

MATH 250a

Fall Semester 2007

Section 2 (J. M. Cushing)

Thursday, October 4

<http://math.arizona.edu/~cushing/250a.html>

# Chapter 7: Integration

Sections 1 – 6

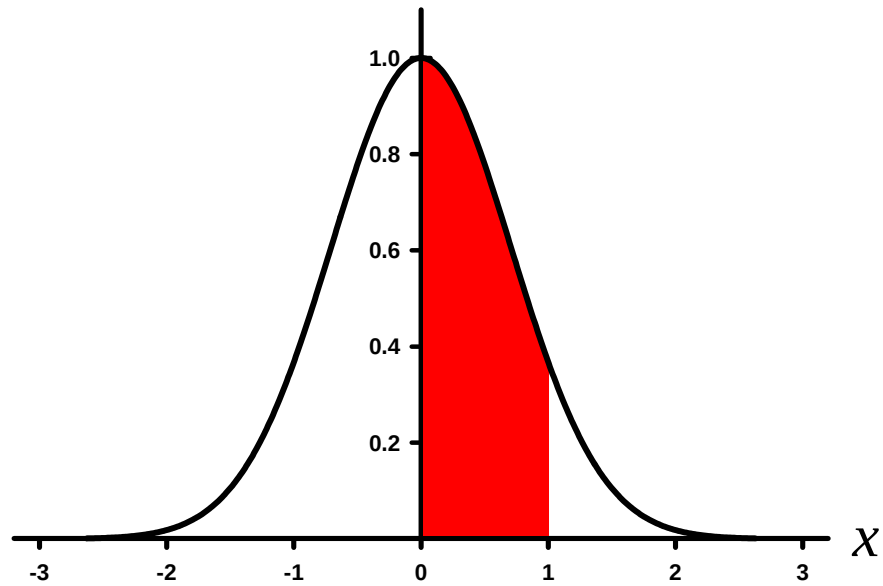
Two main themes:

- ✓ ➤ Calculation of anti-derivatives (integrals)  
One goal is to evaluate integrals using  
the *Fundamental Theorem of Calculus*
- Numerical approximation of integrals

