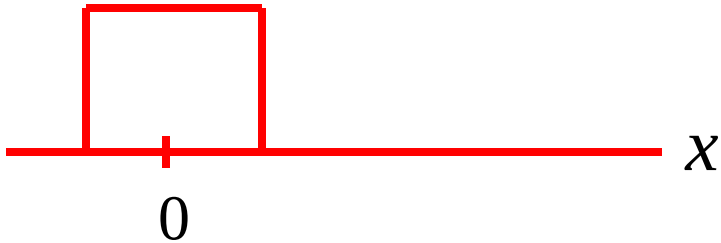
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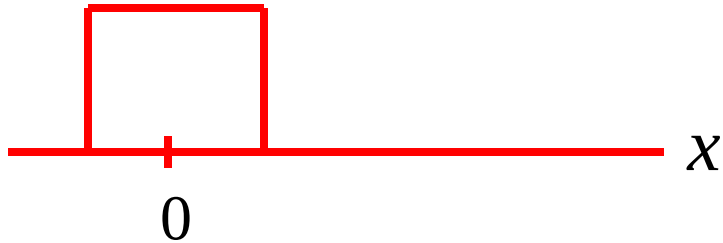
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$$= \left(-v_0 \frac{m}{c} e^{-ct/m} \right)_{t=0}^{t=T}$$

Applications of Definite Integrals

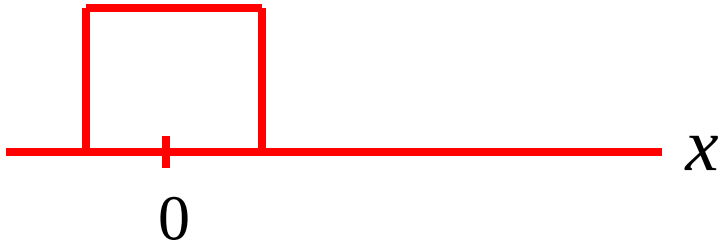
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Applications of Definite Integrals

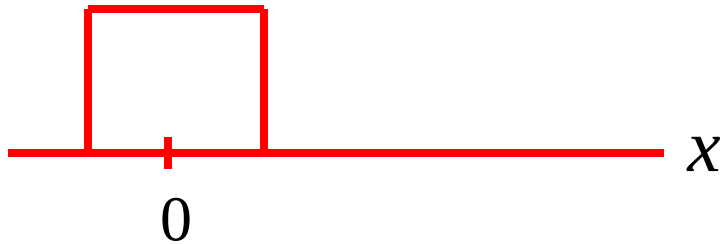
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What happens in the long run?

$$\frac{dx(t)}{dt} = v(t) \quad \text{and} \quad x(0) = 0$$

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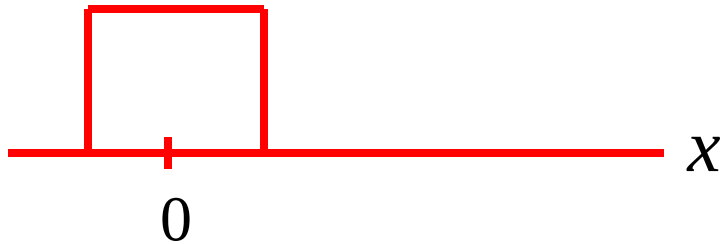
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$$\lim_{T \rightarrow +\infty} x(T) = \lim_{T \rightarrow +\infty} \left(-v_0 \frac{m}{c} e^{-cT/m} + v_0 \frac{m}{c} \right)$$

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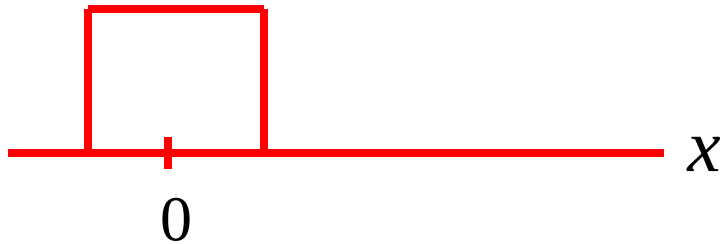
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Object comes to rest at position $v_0 \frac{m}{c}$

