

MATH 250a

Fall Semester 2007

Section 2 (J. M. Cushing)

Tuesday, October 9

<http://math.arizona.edu/~cushing/250a.html>

Chapter 7.7: Improper Integrals

$$\int_a^b f(x) dx,$$

$$f(x) \geq 0 \text{ for } a < x < b$$

- A limit of integration (a or b) is infinite
- The integrand $f(x)$ becomes infinite

- A limit of integration is infinite

If $\lim_{b \rightarrow +\infty} \int_a^b f(x) dx$ is a finite number,

we say $\int_a^\infty f(x) dx$ *converges*

- A limit of integration is infinite

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$$\int_a^\infty f(x) dx = \lim_{b \rightarrow +\infty} \int_a^b f(x) dx.$$

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Otherwise we say $\int_a^\infty f(x) dx$ *diverges*.

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$$\int_a^\infty f(x) dx = \lim_{b \rightarrow +\infty} \int_a^b f(x) dx.$$

Otherwise we say $\int_a^\infty f(x) dx$ *diverges*.

We define

$$\int_{-\infty}^a f(x) dx = \lim_{b \rightarrow -\infty} \int_b^a f(x) dx.$$

in an analogous manner.

- A limit of integration is infinite

Note: there are two basic questions:

1. Does an improper integral converge or diverge ?
2. If it converges, to what does it converge ?

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{x} dx$$

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{x} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x} dx$$

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{x} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x} dx = \lim_{b \rightarrow +\infty} (\ln |x|) \Big|_{x=1}^{x=b}$$

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{x} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x} dx = \lim_{b \rightarrow +\infty} (\ln |x|) \Big|_{x=1}^{x=b} = \lim_{b \rightarrow +\infty} (\ln b)$$

Not finite !

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{x} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x} dx = \lim_{b \rightarrow +\infty} (\ln |x|) \Big|_{x=1}^{x=b} = \lim_{b \rightarrow +\infty} (\ln b)$$

Divergent

Not finite !

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{x^2} dx$$

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Examples

$$\int_1^{+\infty} \frac{1}{x^2} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^2} dx$$

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Examples

$$\int_1^{+\infty} \frac{1}{x^2} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^2} dx = \lim_{b \rightarrow +\infty} \left(-\frac{1}{x} \right) \Big|_{x=1}^b$$

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{x^2} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^2} dx = \lim_{b \rightarrow +\infty} \left(-\frac{1}{x} \right) \Big|_{x=1}^b = \lim_{b \rightarrow +\infty} \left(1 - \frac{1}{b} \right) = 1$$

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{x^2} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^2} dx = \lim_{b \rightarrow +\infty} \left(-\frac{1}{x} \right) \Big|_{x=1}^b = \lim_{b \rightarrow +\infty} \left(1 - \frac{1}{b} \right) = 1$$

Finite !

- A limit of integration is infinite

Examples

$$\boxed{\int_1^{+\infty} \frac{1}{x^2} dx} = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^2} dx = \lim_{b \rightarrow +\infty} \left(-\frac{1}{x} \right) \Big|_{x=1}^b = \lim_{b \rightarrow +\infty} \left(1 - \frac{1}{b} \right) = 1$$

Convergent

Finite !

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{\sqrt{x}} dx$$

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{\sqrt{x}} dx$$

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow +\infty} \left(2\sqrt{x} \right) \Big|_{x=1}^b$$

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow +\infty} \left(2\sqrt{x} \right) \Big|_{x=1}^b = \lim_{b \rightarrow +\infty} (2\sqrt{b} - 2)$$

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow +\infty} \left(2\sqrt{x} \right) \Big|_{x=1}^b = \lim_{b \rightarrow +\infty} (2\sqrt{b} - 2)$$

Not finite !

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{\sqrt{x}} dx = \lim_{b \rightarrow +\infty} \left(2\sqrt{x} \right) \Big|_{x=1}^b = \lim_{b \rightarrow +\infty} (2\sqrt{b} - 2)$$

Divergent

Not finite !