## EXAMPLE

$$\int_0^1 e^{-x^2} dx$$

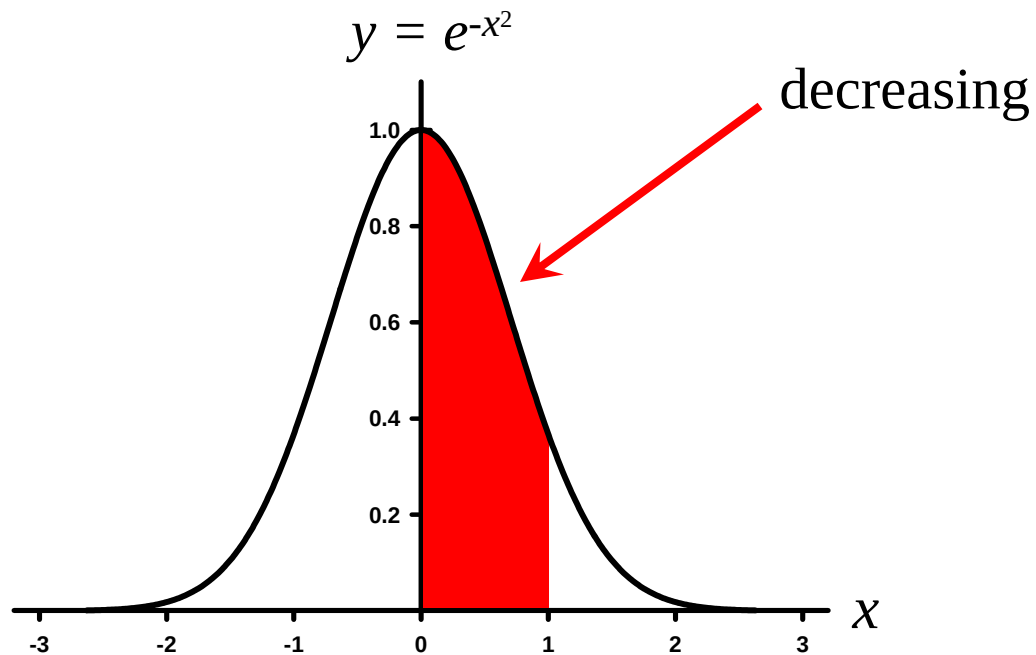
$$y = e^{-x^2}$$



## EXAMPLE

$$\int_0^1 e^{-x^2} dx$$

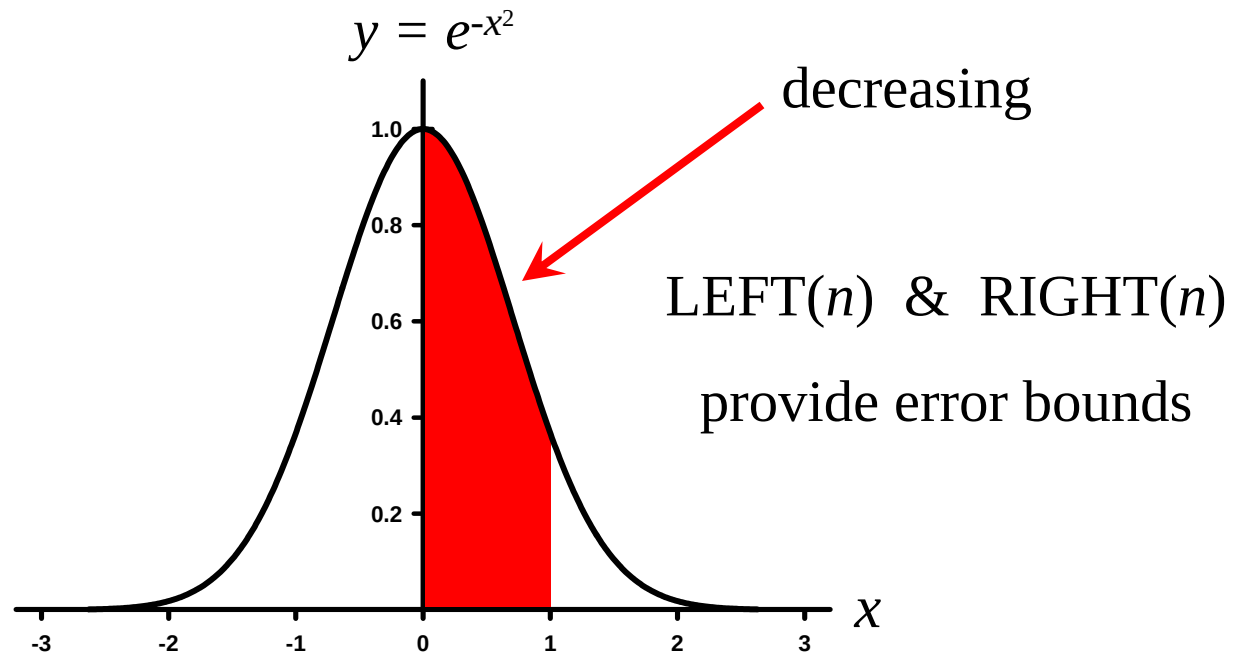
$$\frac{dy}{dx} = -2xe^{-x^2}$$



## EXAMPLE

$$\int_0^1 e^{-x^2} dx$$

$$\frac{dy}{dx} = -2xe^{-x^2}$$

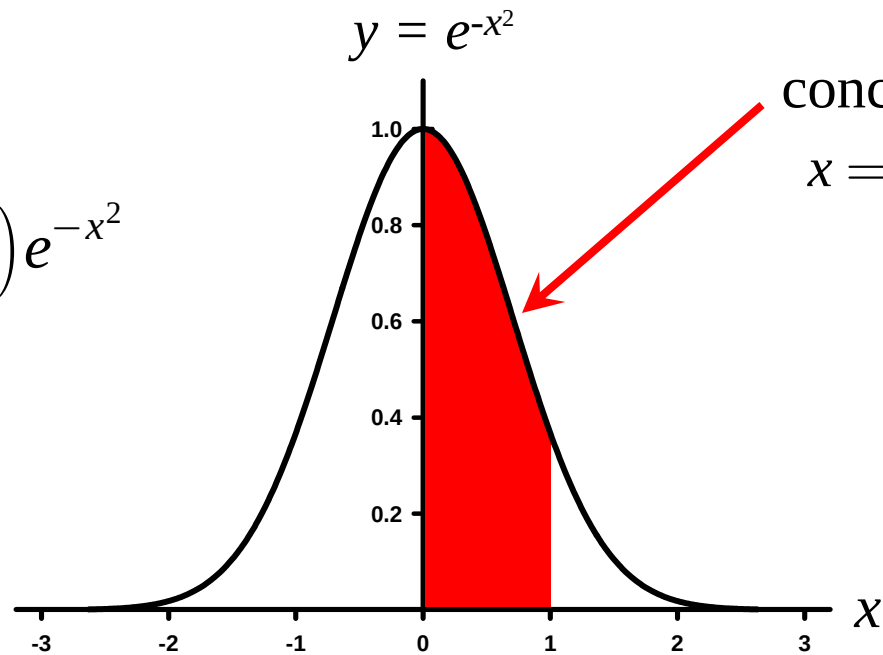


## EXAMPLE

$$\int_0^1 e^{-x^2} dx$$

$$\frac{dy}{dx} = -2xe^{-x^2}$$

$$\frac{dy^2}{dx^2} = 2(2x^2 - 1)e^{-x^2}$$

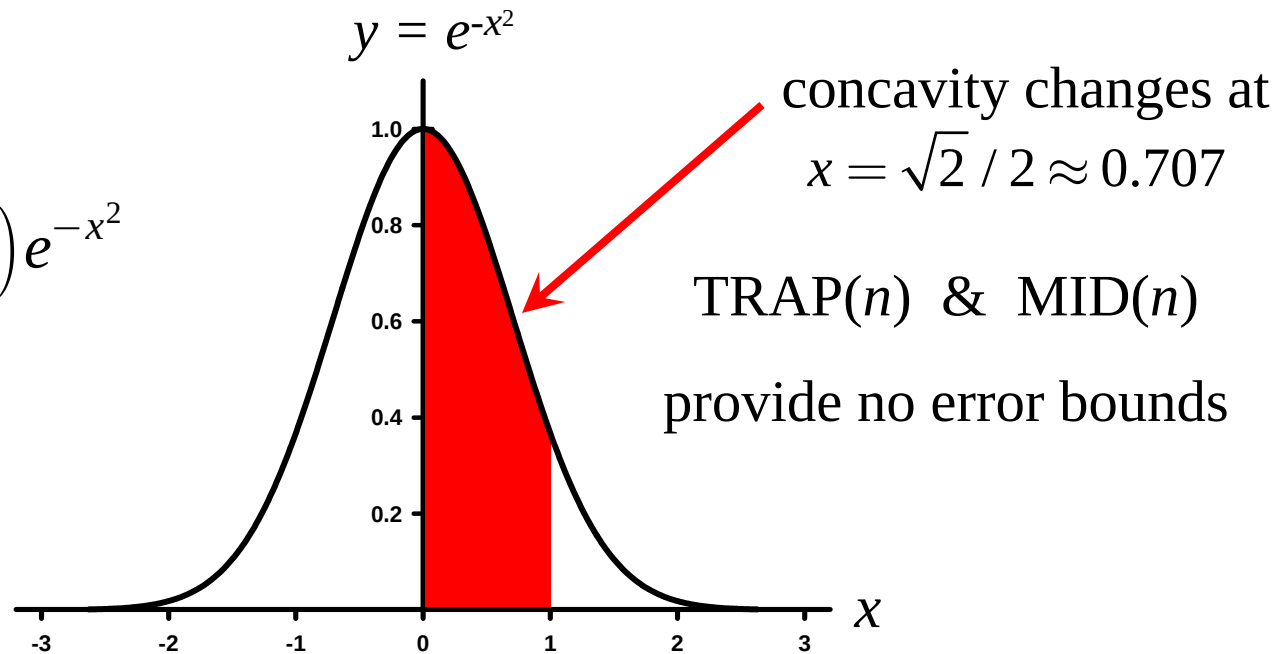


## EXAMPLE

$$\int_0^1 e^{-x^2} dx$$

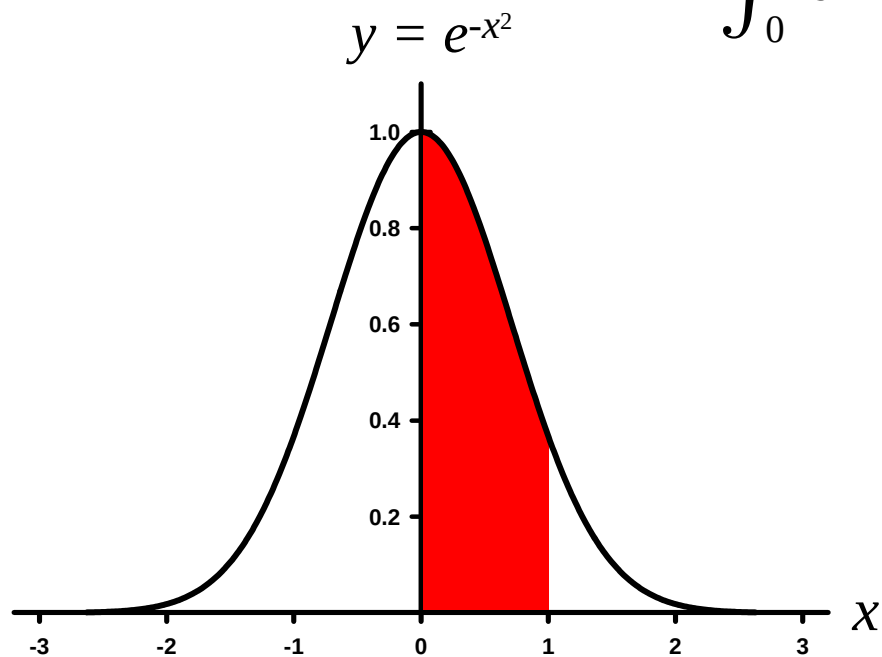
$$\frac{dy}{dx} = -2xe^{-x^2}$$

$$\frac{dy^2}{dx^2} = 2(2x^2 - 1)e^{-x^2}$$



## EXAMPLE

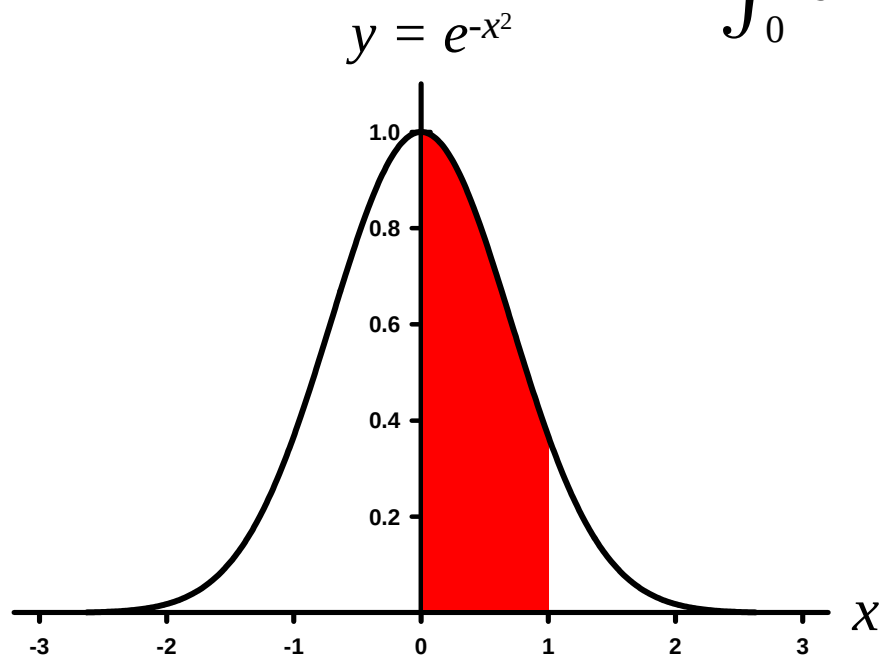
$$\int_0^1 e^{-x^2} dx$$



| $n$ | RIGHT( $n$ ) |
|-----|--------------|
| 2   | 0.5733       |

## EXAMPLE

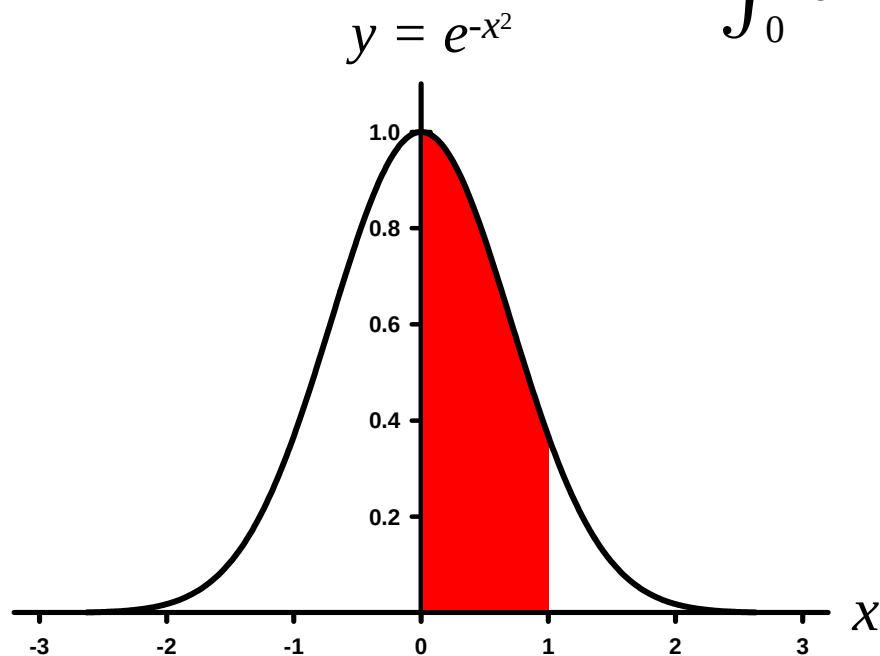
$$\int_0^1 e^{-x^2} dx$$



