

MATH 250a

Fall Semester 2007

Section 2 (J. M. Cushing)

Tuesday, September 18

<http://math.arizona.edu/~cushing/250a.html>

TEST #1

DATE: Thursday, September 20

MATERIAL TO BE COVERED:

Sections 1.2-1.6, 2.3-2.5, 3.1-3.6, 4.1, 4.3, 5.2-5.4

INSTRUCTIONS:

Open book, open notes and calculators are permitted.

Chapter 7: Integration

Sections 1 – 6

Two main themes:

- Calculation of anti-derivatives (integrals)
- Numerical approximation of integrals

Sections 7 - 8

- Improper integrals

Chapter 7: Integration

Sections 1 – 6

Two main themes:

➤ Calculation of anti-derivatives (integrals)

One goal is to evaluate integrals using
the *Fundamental Theorem of Calculus*

Chapter 7: Integration

Sections 1 - 4

➤ Calculation of anti-derivatives (integrals)

1. Learn some fundamental properties of integrals

Chapter 7: Integration

Sections 1 - 4

- Calculation of anti-derivatives (integrals)
 1. Learn some fundamental properties of integrals
 2. Use known differentiation formulas “backwards”

Chapter 7: Integration

Sections 1 - 4

➤ Calculation of anti-derivatives (integrals)

1. Learn some fundamental properties of integrals
2. Use known differentiation formulas “backwards”

$f(x)$	Derivative of $f(x)$
$\sin x$	$\cos x$
x^2	$2x$
$\tan x$	$\frac{1}{1+x^2}$

Chapter 7: Integration

Sections 1 - 4

- Calculation of anti-derivatives (integrals)
 1. Learn some fundamental properties of integrals
 2. Use known differentiation formulas “backwards”

Integral of $f(x)$	$f(x)$
$\sin x$	$\cos x$
x^2	$2x$
$\tan x$	$\frac{1}{1+x^2}$

Chapter 7: Integration

Sections 1 - 4

- Calculation of anti-derivatives (integrals)
 1. Learn some fundamental properties of integrals
 2. Use known differentiation formulas “backwards”
 3. Use differentiation rules “backwards”

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chain rule $\longleftrightarrow$ substitution

Chapter 7: Integration

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- Calculation of anti-derivatives (integrals)
 1. Learn some fundamental properties of integrals
 2. Use known differentiation formulas “backwards”
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 - chain rule $\longleftrightarrow$ substitution
 - product rule $\longleftrightarrow$ integration by parts

Chapter 7: Integration

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 1. Learn some fundamental properties of integrals
 2. Use known differentiation formulas “backwards”
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 - e.g., partial fraction decomposition

Chapter 7: Integration

Sections 1 - 4

- Calculation of anti-derivatives (integrals)
 1. Learn some fundamental properties of integrals
 2. Use known differentiation formulas “backwards”
 3. Use differentiation rules “backwards”
 - chain rule $\longleftrightarrow$ substitution
 - product rule $\longleftrightarrow$ integration by parts
 4. Use algebraic manipulations
 - e.g., partial fraction decomposition
 5. Use tables of integrals

Some Notation

If $F(x)$ is an *anti-derivative* of $f(x)$,

that is to say, if $\frac{dF(x)}{dx} = f(x)$,

then we write

$$F(x) = \int f(x) dx$$

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Note

Sometimes $F(x)$ is called an *integral* of $f(x)$
and anti-differentiation is called *integration*.

1. Some fundamental properties of integrals

An anti-derivative of a function is not unique
(i.e., there is not just one).

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(i.e., there is not just one).

If $F(x)$ is an anti-derivative of $f(x)$,
then so is $F(x) + c$ for any constant c .

(why ?)

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the constant of integration

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Examples

$$\int 2x \, dx = x^2$$

$$\int \frac{1}{1+x^2} \, dx = \arctan x.$$

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Examples

$$\int 2x \, dx = x^2 + c$$

$$\int \frac{1}{1+x^2} \, dx = \arctan x + c$$

1. Some fundamental properties of integrals

Since differentiation is a “linear operator”

$$\frac{d}{dx}(c_1 f(x) + c_2 g(x)) = c_1 \frac{df(x)}{dx} + c_2 \frac{dg(x)}{dx}$$

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so is anti-differentiation is a “linear operator”

$$\int (c_1 f(x) + c_2 g(x)) dx = c_1 \int f(x) dx + c_2 \int g(x) dx$$

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$$\int (c_1 f(x) + c_2 g(x)) dx = c_1 \int f(x) dx + c_2 \int g(x) dx$$