- A limit of integration is infinite

Examples

For $p > 0$, but $p \neq 1$

$$\int_1^{+\infty} \frac{1}{x^p} dx :$$

- A limit of integration is infinite

Examples

For $p > 0$, but $p \neq 1$

$$\int_1^{+\infty} \frac{1}{x^p} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^p} dx$$

- A limit of integration is infinite

Examples

For $p > 0$, but $p \neq 1$

$$\int_1^{+\infty} \frac{1}{x^p} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^p} dx = \lim_{b \rightarrow +\infty} \left(\frac{x^{1-p}}{1-p} \right) \bigg|_{x=1}^b$$

- A limit of integration is infinite

Examples

For $p > 0$, but $p \neq 1$

$$\int_1^{+\infty} \frac{1}{x^p} dx = \lim_{b \rightarrow +\infty} \int_1^b \frac{1}{x^p} dx = \lim_{b \rightarrow +\infty} \left(\frac{x^{1-p}}{1-p} \right) \bigg|_{x=1}^b = \lim_{b \rightarrow +\infty} \left(\frac{b^{1-p}}{1-p} - \frac{1}{1-p} \right)$$

- A limit of integration is infinite

Examples

For $p > 0$, but $p \neq 1$

$$\int_1^{+\infty} \frac{1}{x^p} dx = \lim_{b \rightarrow +\infty} \left(\frac{b^{1-p}}{1-p} - \frac{1}{1-p} \right) = \begin{cases} \text{infinite if } 0 < p < 1 \\ \text{finite if } 1 < p \end{cases}$$

- A limit of integration is infinite

Examples

For $p > 0$, but $p \neq 1$

$$\int_1^{+\infty} \frac{1}{x^p} dx = \lim_{b \rightarrow +\infty} \left(\frac{b^{1-p}}{1-p} - \frac{1}{1-p} \right) = \begin{cases} \text{infinite if } 0 < p < 1 \\ \text{finite if } 1 < p \end{cases}$$



and equals

$$-\frac{1}{1-p} = \frac{1}{p-1}$$

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{x^p} dx = \begin{cases} \text{diverges if } 0 < p < 1 \\ \text{converges to } \frac{1}{p-1} \text{ if } 1 < p \end{cases}$$

- A limit of integration is infinite

Examples

$$\int_1^{+\infty} \frac{1}{x^p} dx = \begin{cases} \text{diverges if } 0 < p \leq 1 \\ \text{converges to } \frac{1}{p-1} \text{ if } 1 < p \end{cases}$$

- A limit of integration is infinite

Examples

for $a > 0$

$$\int_0^{\infty} e^{-ax} dx$$

- A limit of integration is infinite

Examples

for $a > 0$

$$\int_0^{\infty} e^{-ax} dx = \lim_{b \rightarrow +\infty} \int_0^b e^{-ax} dx =$$

- A limit of integration is infinite

Examples

for $a > 0$

$$\int_0^{\infty} e^{-ax} dx = \lim_{b \rightarrow +\infty} \int_0^b e^{-ax} dx = \lim_{b \rightarrow +\infty} \left(-\frac{1}{a} e^{-ax} \right)_{x=0}^{x=b}$$

- A limit of integration is infinite

Examples

for $a > 0$

$$\begin{aligned}\int_0^{\infty} e^{-ax} dx &= \lim_{b \rightarrow +\infty} \int_0^b e^{-ax} dx = \lim_{b \rightarrow +\infty} \left(-\frac{1}{a} e^{-ax} \right)_{x=0}^{x=b} \\ &= \lim_{b \rightarrow +\infty} \left(-\frac{1}{a} e^{-ab} - \left(-\frac{1}{a} \right) \right)\end{aligned}$$

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Examples

for $a > 0$

$$\begin{aligned}\int_0^{\infty} e^{-ax} dx &= \lim_{b \rightarrow +\infty} \int_0^b e^{-ax} dx = \lim_{b \rightarrow +\infty} \left(-\frac{1}{a} e^{-ax} \right)_{x=0}^{x=b} \\ &= \lim_{b \rightarrow +\infty} \left(-\frac{1}{a} e^{-ab} - \left(-\frac{1}{a} \right) \right) \\ &= \frac{1}{a}\end{aligned}$$

Convergent !

- A limit of integration is infinite

Examples

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$$

- A limit of integration is infinite

Examples

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx = \int_{-\infty}^0 \frac{1}{1+x^2} dx + \int_0^{\infty} \frac{1}{1+x^2} dx$$

This is *by definition*.