| $n$ | RIGHT( $n$ ) | LEFT( $n$ ) |
|-----|--------------|-------------|
| 2   | 0.5733       | 0.8894      |

## EXAMPLE

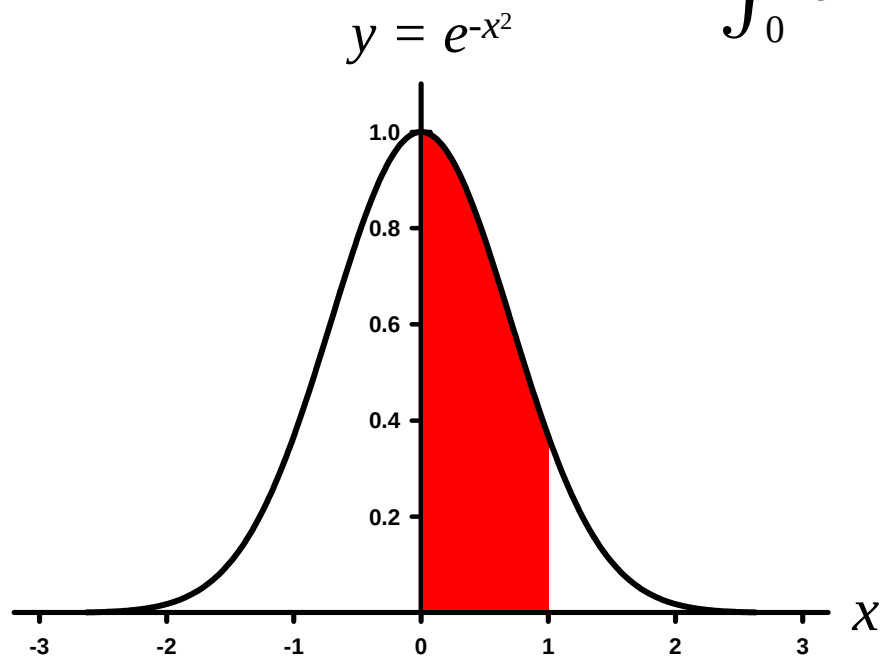
$$\int_0^1 e^{-x^2} dx$$



| $n$ | RIGHT( $n$ ) | LEFT( $n$ ) |
|-----|--------------|-------------|
| 2   | 0.5733       | 0.8894      |
| 4   | 0.6640       | 0.8220      |

## EXAMPLE

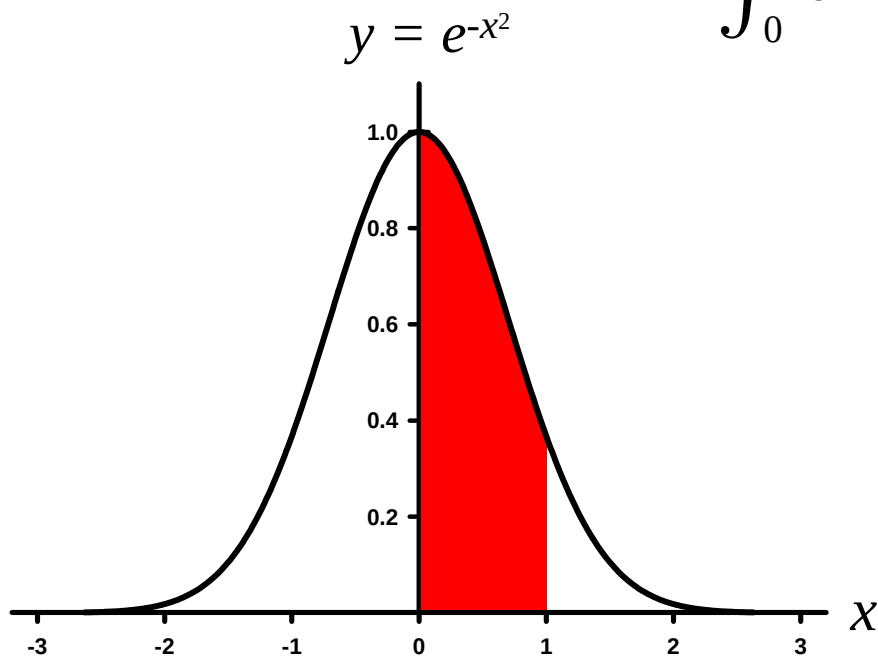
$$\int_0^1 e^{-x^2} dx$$



| $n$ | RIGHT( $n$ ) | LEFT( $n$ ) |
|-----|--------------|-------------|
| 2   | 0.5733       | 0.8894      |
| 4   | 0.6640       | 0.8220      |
| 8   | 0.7064       | 0.7854      |

## EXAMPLE

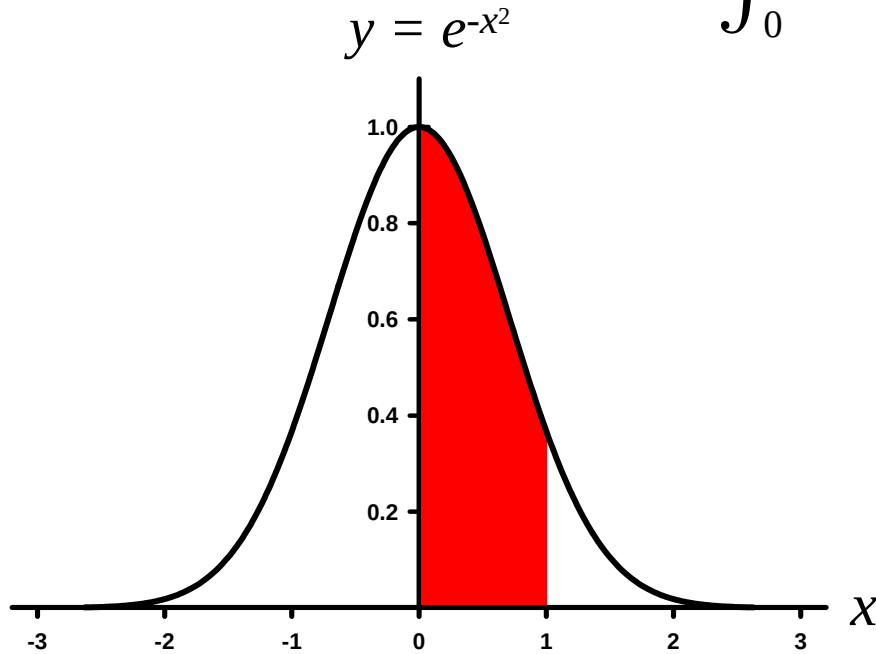
$$\int_0^1 e^{-x^2} dx$$



| $n$ | RIGHT( $n$ ) | LEFT( $n$ ) |
|-----|--------------|-------------|
| 2   | 0.5733       | 0.8894      |
| 4   | 0.6640       | 0.8220      |
| 8   | 0.7064       | 0.7854      |
| 16  | 0.7268       | 0.7663      |

