

MATH 250a

Fall Semester 2007

Section 2 (J. M. Cushing)

Tuesday, September 25

<http://math.arizona.edu/~cushing/250a.html>

Chapter 7: Integration

Sections 1 - 4

- Calculation of anti-derivatives (integrals)
 - ✓ 1. Learn some fundamental properties of integrals
 - ✓ 2. Use known differentiation formulas “backwards”
 - 3. Use differentiation rules “backwards”
 - ✓ chain rule $\longleftrightarrow$ substitution
 - product rule $\longleftrightarrow$ integration by parts
 - 4. Use algebraic manipulations
e.g., partial fraction decomposition
 - 5. Use tables of integrals

3. Use known differentiation formulas “backwards”

We saw how the chain rule (used in reverse) motivates

the substitution method

(or change of variable method)

for calculating integrals.

Another substitution that is often useful involves trig functions.

Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{9-x^2}} dx$$

Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{9-x^2}} dx$$

$$x = 3\sin w$$

Trigonometric Substitutions

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$$\int \frac{1}{\sqrt{9-x^2}} dx$$

$$x = 3\sin w \quad \text{or} \quad w = \arcsin(x/3), \quad -\pi/2 \leq w \leq \pi/2$$

Trigonometric Substitutions

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$$\frac{dx}{dw} = 3 \cos w \quad \Rightarrow \quad dx = 3 \cos w \, dw$$

Trigonometric Substitutions

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$x = 3 \sin w$ or $w = \arcsin(x/3)$, $-\pi/2 \leq w \leq \pi/2$

$$\frac{dx}{dw} = 3 \cos w \implies dx = 3 \cos w dw$$

Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{9-x^2}} dx = \int \frac{1}{\sqrt{9-9\sin^2 w}} 3\cos w dw$$

$$x = 3\sin w \quad \text{or} \quad w = \arcsin(x/3), \quad -\pi/2 \leq w \leq \pi/2$$

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Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{9-x^2}} dx = \int \frac{1}{3\sqrt{1-\sin^2 w}} 3\cos w dw$$

$$x = 3\sin w \quad \text{or} \quad w = \arcsin(x/3), \quad -\pi/2 \leq w \leq \pi/2$$

$$\frac{dx}{dw} = 3\cos w \quad \Rightarrow \quad dx = 3\cos w dw$$

Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{9-x^2}} dx = \int \frac{1}{\sqrt{\cos^2 w}} \cos w dw$$

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An aside: $\sqrt{x^2}$ is not identical to x , at least for all x .

More correctly, $\sqrt{x^2} = |x|$

Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{9-x^2}} dx = \int \frac{1}{\sqrt{\cos^2 w}} \cos w dw$$

$$x = 3 \sin w \quad \text{or} \quad w = \arcsin(x/3), \quad -\pi/2 \leq w \leq \pi/2 \Rightarrow \cos w \geq 0$$

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$$\frac{dx}{dw} = 3 \cos w \Rightarrow dx = 3 \cos w dw \quad \Rightarrow \sqrt{\cos^2 w} = \cos w$$

Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{9-x^2}} dx = \int \frac{1}{\sqrt{\cos^2 w}} \cos w dw = \int 1 dw$$

$$x = 3 \sin w \quad \text{or} \quad w = \arcsin(x/3), \quad -\pi/2 \leq w \leq \pi/2$$

$$\frac{dx}{dw} = 3 \cos w \quad \Rightarrow \quad dx = 3 \cos w dw$$

Trigonometric Substitutions

Example

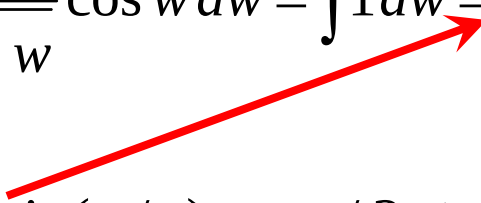
$$\int \frac{1}{\sqrt{9-x^2}} dx = \int \frac{1}{\sqrt{\cos^2 w}} \cos w dw = \int 1 dw = w + c$$

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Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{9-x^2}} dx = \arcsin\left(\frac{1}{3}x\right) + c$$

Substitutions utilizing trig functions are suggested for integrals involving quadratic polynomials in their denominators.

(usually best to “complete the square” first, however)

Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{1-4x-x^2}} dx$$

Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{1-4x-x^2}} dx = \int \frac{1}{\sqrt{5-(2+x)^2}} dx$$

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$$2+x = \sqrt{5} \sin w, \quad -\pi/2 \leq w \leq \pi/2$$

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Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{1-4x-x^2}} dx = \int \frac{1}{\sqrt{5-(2+x)^2}} dx = \int \frac{1}{\sqrt{5-5\sin^2 w}} \sqrt{5} \cos w dw$$

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$$2+x = \sqrt{5} \sin w, \quad -\pi/2 \leq w \leq \pi/2 \quad \Rightarrow \quad w = \arcsin\left(\frac{2+x}{\sqrt{5}}\right)$$


$$\frac{dx}{dw} = \sqrt{5} \cos w \quad \Rightarrow \quad dx = \sqrt{5} \cos w dw$$