- A limit of integration is infinite

Examples

$$\begin{aligned}\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx &= \int_{-\infty}^0 \frac{1}{1+x^2} dx + \int_0^{\infty} \frac{1}{1+x^2} dx \\ &= \lim_{b \rightarrow -\infty} \int_b^0 \frac{1}{1+x^2} dx + \lim_{b \rightarrow +\infty} \int_0^b \frac{1}{1+x^2} dx\end{aligned}$$

- A limit of integration is infinite

Examples

$$\begin{aligned}\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx &= \int_{-\infty}^0 \frac{1}{1+x^2} dx + \int_0^{\infty} \frac{1}{1+x^2} dx \\ &= \lim_{b \rightarrow -\infty} \int_b^0 \frac{1}{1+x^2} dx + \lim_{b \rightarrow +\infty} \int_0^b \frac{1}{1+x^2} dx \\ &= \lim_{b \rightarrow -\infty} (\arctan 0 - \arctan b)\end{aligned}$$

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Examples

$$\begin{aligned}\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx &= \int_{-\infty}^0 \frac{1}{1+x^2} dx + \int_0^{\infty} \frac{1}{1+x^2} dx \\&= \lim_{b \rightarrow -\infty} \int_b^0 \frac{1}{1+x^2} dx + \lim_{b \rightarrow +\infty} \int_0^b \frac{1}{1+x^2} dx \\&= \lim_{b \rightarrow -\infty} (\arctan 0 - \arctan b) + \lim_{b \rightarrow +\infty} (\arctan b - \arctan 0)\end{aligned}$$

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$$\begin{aligned}\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx &= \int_{-\infty}^0 \frac{1}{1+x^2} dx + \int_0^{\infty} \frac{1}{1+x^2} dx \\&= \lim_{b \rightarrow -\infty} \int_b^0 \frac{1}{1+x^2} dx + \lim_{b \rightarrow +\infty} \int_0^b \frac{1}{1+x^2} dx \\&= \lim_{b \rightarrow -\infty} (\arctan 0 - \arctan b) + \lim_{b \rightarrow +\infty} (\arctan b - \arctan 0) \\&= -\lim_{b \rightarrow -\infty} (\arctan b) + \lim_{b \rightarrow +\infty} (\arctan b)\end{aligned}$$

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$$\begin{aligned}\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx &= \int_{-\infty}^0 \frac{1}{1+x^2} dx + \int_0^{\infty} \frac{1}{1+x^2} dx \\&= \lim_{b \rightarrow -\infty} \int_b^0 \frac{1}{1+x^2} dx + \lim_{b \rightarrow +\infty} \int_0^b \frac{1}{1+x^2} dx \\&= \lim_{b \rightarrow -\infty} (\arctan 0 - \arctan b) + \lim_{b \rightarrow +\infty} (\arctan b - \arctan 0) \\&= -\lim_{b \rightarrow -\infty} (\arctan b) + \lim_{b \rightarrow +\infty} (\arctan b) \\&= -\left(-\frac{\pi}{2}\right) + \frac{\pi}{2}\end{aligned}$$

- A limit of integration is infinite

Examples

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx = \pi$$

convergent

- The integrand becomes infinite

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Suppose $f(x)$ is positive on $a \leq x < b$ and tends to infinity as $x \rightarrow b -$.

- The integrand becomes infinite

Suppose $f(x)$ is positive on $a \leq x < b$ and tends to infinity as $x \rightarrow b -$.

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If $f(x)$ tends to infinity as $x \rightarrow a +$ we define

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in an analogous manner.

- The integrand becomes infinite

Examples

$$\int_0^1 \frac{1}{x} dx$$

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Examples

$$\int_0^1 \frac{1}{x} dx = \lim_{c \rightarrow 0^+} \int_c^1 \frac{1}{x} dx$$

- The integrand becomes infinite

Examples

$$\int_0^1 \frac{1}{x} dx = \lim_{c \rightarrow 0^+} \int_c^1 \frac{1}{x} dx = \lim_{c \rightarrow 0^+} \left(\ln |x| \right)_{x=c}^{x=1}$$

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Examples

$$\int_0^1 \frac{1}{x} dx = \lim_{c \rightarrow 0^+} \int_c^1 \frac{1}{x} dx = \lim_{c \rightarrow 0^+} \left(\ln |x| \right)_{x=c}^{x=1} = \lim_{c \rightarrow 0^+} (0 - \ln c)$$