## EXAMPLE

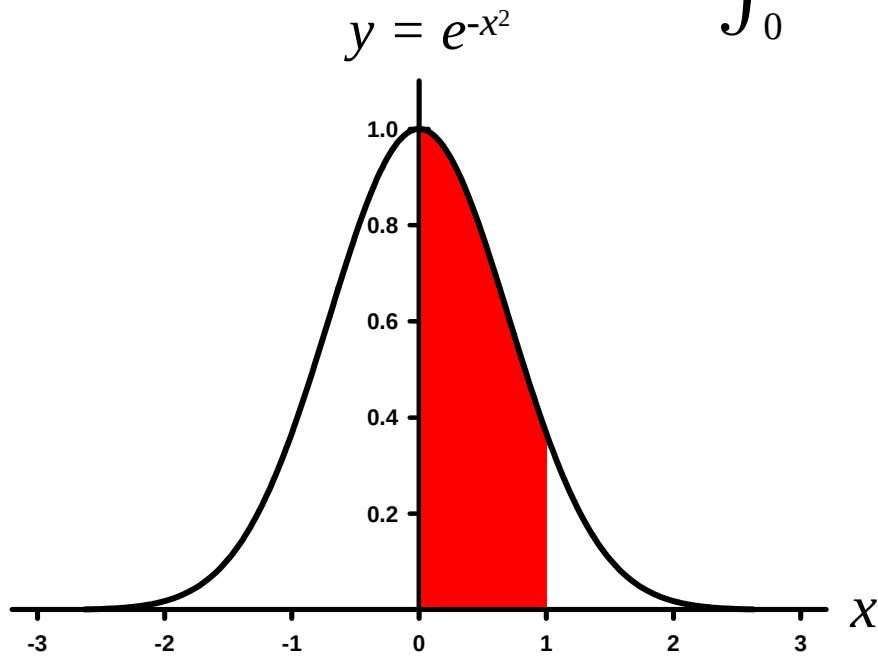
$$\int_0^1 e^{-x^2} dx$$



| $n$ | RIGHT( $n$ ) | LEFT( $n$ ) |
|-----|--------------|-------------|
| 2   | 0.5733       | 0.8894      |
| 4   | 0.6640       | 0.8220      |
| 8   | 0.7064       | 0.7854      |
| 16  | 0.7268       | 0.7663      |
| 32  | 0.7369       | 0.7566      |
| 64  | 0.7419       | 0.7517      |
| 128 | 0.7444       | 0.7493      |
| 256 | 0.7456       | 0.7481      |
| 512 | 0.7462       | 0.7474      |

## EXAMPLE

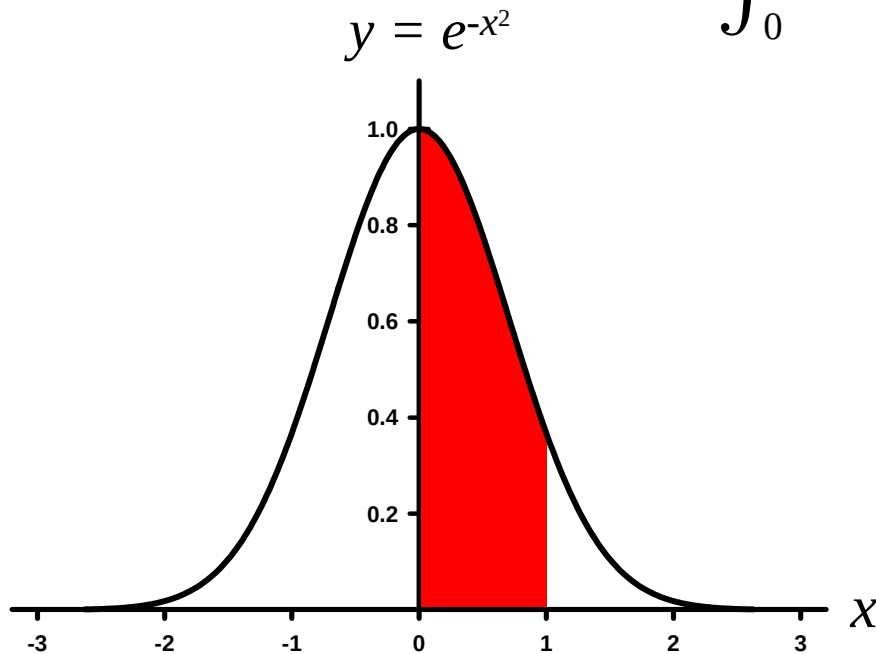
$$\int_0^1 e^{-x^2} dx$$



| $n$ | TRAP( $n$ ) |
|-----|-------------|
| 2   | 0.7314      |
| 4   | 0.7430      |
| 8   | 0.7459      |
| 16  | 0.7466      |
| 32  | 0.7468      |
| 64  | 0.7468      |
| 128 | 0.7468      |
| 256 | 0.7468      |
| 512 | 0.7468      |

## EXAMPLE

$$\int_0^1 e^{-x^2} dx$$



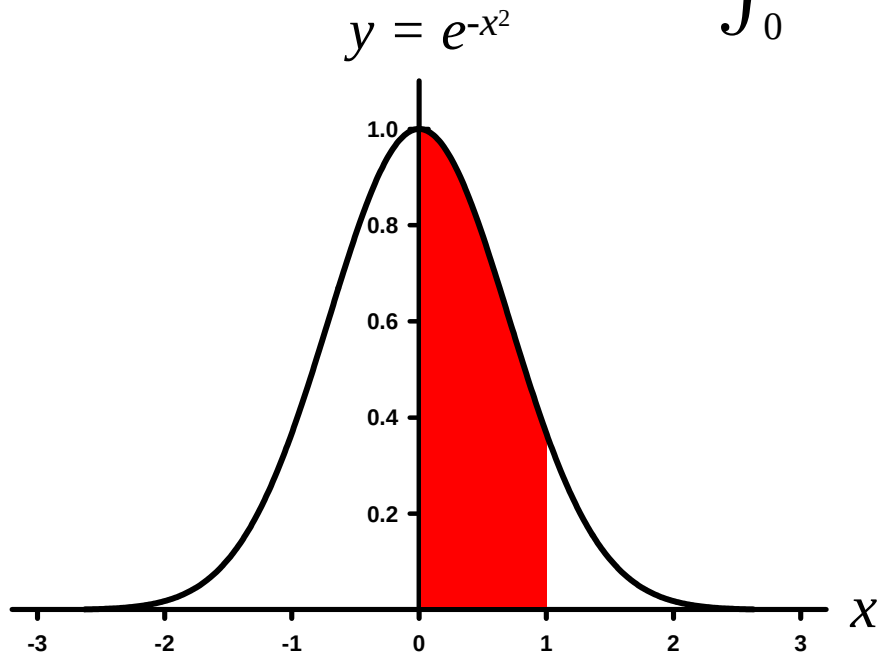
Decide on “stopping rule”

Number of digits (left to right)  
that have “stabilized”

| $n$ | TRAP( $n$ ) |
|-----|-------------|
| 2   | 0.7314      |
| 4   | 0.7430      |
| 8   | 0.7459      |
| 16  | 0.7466      |
| 32  | 0.7468      |
| 64  | 0.7468      |
| 128 | 0.7468      |
| 256 | 0.7468      |
| 512 | 0.7468      |