Trigonometric Substitutions

Example

$$\int \frac{1}{\sqrt{1-4x-x^2}} dx = \arcsin\left(\frac{2+x}{\sqrt{5}}\right) + c$$

3. Use known differentiation formulas “backwards”

The *integration by parts* method (Section 7.2)
is based on using the *product rule* backwards

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$$(uv)' = uv' + u'v$$

$$uv' = (uv)' - u'v$$

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The *integration by parts* method (Section 7.2)
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$$(uv)' = uv' + u'v$$

$$uv' = (uv)' - u'v$$

same as ... $\int u v' dx = uv - \int u' v dx$

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Integration by Parts formula

$$\int u v' dx = uv - \int u' v dx$$

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Integration by Parts formula

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Basic idea :

We can calculate an integral recognizable as in being this form,

3. Use known differentiation formulas “backwards”

The *integration by parts* method (Section 7.2) is based on using the *product rule* backwards

Integration by Parts formula

$$\int u v' dx = uv - \int u' v dx$$

Basic idea :

We can calculate an integral recognizable as in being this form,
by calculating this associated integral .

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x \ln x dx$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

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Integrand is a product !

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Which factor should play
the role of u ?

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Example

$$\int x \ln x dx$$

Which factor should play
the role of u ?

We will need u , u' and v
to calculate right hand side.

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x \ln x dx$$

Which factor should play
the role of u ?

If we choose $u = x$,

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x \ln x dx$$

Which factor should play
the role of u ?

If we choose $u = x$, then $u' = 1$ and

$$v' = \ln x$$

and we must integrate $\ln x$ to find v .

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int (\ln x) x dx$$

Which factor should play
the role of u ?

If we choose $u = \ln x$, then $u' = 1/x$ and

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Which factor should play
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$$v = \frac{1}{2} x^2$$

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$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int (\ln x) x dx = (\ln x) \frac{1}{2} x^2 - \int \left(\frac{1}{x} \right) \frac{1}{2} x^2 dx$$

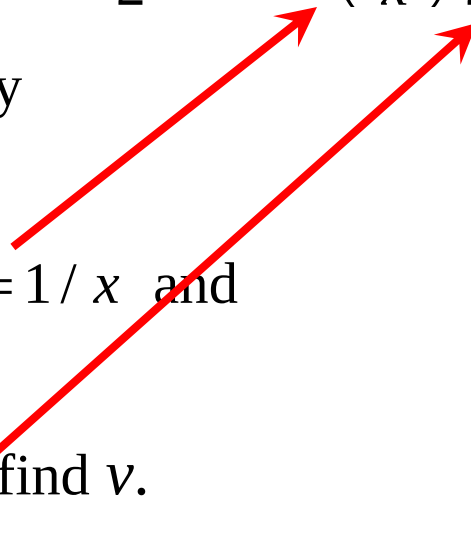
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Example

$$\begin{aligned}\int (\ln x) x dx &= (\ln x) \frac{1}{2} x^2 - \int \left(\frac{1}{x} \right) \frac{1}{2} x^2 dx \\ &= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x dx\end{aligned}$$

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Example

$$\begin{aligned}\int (\ln x) x dx &= (\ln x) \frac{1}{2} x^2 - \int \left(\frac{1}{x} \right) \frac{1}{2} x^2 dx \\ &= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x dx \\ &= \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + c\end{aligned}$$

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Example

$$\int (\ln x) x dx$$

$$u = \ln x, \quad v' = x$$

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Summary:

$$u' = \frac{1}{x}, \quad v = \frac{1}{2} x^2$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

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What was key in making this choice for u work ?

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$$\int (\ln x) x dx = (\ln x) \frac{1}{2} x^2 - \int \left(\frac{1}{x} \right) \frac{1}{2} x^2 dx$$

$$u = \ln x, \quad v' = x$$

We can calculate

$$u' \text{ and } v \longrightarrow u' = \frac{1}{x}, \quad v = \frac{1}{2} x^2$$

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$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int (\ln x) x dx = (\ln x) \frac{1}{2} x^2 - \int \left(\frac{1}{x} \right) \frac{1}{2} x^2 dx$$

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We can calculate

u' and v

$$u' = \frac{1}{x}, \quad v = \frac{1}{2} x^2$$

u' and v

are "simpler" than u and v'

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Hints on How to Choose u and v' :

(page 322 of text)

- Whatever you let v' be, you need to be able to find v .
- It helps if u' is simpler (or at least no more complicated) than u .
- It helps if v is simpler (or at least no more complicated) than v' .