- The integrand becomes infinite

Examples

$$\int_0^1 \frac{1}{x} dx = \lim_{c \rightarrow 0^+} \int_c^1 \frac{1}{x} dx = \lim_{c \rightarrow 0^+} (\ln |x|)_{x=c}^{x=1} = \lim_{c \rightarrow 0^+} (0 - \ln c)$$

not finite

Divergent

- The integrand becomes infinite

Examples

For $p > 0$, $n \neq 1$

$$\int_0^1 \frac{1}{x^p} dx$$

- The integrand becomes infinite

Examples

For $p > 0, p \neq 1$

$$\int_0^1 \frac{1}{x^p} dx = \lim_{c \rightarrow 0^+} \int_c^1 \frac{1}{x^p} dx$$

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Examples

For $p > 0, p \neq 1$

$$\int_0^1 \frac{1}{x^p} dx = \lim_{c \rightarrow 0^+} \int_c^1 \frac{1}{x^p} dx = \lim_{c \rightarrow 0^+} \left(\frac{x^{1-p}}{1-p} \right)_{x=c}^{x=1}$$

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Examples

For $p > 0, p \neq 1$

$$\int_0^1 \frac{1}{x^p} dx = \lim_{c \rightarrow 0^+} \int_c^1 \frac{1}{x^p} dx = \lim_{c \rightarrow 0^+} \left(\frac{x^{1-p}}{1-p} \right)_{x=c}^{x=1} = \lim_{c \rightarrow 0^+} \left(\frac{1}{1-p} - \frac{c^{1-p}}{1-p} \right)$$

- The integrand becomes infinite

Examples

$$\int_0^1 \frac{1}{x^p} dx = \lim_{c \rightarrow 0^+} \left(\frac{1}{1-p} - \frac{c^{1-p}}{1-p} \right)$$

- The integrand becomes infinite

Examples

$$\int_0^1 \frac{1}{x^p} dx = \lim_{c \rightarrow 0^+} \left(\frac{1}{1-p} - \frac{c^{1-p}}{1-p} \right) = \begin{cases} \text{not finite if } 1 < p \end{cases}$$

- The integrand becomes infinite

Examples

$$\int_0^1 \frac{1}{x^p} dx = \lim_{c \rightarrow 0^+} \left(\frac{1}{1-p} - \frac{c^{1-p}}{1-p} \right) = \begin{cases} \text{not finite if } 1 < p \\ \frac{1}{1-p} & \text{if } 0 < p < 1 \end{cases}$$

- The integrand becomes infinite

Examples

$$\int_0^1 \frac{1}{x^p} dx = \lim_{c \rightarrow 0^+} \left(\frac{1}{1-p} - \frac{c^{1-p}}{1-p} \right) = \begin{cases} \text{divergent if } 1 < p \\ \frac{1}{1-p} \text{ if } 0 < p < 1 \end{cases}$$

- The integrand becomes infinite

Examples

$$\int_0^1 \frac{1}{x^p} dx = \lim_{c \rightarrow 0^+} \left(\frac{1}{1-p} - \frac{c^{1-p}}{1-p} \right) = \begin{cases} \text{divergent if } 1 \leq p \\ \frac{1}{1-p} \text{ if } 0 < p < 1 \end{cases}$$

- The integrand becomes infinite

Examples

$$\int_5^{10} \frac{1}{(2x-10)^{2/3}} dx$$

- The integrand becomes infinite

Examples

$$\int_5^{10} \frac{1}{(2x-10)^{2/3}} dx = \lim_{c \rightarrow 5^+} \int_c^{10} \frac{1}{(2x-10)^{2/3}} dx$$

- The integrand becomes infinite

Examples

$$\int_5^{10} \frac{1}{(2x-10)^{2/3}} dx = \lim_{c \rightarrow 5^+} \int_c^{10} \frac{1}{(2x-10)^{2/3}} dx$$

Use substitution $w = 2x - 10$.

- The integrand becomes infinite

Examples

$$\int_5^{10} \frac{1}{(2x-10)^{2/3}} dx = \lim_{c \rightarrow 5^+} \int_c^{10} \frac{1}{(2x-10)^{2/3}} dx = \lim_{c \rightarrow 5^+} \frac{3}{2} (2x-10)^{1/3} \bigg|_{x=c}^{x=10}$$

- The integrand becomes infinite

Examples

$$\begin{aligned}\int_5^{10} \frac{1}{(2x-10)^{2/3}} dx &= \lim_{c \rightarrow 5^+} \int_c^{10} \frac{1}{(2x-10)^{2/3}} dx = \lim_{c \rightarrow 5^+} \frac{3}{2} (2x-10)^{1/3} \bigg|_{x=c}^{x=10} \\ &= \lim_{c \rightarrow 5^+} \left(\frac{3}{2} (20-10)^{1/3} - \frac{3}{2} (2c-10)^{1/3} \right)\end{aligned}$$