## EXAMPLE

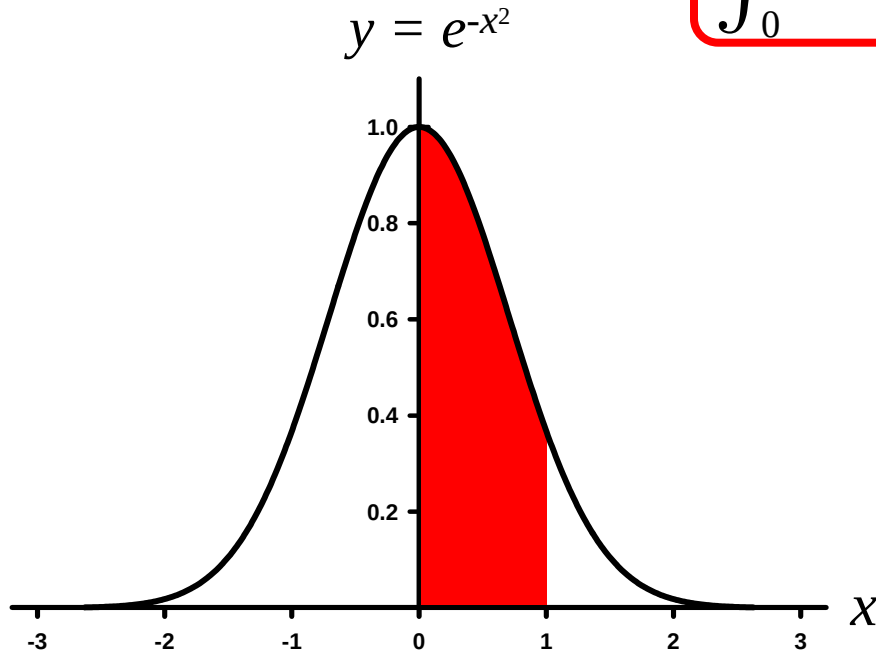
$$\int_0^1 e^{-x^2} dx$$



| $n$ | TRAP( $n$ ) | MID( $n$ ) |
|-----|-------------|------------|
| 2   | 0.7314      | 0.7546     |
| 4   | 0.7430      | 0.7487     |
| 8   | 0.7459      | 0.7473     |
| 16  | 0.7466      | 0.7469     |
| 32  | 0.7468      | 0.7469     |
| 64  | 0.7468      | 0.7468     |
| 128 | 0.7468      | 0.7468     |
| 256 | 0.7468      | 0.7468     |
| 512 | 0.7468      | 0.7468     |

## EXAMPLE

$$\int_0^1 e^{-x^2} dx \approx 0.7468$$



| $n$ | TRAP( $n$ ) | MID( $n$ ) |
|-----|-------------|------------|
| 2   | 0.7314      | 0.7546     |
| 4   | 0.7430      | 0.7487     |
| 8   | 0.7459      | 0.7473     |
| 16  | 0.7466      | 0.7469     |
| 32  | 0.7468      | 0.7469     |
| 64  | 0.7468      | 0.7468     |
| 128 | 0.7468      | 0.7468     |
| 256 | 0.7468      | 0.7468     |
| 512 | 0.7468      | 0.7468     |

Assuming 0.7468 is the correct answer, note that

- TRAP & MID provide more accuracy for each  $n$

| $n$ | RIGHT( $n$ ) | LEFT( $n$ ) |
|-----|--------------|-------------|
| 2   | 0.5733       | 0.8894      |
| 4   | 0.6640       | 0.8220      |
| 8   | 0.7064       | 0.7854      |
| 16  | 0.7268       | 0.7663      |
| 32  | 0.7369       | 0.7566      |
| 64  | 0.7419       | 0.7517      |
| 128 | 0.7444       | 0.7493      |
| 256 | 0.7456       | 0.7481      |
| 512 | 0.7462       | 0.7474      |

| $n$ | TRAP( $n$ ) | MID( $n$ ) |
|-----|-------------|------------|
| 2   | 0.7314      | 0.7546     |
| 4   | 0.7430      | 0.7487     |
| 8   | 0.7459      | 0.7473     |
| 16  | 0.7466      | 0.7469     |
| 32  | 0.7468      | 0.7469     |
| 64  | 0.7468      | 0.7468     |
| 128 | 0.7468      | 0.7468     |
| 256 | 0.7468      | 0.7468     |
| 512 | 0.7468      | 0.7468     |

Assuming 0.7468 is the correct answer, note that

- TRAP & MID provide more accuracy for each  $n$
- TRAP & MID *converge faster* as  $n$  increases

| $n$ | RIGHT( $n$ ) | LEFT( $n$ ) |
|-----|--------------|-------------|
| 2   | 0.5733       | 0.8894      |
| 4   | 0.6640       | 0.8220      |
| 8   | 0.7064       | 0.7854      |
| 16  | 0.7268       | 0.7663      |
| 32  | 0.7369       | 0.7566      |
| 64  | 0.7419       | 0.7517      |
| 128 | 0.7444       | 0.7493      |
| 256 | 0.7456       | 0.7481      |
| 512 | 0.7462       | 0.7474      |

| $n$ | TRAP( $n$ ) | MID( $n$ ) |
|-----|-------------|------------|
| 2   | 0.7314      | 0.7546     |
| 4   | 0.7430      | 0.7487     |
| 8   | 0.7459      | 0.7473     |
| 16  | 0.7466      | 0.7469     |
| 32  | 0.7468      | 0.7469     |
| 64  | 0.7468      | 0.7468     |
| 128 | 0.7468      | 0.7468     |
| 256 | 0.7468      | 0.7468     |
| 512 | 0.7468      | 0.7468     |

Assuming 0.7468 is the correct answer, note that

- TRAP & MID provide more accuracy at each step
- TRAP & MID converge faster as  $n$  increases

| $n$ | LEFT( $n$ ) | ERROR( $n$ )           |
|-----|-------------|------------------------|
| 2   | 0.8894      | $1.426 \times 10^{-1}$ |
| 4   | 0.8220      | $7.520 \times 10^{-2}$ |
| 8   | 0.7854      | $3.860 \times 10^{-2}$ |
| 16  | 0.7663      | $1.950 \times 10^{-2}$ |
| 32  | 0.7566      | $9.800 \times 10^{-3}$ |
| 64  | 0.7517      | $4.900 \times 10^{-3}$ |
| 128 | 0.7493      | $2.500 \times 10^{-3}$ |
| 256 | 0.7481      | $1.300 \times 10^{-3}$ |
| 512 | 0.7474      | $6.000 \times 10^{-3}$ |

| $n$ | TRAP( $n$ ) | ERROR( $n$ )           |
|-----|-------------|------------------------|
| 2   | 0.7314      | $1.540 \times 10^{-2}$ |
| 4   | 0.7430      | $3.800 \times 10^{-3}$ |
| 8   | 0.7459      | $9.000 \times 10^{-3}$ |
| 16  | 0.7466      | $2.000 \times 10^{-3}$ |
| 32  | 0.7468      | 0.0000                 |
| 64  | 0.7468      | 0.0000                 |
| 128 | 0.7468      | 0.0000                 |
| 256 | 0.7468      | 0.0000                 |
| 512 | 0.7468      | 0.0000                 |

Assuming 0.7468 is the correct answer, note that

- TRAP & MID provide more accuracy at each step
- TRAP & MID converge faster as  $n$  increases

| $n$ | LEFT( $n$ ) | %ERROR( $n$ ) |
|-----|-------------|---------------|
| 2   | 0.8894      | 19.095        |
| 4   | 0.8220      | 10.070        |
| 8   | 0.7854      | 5.169         |
| 16  | 0.7663      | 2.611         |
| 32  | 0.7566      | 1.312         |
| 64  | 0.7517      | 0.6561        |
| 128 | 0.7493      | 0.3348        |
| 256 | 0.7481      | 0.1741        |
| 512 | 0.7474      | 0.0803        |

| $n$ | TRAP( $n$ ) | %ERROR( $n$ ) |
|-----|-------------|---------------|
| 2   | 0.7314      | 2.062         |
| 4   | 0.7430      | 0.5088        |
| 8   | 0.7459      | 0.1205        |
| 16  | 0.7466      | 0.0268        |
| 32  | 0.7468      | 0.0000        |
| 64  | 0.7468      | 0.0000        |
| 128 | 0.7468      | 0.0000        |
| 256 | 0.7468      | 0.0000        |
| 512 | 0.7468      | 0.0000        |