$$\frac{dx(t)}{dt} = v(t) \quad \text{and} \quad x(0) = 0$$

Applications of Definite Integrals

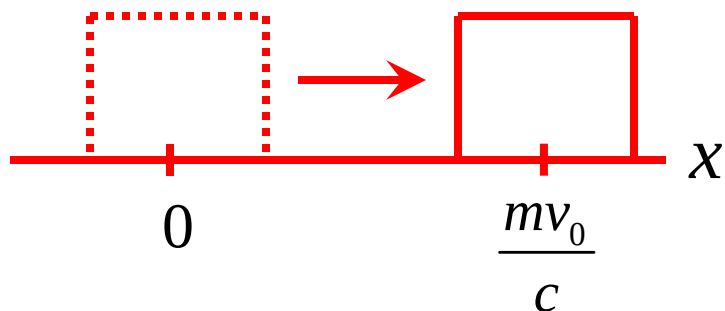
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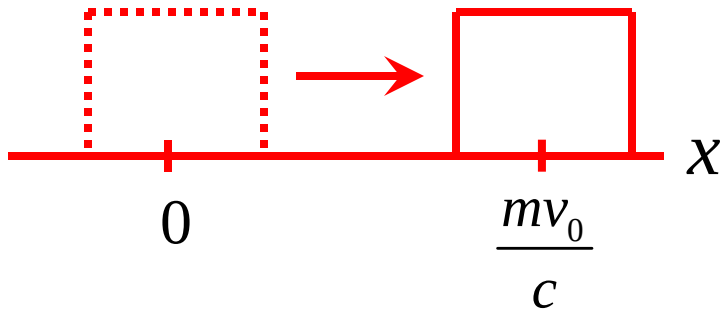
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How does distance depend on

Weight of the object?

Magnitude of initial push?

Properties of the surface?

Applications of Definite Integrals

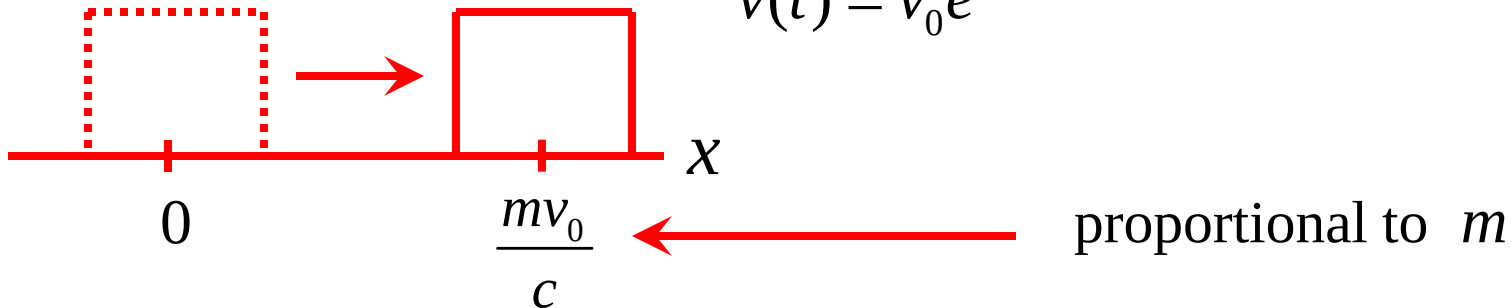
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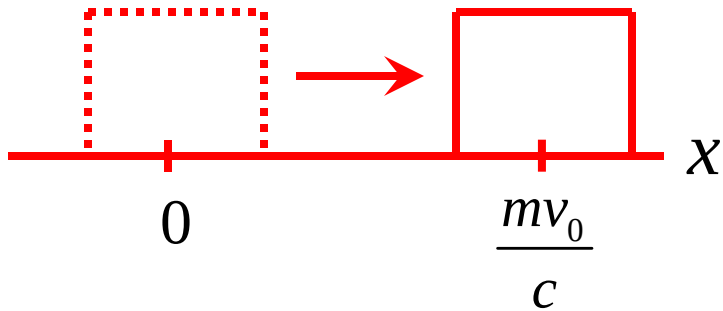
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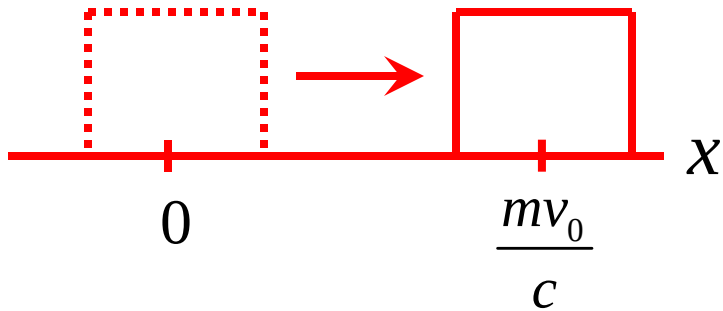
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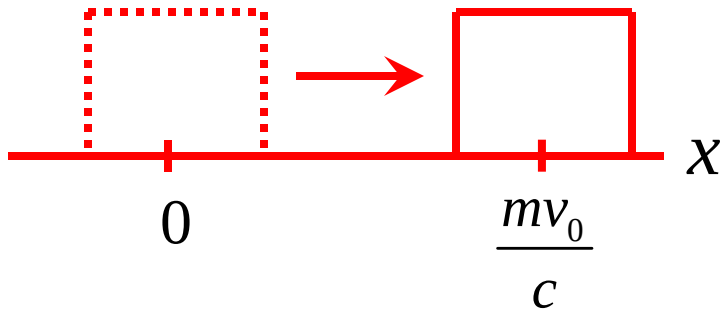
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inversely proportional to
the coefficient of friction c