- The integrand becomes infinite

Examples

$$\begin{aligned}\int_5^{10} \frac{1}{(2x-10)^{2/3}} dx &= \lim_{c \rightarrow 5^+} \int_c^{10} \frac{1}{(2x-10)^{2/3}} dx = \lim_{c \rightarrow 5^+} \frac{3}{2} (2x-10)^{1/3} \bigg|_{x=c}^{x=10} \\ &= \lim_{c \rightarrow 5^+} \left(\frac{3}{2} (20-10)^{1/3} - \frac{3}{2} (2c-10)^{1/3} \right) \\ &= \frac{3}{2} 10^{1/3} - \frac{3}{2} (0)^{1/3}\end{aligned}$$

- The integrand becomes infinite

Examples

$$\int_5^{10} \frac{1}{(2x-10)^{2/3}} dx = \frac{3}{2} 10^{1/3} \quad \text{converges !}$$

For improper integrals here are two basic questions:

1. Does an improper integral converge or diverge ?
2. If it converges, to what does it converge ?

For improper integrals here are two basic questions:

1. Does an improper integral converge or diverge ?
2. If it converges, to what does it converge ?

In examples so far we've answered both questions.

Often one can answer #1 even when one cannot answer #2.

Does an improper integral converge or diverge ?

A powerful method for answering this question is the *Comparison Test*.

$$0 \leq f(x) \leq g(x)$$

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Area under $g(x)$ contains
area under $f(x)$.

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A powerful method for answering this question is the *Comparison Test*.

$$0 \leq f(x) \leq g(x) \text{ and } \int_a^b g(x) dx \text{ converges} \Rightarrow \int_a^b f(x) dx \text{ converges}$$

Area under $g(x)$ contains
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Does an improper integral converge or diverge ?

A powerful method for answering this question is the *Comparison Test*.

$$0 \leq f(x) \leq g(x) \text{ and } \int_a^b g(x) dx \text{ converges} \Rightarrow \int_a^b f(x) dx \text{ converges}$$

In applications, $\int_a^b f(x) dx$ is under study and $f(x)$ is given.

Goal is to find a test function $g(x)$.

Example

$$\int_1^{\infty} \frac{1}{x^2 + x + e^x} dx \quad \text{converge or diverge?}$$

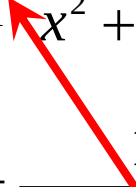
Example

$$\int_1^{\infty} \frac{1}{x^2 + x + e^x} dx \quad \text{converge or diverge?}$$

$$0 \leq f(x) = \frac{1}{x^2 + x + e^x}$$

Example

$$\int_1^{\infty} \frac{1}{x^2 + x + e^x} dx \quad \text{converge or diverge?}$$

$$0 \leq f(x) = \frac{1}{x^2 + \textcircled{x} + e^x}$$


x is positive

Example

$$\int_1^{\infty} \frac{1}{x^2 + x + e^x} dx \quad \text{converge or diverge?}$$

$$0 \leq f(x) = \frac{1}{x^2 + x + e^x} \leq \frac{1}{x^2}$$

Example

$$\int_1^{\infty} \frac{1}{x^2 + x + e^x} dx \quad \text{converge or diverge?}$$

$$0 \leq f(x) = \frac{1}{x^2 + x + e^x} \leq \frac{1}{x^2} = g(x)$$

Example

$$\int_1^{\infty} \frac{1}{x^2 + x + e^x} dx \quad \text{converge or diverge?}$$

$$0 \leq f(x) = \frac{1}{x^2 + x + e^x} \leq \frac{1}{x^2} = g(x) \quad \text{and} \quad \int_1^{\infty} \frac{1}{x^2} dx \text{ converges.}$$

Example

$$\int_1^{\infty} \frac{1}{x^2 + x + e^x} dx \quad \text{converge or diverge?}$$

$$0 \leq f(x) = \frac{1}{x^2 + x + e^x} \leq \frac{1}{x^2} = g(x) \quad \text{and} \quad \int_1^{\infty} \frac{1}{x^2} dx \text{ converges.}$$

$$p = 2 > 1$$

Example

$$\int_1^{\infty} \frac{1}{x^2 + x + e^x} dx \text{ is convergent}$$



Comparison Test implies