Number of steps doubles.  
Error decreases by a factor of  $\sim 1/2$ .

| $n$ | LEFT( $n$ ) | %ERROR( $n$ ) |
|-----|-------------|---------------|
| 2   | 0.8894      | 19.095        |
| 4   | 0.8220      | 10.070        |
| 8   | 0.7854      | 5.169         |
| 16  | 0.7663      | 2.611         |
| 32  | 0.7566      | 1.312         |
| 64  | 0.7517      | 0.6561        |
| 128 | 0.7493      | 0.3348        |
| 256 | 0.7481      | 0.1741        |
| 512 | 0.7474      | 0.0803        |

| $n$ | TRAP( $n$ ) | %ERROR( $n$ ) |
|-----|-------------|---------------|
| 2   | 0.7314      | 2.062         |
| 4   | 0.7430      | 0.5088        |
| 8   | 0.7459      | 0.1205        |
| 16  | 0.7466      | 0.0268        |
| 32  | 0.7468      | 0.0000        |
| 64  | 0.7468      | 0.0000        |
| 128 | 0.7468      | 0.0000        |
| 256 | 0.7468      | 0.0000        |
| 512 | 0.7468      | 0.0000        |

Number of steps doubles.  
Error decreases by a factor of  $\sim 1/2$ .

| $n$ | LEFT( $n$ ) | %ERROR( $n$ ) |
|-----|-------------|---------------|
| 2   | 0.8894      | 19.095        |
| 4   | 0.8220      | 10.070        |
| 8   | 0.7854      | 5.169         |
| 16  | 0.7663      | 2.611         |
| 32  | 0.7566      | 1.312         |
| 64  | 0.7517      | 0.6561        |
| 128 | 0.7493      | 0.3348        |
| 256 | 0.7481      | 0.1741        |
| 512 | 0.7474      | 0.0803        |

Number of steps doubles. Error  
decreases by a factor of  $\sim 1/4$ .

| $n$ | TRAP( $n$ ) | %ERROR( $n$ ) |
|-----|-------------|---------------|
| 2   | 0.7314      | 2.062         |
| 4   | 0.7430      | 0.5088        |
| 8   | 0.7459      | 0.1205        |
| 16  | 0.7466      | 0.0268        |
| 32  | 0.7468      | 0.0000        |
| 64  | 0.7468      | 0.0000        |
| 128 | 0.7468      | 0.0000        |
| 256 | 0.7468      | 0.0000        |
| 512 | 0.7468      | 0.0000        |

*Step size decreases by 1/2.*

Error decreases by a factor of  $\sim 1/2$  .

| $n$ | LEFT( $n$ ) | %ERROR( $n$ ) |
|-----|-------------|---------------|
| 2   | 0.8894      | 19.095        |
| 4   | 0.8220      | 10.070        |
| 8   | 0.7854      | 5.169         |
| 16  | 0.7663      | 2.611         |
| 32  | 0.7566      | 1.312         |
| 64  | 0.7517      | 0.6561        |
| 128 | 0.7493      | 0.3348        |
| 256 | 0.7481      | 0.1741        |
| 512 | 0.7474      | 0.0803        |

*Step size decreases by 1/2.*

Error decreases by a factor of  $\sim 1/4$  .

| $n$ | TRAP( $n$ ) | %ERROR( $n$ ) |
|-----|-------------|---------------|
| 2   | 0.7314      | 2.062         |
| 4   | 0.7430      | 0.5088        |
| 8   | 0.7459      | 0.1205        |
| 16  | 0.7466      | 0.0268        |
| 32  | 0.7468      | 0.0000        |
| 64  | 0.7468      | 0.0000        |
| 128 | 0.7468      | 0.0000        |
| 256 | 0.7468      | 0.0000        |
| 512 | 0.7468      | 0.0000        |

Put another way:

$$\text{Error ratio} = E(n) / E(2n) \approx 2$$

| $n$ | LEFT( $n$ ) | %ERROR( $n$ ) |
|-----|-------------|---------------|
| 2   | 0.8894      | 19.095        |
| 4   | 0.8220      | 10.070        |
| 8   | 0.7854      | 5.169         |
| 16  | 0.7663      | 2.611         |
| 32  | 0.7566      | 1.312         |
| 64  | 0.7517      | 0.6561        |
| 128 | 0.7493      | 0.3348        |
| 256 | 0.7481      | 0.1741        |
| 512 | 0.7474      | 0.0803        |

Put another way:

$$\text{Error ratio} = E(n) / E(2n) \approx 4$$

| $n$ | TRAP( $n$ ) | %ERROR( $n$ ) |
|-----|-------------|---------------|
| 2   | 0.7314      | 2.062         |
| 4   | 0.7430      | 0.5088        |
| 8   | 0.7459      | 0.1205        |
| 16  | 0.7466      | 0.0268        |
| 32  | 0.7468      | 0.0000        |
| 64  | 0.7468      | 0.0000        |
| 128 | 0.7468      | 0.0000        |
| 256 | 0.7468      | 0.0000        |
| 512 | 0.7468      | 0.0000        |

Put another way:

$$\text{Error ratio} = E(n) / E(2n) \approx 2$$

| $n$ | LEFT( $n$ ) | Error Ratio |
|-----|-------------|-------------|
| 2   | 0.8894      | --          |
| 4   | 0.8220      | 1.896       |
| 8   | 0.7854      | 1.948       |
| 16  | 0.7663      | 1.980       |
| 32  | 0.7566      | 1.990       |
| 64  | 0.7517      | 2.000       |
| 128 | 0.7493      | 1.960       |
| 256 | 0.7481      | 1.923       |
| 512 | 0.7474      | 2.167       |

Put another way:

$$\text{Error ratio} = E(n) / E(2n) \approx 4$$

| $n$ | TRAP( $n$ ) | Error Ratio |
|-----|-------------|-------------|
| 2   | 0.7314      | --          |
| 4   | 0.7430      | 4.053       |
| 8   | 0.7459      | 4.222       |
| 16  | 0.7466      | 4.500       |
| 32  | 0.7468      |             |
| 64  | 0.7468      |             |
| 128 | 0.7468      |             |
| 256 | 0.7468      |             |
| 512 | 0.7468      |             |

“Step size”  $1/n$  decreases by  $1/2$   
Error decreases by factor of  $1/2$

“Step size”  $1/n$  decreases by  $1/2$   
Error decreases by factor of  $1/4$

These observations are not peculiar to this example.

“Step size”  $1/n$  decreases by  $1/2$   
Error decreases by factor of  $1/2$

“Step size”  $1/n$  decreases by  $1/2$   
Error decreases by factor of  $1/4$

These observations are not peculiar to this example.

LEFT & RIGHT are “*first order*” methods:

$$|\text{Error}| \leq \text{constant} \times \frac{1}{n}$$

“Step size”  $1/n$  decreases by  $1/2$   
Error decreases by factor of  $1/2$

“Step size”  $1/n$  decreases by  $1/2$   
Error decreases by factor of  $1/4$

These observations are not peculiar to this example.

LEFT & RIGHT are “*first order*” methods:

$$|\text{Error}| \leq \text{constant} \times \frac{1}{n}$$

Errors decrease at (roughly) the *same rate* that the step size decreases

“Step size”  $1/n$  decreases by  $1/2$   
Error decreases by factor of  $1/2$

“Step size”  $1/n$  decreases by  $1/2$   
Error decreases by factor of  $1/4$

These observations are not peculiar to this example.

TRAP & MID are “*second order*” methods:

$$|\text{Error}| \leq \text{constant} \times \left(\frac{1}{n}\right)^2$$

Errors decrease as (roughly) the *square* of the step size decreases.

LEFT & RIGHT

$$|\text{Error}| \leq \text{constant} \times \frac{1}{n}$$

TRAP & MID

$$|\text{Error}| \leq \text{constant} \times \left(\frac{1}{n}\right)^2$$

## LEFT & RIGHT

$$|\text{Error}| \leq \text{constant} \times \frac{1}{n}$$

Increase  $n$  by a factor of 10,  
decreases error by a factor of 1/10.