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Example

$$\int x e^x dx$$

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$$\begin{array}{l} u = e^x, \\ u' = e^x, \end{array} \quad \boxed{\begin{array}{l} v' = x \\ v = \frac{1}{2}x^2 \end{array}}$$

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Example

$$\begin{aligned} u &= x, & v' &= e^x \\ u' &= 1, & v &= e^x \end{aligned}$$

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Example

$$\begin{aligned} u &= x, & v' &= e^x \\ u' &= 1, & v &= e^x \end{aligned}$$

$$\begin{aligned} \int x e^x dx &= x e^x - \int e^x dx \\ &= x e^x - e^x + c \end{aligned}$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int \ln x dx :$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int \ln x dx = \int (1)(\ln x) dx$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int \ln x dx = \int (1)(\ln x) dx$$

$$u = \ln x, \quad v' = 1$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int \ln x dx = \int (1)(\ln x) dx$$

$$u = \ln x, \quad v' = 1$$

$$u' = \frac{1}{x}, \quad v = x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int \ln x dx = \int (1)(\ln x) dx$$

$$u = \ln x, \quad v' = 1$$

$$u' = \frac{1}{x}, \quad v = x$$

$$\int \ln x dx = (\ln x)x - \int \frac{1}{x} x dx$$

3. Use known differentiation formulas “backwards”

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Example

$$\int \ln x dx = \int (1)(\ln x) dx$$

$$u = \ln x, \quad v' = 1$$

$$u' = \frac{1}{x}, \quad v = x$$

$$\int \ln x dx = (\ln x)x - \int \frac{1}{x} x dx = x \ln x - \int 1 dx$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int \ln x dx = \int (1)(\ln x) dx$$

$$u = \ln x, \quad v' = 1$$

$$u' = \frac{1}{x}, \quad v = x$$

$$\begin{aligned} \int \ln x dx &= (\ln x)x - \int \frac{1}{x} x dx = x \ln x - \int 1 dx \\ &= x \ln x - x + c \end{aligned}$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx$$

$$u = x^2, \quad v' = e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx$$

$$u = x^2, \quad v' = e^x$$

$$u' = 2x, \quad v = e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x$$

$$u = x^2, \quad v' = e^x$$

$$u' = 2x, \quad v = e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx$$

$$u = x^2, \quad v' = e^x$$

$$u' = 2x, \quad v = e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx$$

$$u = x^2, \quad v' = e^x$$

$$u' = 2x, \quad v = e^x$$

Previous example

$$\int x e^x dx = x e^x - e^x + c$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2(x e^x - e^x + c)$$

$$u = x^2, \quad v' = e^x$$

$$u' = 2x, \quad v = e^x$$

Previous example

$$\int x e^x dx = x e^x - e^x + c$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2x e^x + 2e^x - 2c$$

$$u = x^2, \quad v' = e^x$$

$$u' = 2x, \quad v = e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2x e^x + 2e^x - 2c$$

$$u = x^2, \quad v' = e^x$$

$$u' = 2x, \quad v = e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2x e^x + 2e^x + c$$

$$u = x^2, \quad v' = e^x$$

$$u' = 2x, \quad v = e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2x e^x + 2e^x + c$$

$$\int x^3 e^x dx :$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2x e^x + 2e^x + c$$

$$\int x^3 e^x dx :$$

$$u = x^3, \quad v' = e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2x e^x + 2e^x + c$$

$$\int x^3 e^x dx :$$

$$u = x^3, \quad v' = e^x$$

$$u' = 3x^2, \quad v = e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2x e^x + 2e^x + c$$

$$\int x^3 e^x dx = x^3 e^x$$

$$u = x^3, \quad v' = e^x$$

$$u' = 3x^2, \quad v = e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2x e^x + 2e^x + c$$

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx$$

$$u = x^3, \quad v' = e^x$$

$$u' = 3x^2, \quad v = e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2x e^x + 2e^x + c$$

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx$$

$$u = x^3, \quad v' = e^x$$

$$u' = 3x^2, \quad v = e^x$$

Previous example

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2x e^x + 2e^x + c$$

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx$$


$$u = x^3, \quad v' = e^x$$

$$u' = 3x^2, \quad v = e^x$$

Previous example

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx = x^2 e^x - 2x e^x + 2e^x + c$$

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx$$

$$u = x^3, \quad v' = e^x$$

$$u' = 3x^2, \quad v = e^x$$

$$= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + c$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx$$

See a pattern ?

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx$$

See a pattern ?

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx$$

$$\int x^4 e^x dx =$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx$$

See a pattern ?

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx$$

$$\int x^4 e^x dx = x^4 e^x - 4 \int x^3 e^x dx$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx$$

See a pattern ?

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx$$

$$\int x^4 e^x dx = x^4 e^x - 4 \int x^3 e^x dx$$

$$\int x^n e^x dx =$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx$$

See a pattern ?

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx$$

$$\int x^4 e^x dx = x^4 e^x - 4 \int x^3 e^x dx$$

$$\int x^n e^x dx = x^n e^x$$

3. Use known differentiation formulas “backwards”

$$\int u v' dx = uv - \int u' v dx$$

Example

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx$$

See a pattern ?

$$\int x^3 e^x dx = x^3 e^x - 3 \int x^2 e^x dx$$

$$\int x^4 e^x dx = x^4 e^x - 4 \int x^3 e^x dx$$

$$\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx \quad \text{for any power } n !$$