## TRAP & MID

$$|\text{Error}| \leq \text{constant} \times \left(\frac{1}{n}\right)^2$$

## LEFT & RIGHT

$$|\text{Error}| \leq \text{constant} \times \frac{1}{n}$$

Increase  $n$  by a factor of 10,  
decreases error by a factor of 1/10.

10 times the work gains  
1 decimal of accuracy.

## TRAP & MID

$$|\text{Error}| \leq \text{constant} \times \left(\frac{1}{n}\right)^2$$

## LEFT & RIGHT

$$|\text{Error}| \leq \text{constant} \times \frac{1}{n}$$

Increase  $n$  by a factor of 10,  
decreases error by a factor of 1/10.

10 times the work gains  
1 decimal of accuracy.

## TRAP & MID

$$|\text{Error}| \leq \text{constant} \times \left(\frac{1}{n}\right)^2$$

Increase  $n$  by a factor of 10,  
decreases error by a factor of 1/100.

## LEFT & RIGHT

$$|\text{Error}| \leq \text{constant} \times \frac{1}{n}$$

Increase  $n$  by a factor of 10,  
decreases error by a factor of 1/10.

10 times the work gains  
1 decimal of accuracy.

## TRAP & MID

$$|\text{Error}| \leq \text{constant} \times \left(\frac{1}{n}\right)^2$$

Increase  $n$  by a factor of 10,  
decreases error by a factor of 1/100.

10 times the work gains  
2 decimals of accuracy.

LEFT & RIGHT

$$|\text{Error}| \leq \text{constant} \times \frac{1}{n}$$

TRAP & MID

$$|\text{Error}| \leq \text{constant} \times \left(\frac{1}{n}\right)^2$$

A  $p^{\text{th}}$  order method would gain  
 $p$  decimals of accuracy  
for 10 times the work.

$$|\text{Error}| \leq \text{constant} \times \left(\frac{1}{n}\right)^p$$

## SIMPSON'S RULE

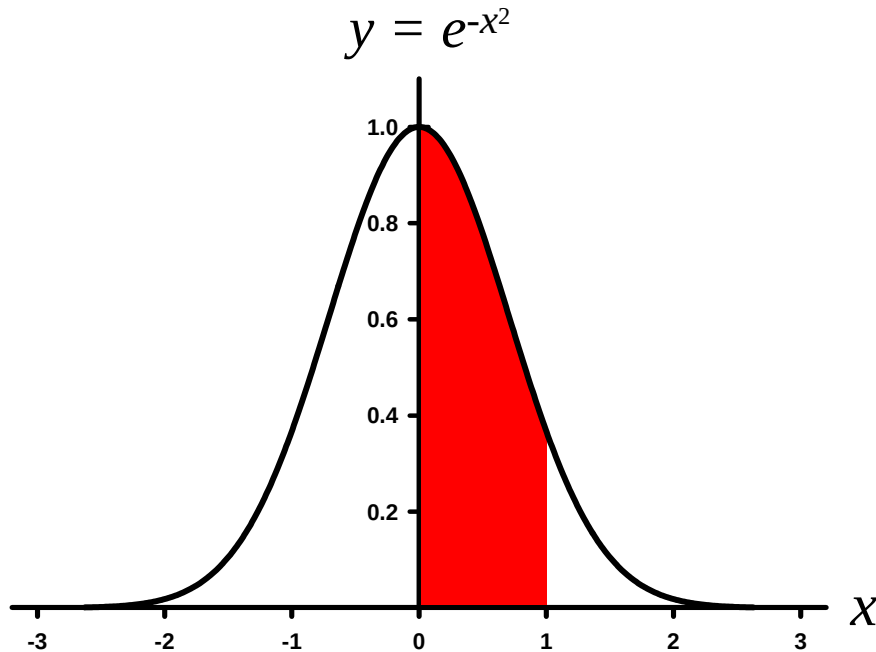
An order 4 method

$$\text{SIMP}(n) = \frac{2 \cdot \text{MID}(n) + \text{TRAP}(n)}{3}$$

... a weighted average of MID and TRAP

# SIMPSON'S RULE

An order 4 method



$$\int_0^1 e^{-x^2} dx$$

| $n$ | SIMP( $n$ ) |
|-----|-------------|
| 2   | 0.7469      |
| 4   | 0.7468      |
| 8   | 0.7468      |
| 16  | 0.7468      |
| 32  | 0.7469      |
| 64  | 0.7468      |
| 128 | 0.7468      |
| 256 | 0.7468      |
| 512 | 0.7468      |

## Section 7: Improper Integrals

## Section 7: Improper Integrals

### Examples from Probability Theory

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$

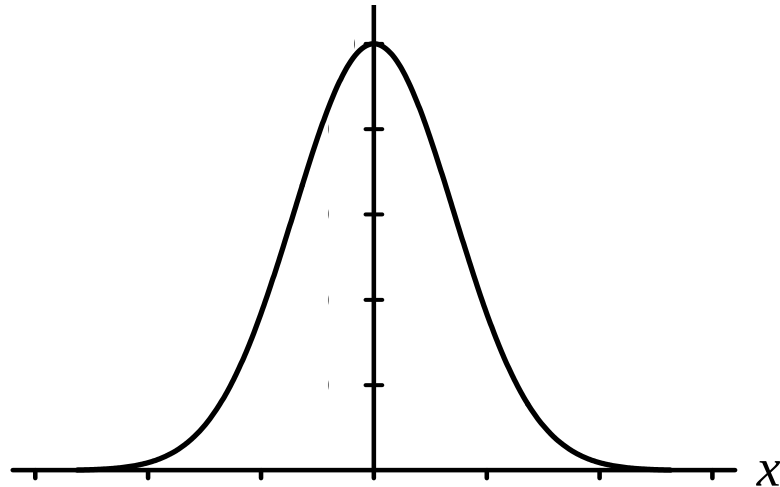
## Section 7: Improper Integrals

### Examples from Probability Theory

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$

The famous “bell curve”.

Describes what’s called a *normal probability distribution*



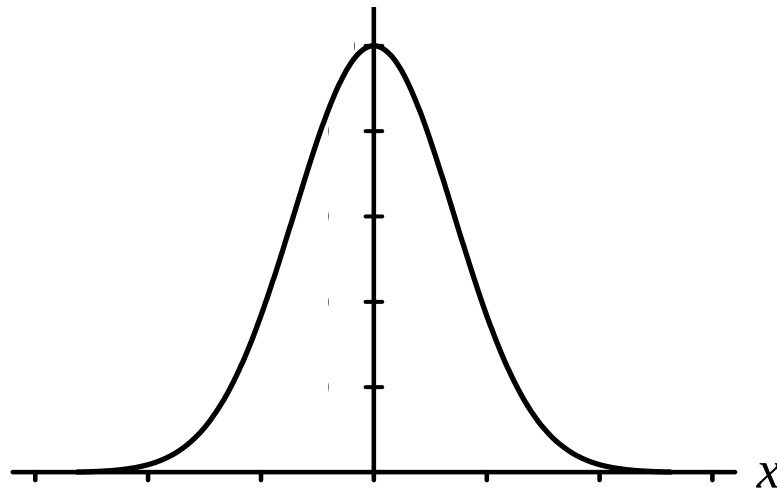
## Section 7: Improper Integrals

### Examples from Probability Theory

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$

The famous “bell curve”.

Describes what's called a *normal probability distribution*  
for a random variable  $x$  varying on the infinite interval  $-\infty < x < +\infty$



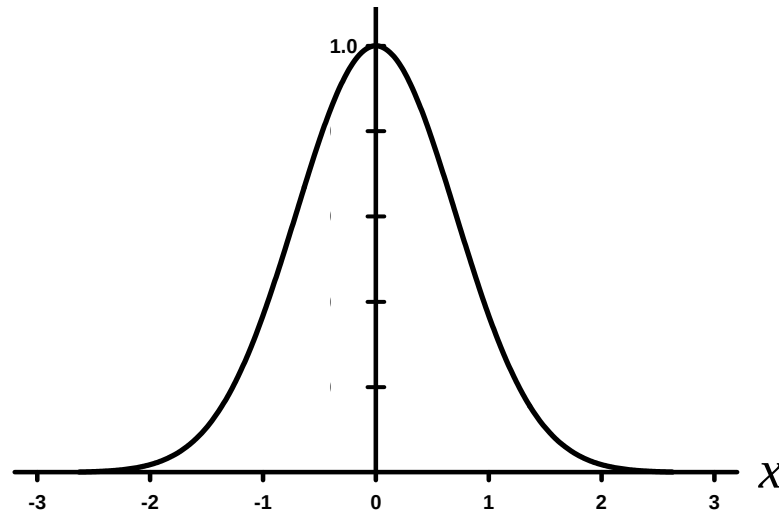
## Section 7: Improper Integrals

### Examples from Probability Theory

$$\int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} dx = \begin{cases} \text{probability random variable } x \\ \text{lies between } a \text{ and } b \end{cases}$$

The famous “bell curve”.

Describes what's called a *normal probability distribution*  
for a random variable  $x$  varying on the infinite interval  $-\infty < x < +\infty$



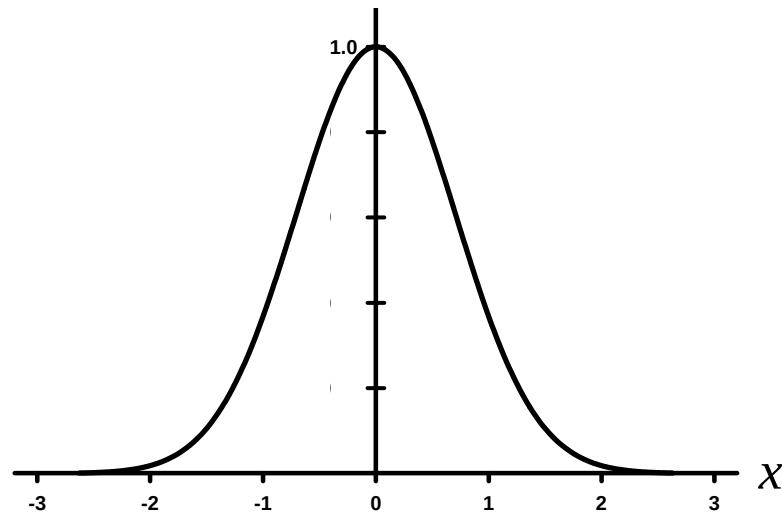
## Section 7: Improper Integrals

### Examples from Probability Theory

$$\int_a^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} dx = \begin{cases} \text{probability random variable } x \\ \text{is greater than } a \end{cases}$$

The famous “bell curve”.

Describes what's called a *normal probability distribution*  
for a random variable  $x$  varying on the infinite interval  $-\infty < x < +\infty$



## Section 7: Improper Integrals

### Examples from Probability Theory

$$\int_a^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} dx = \begin{cases} \text{probability random variable } x \\ \text{is greater than } a \end{cases}$$

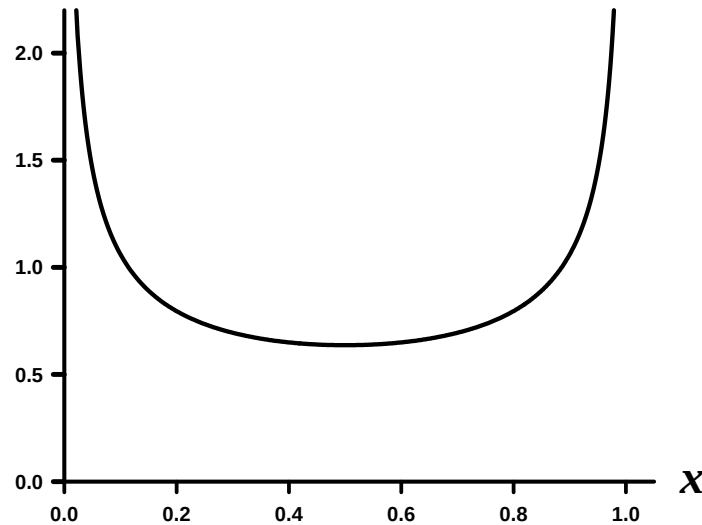
One kind of improper integral:  
integration over an *infinite* interval

## Section 7: Improper Integrals

### Examples from Probability Theory

$$\frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}}$$

This is an example of a *beta probability distribution* for a random variable  $x$  varying on the finite interval  $0 < x < 1$

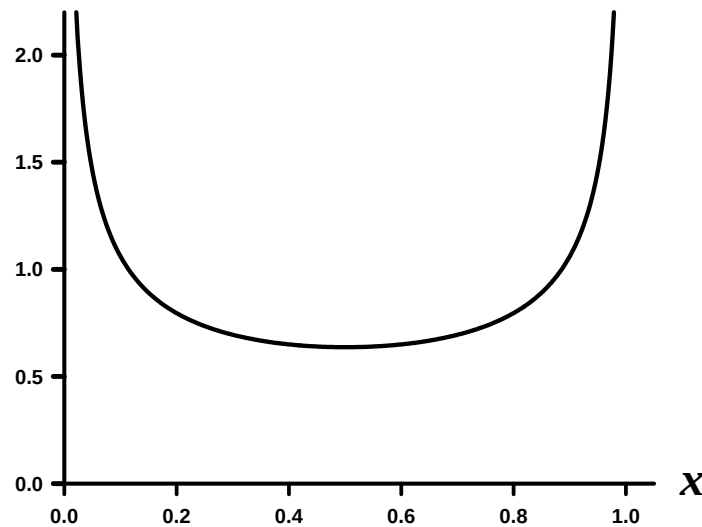


## Section 7: Improper Integrals

### Examples from Probability Theory

$$\int_a^b \frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}} dx = \begin{cases} \text{probability random variable } x \\ \text{lies between } a \text{ and } b \end{cases}$$

This is an example of a *beta probability distribution* for a random variable  $x$  varying on the finite interval  $0 < x < 1$

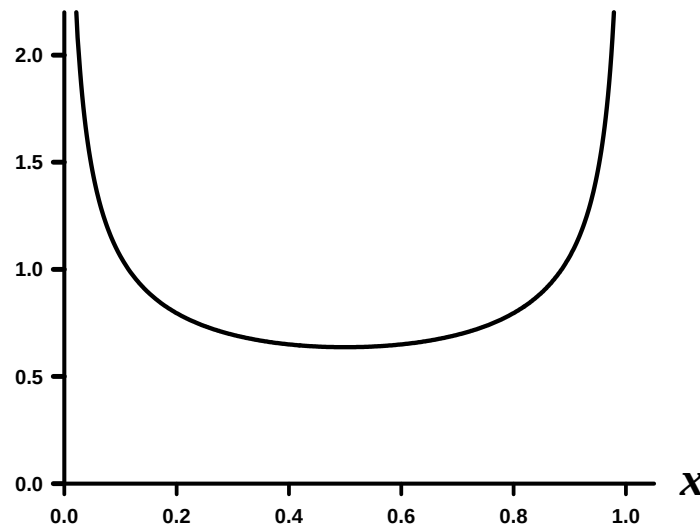


## Section 7: Improper Integrals

### Examples from Probability Theory

$$\int_0^{1/2} \frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}} dx = \begin{cases} \text{probability random variable } x \\ \text{lies between } 0 \text{ and } 1/2 \end{cases}$$

This is an example of a *beta probability distribution* for a random variable  $x$  varying on the finite interval  $0 < x < 1$



## Section 7: Improper Integrals

### Examples from Probability Theory

$$\int_0^{1/2} \frac{1}{\pi} \frac{1}{\sqrt{x(1-x)}} dx = \begin{cases} \text{probability random variable } x \\ \text{lies between } 0 \text{ and } 1/2 \end{cases}$$

This is an example of a *beta probability distribution* for a random variable  $x$  varying on the finite interval  $0 < x < 1$

Another kind of improper integral:  
integrand becomes infinite at endpoint

## Section 7: Improper Integrals

Text book considers two types:

- A limit of integration is infinite
- The integrand becomes infinite