In particular,

$$\int cf(x) dx = c \int f(x) dx$$

Example

$$\int (mx + b) dx = m \int x dx + b \int 1 dx$$

Example

$$\begin{aligned}\int (mx + b) dx &= m \int x dx + b \int 1 dx \\ &= m \frac{1}{2} x^2 + bx\end{aligned}$$

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Example

Linear operator principle also works for longer sums

$$\int (c_2 x^2 + c_1 x + c_0) dx$$

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$$\int (c_2 x^2 + c_1 x + c_0) dx = c_2 \int x^2 dx + c_1 \int x dx + c_0 \int 1 dx$$

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Linear operator principle also works for longer sums

$$\begin{aligned}\int (c_2 x^2 + c_1 x + c_0) dx &= c_2 \int x^2 dx + c_1 \int x dx + c_0 \int 1 dx \\ &= c_2 \frac{1}{3} x^3 + c_1 \frac{1}{2} x^2 + c_0 x\end{aligned}$$

Example

Linear operator principle also works for longer sums

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Example

$$\int \frac{3 + 2x + 2x^3}{1 + x^2} dx =$$

Example

$$\int \frac{3 + 2x + 2x^3}{1 + x^2} dx = \int \left(\frac{3 + 2x(1 + x^2)}{1 + x^2} \right) dx$$

Example

$$\begin{aligned}\int \frac{3 + 2x + 2x^3}{1 + x^2} dx &= \int \left(\frac{3 + 2x(1 + x^2)}{1 + x^2} \right) dx \\ &= \int \left(\frac{3}{1 + x^2} + 2x \right) dx\end{aligned}$$

Example

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Example

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Example

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3. Use known differentiation formulas “backwards”

The *substitution method* discussed in Section 7.1
is based on using the *chain rule* backwards

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same as ... $\int f'(g(x)) g'(x) dx = f(g(x))$

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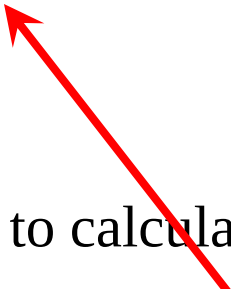
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$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

Given an integral to calculate,
can you recognize it as being in this form ?



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$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

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Example

$$\int 3(\sin x)^2 \cos x dx$$

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Integrand is a product !

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Example

$$\int \boxed{3(\sin x)^2} \cos x dx$$

One factor is a
composite function !

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$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

 Example

$$\int 3(\sin x)^2 \cos x dx$$


Other factor is the
derivative of the
“inside” function !

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$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

Example

$$\int 3(\sin x)^2 \cos x dx$$

Need $g(x)$ and $f(w)$ to calculate.



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Example

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$$g(x) = \sin x$$

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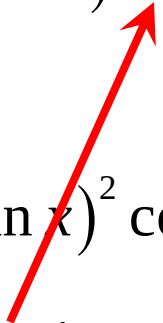
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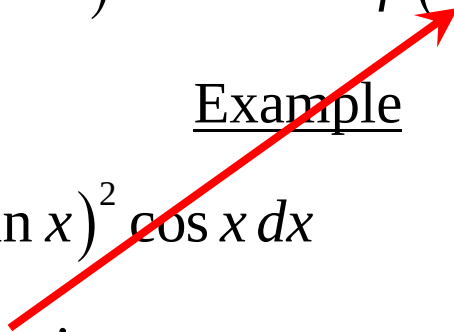
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$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

$$\int f'(\sin x) \cos x \, dx = f(\sin x) + c$$



Example

$$\int 3(\sin x)^2 \cos x \, dx$$

$$g(x) = \sin x$$

$$f'(w) = 3w^2$$

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Example

$$\int 3(\sin x)^2 \cos x \, dx$$

$$g(x) = \sin x$$

$$f'(w) = 3w^2 \quad \Rightarrow \quad f(w) = w^3$$

3. Use known differentiation formulas “backwards”

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$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

$$\int f'(\sin x) \cos x \, dx = (\sin x)^3 + c$$

Example

$$\int 3(\sin x)^2 \cos x \, dx$$

$$g(x) = \sin x$$

$$f'(w) = 3w^2 \quad \Rightarrow \quad f(w) = w^3$$



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$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

Example

$$\int 3(\sin x)^2 \cos x dx = \sin^3 x + c$$

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The *substitution method* discussed in Section 7.1 is based on using the *chain rule* backwards

$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

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Example

$$\int 3(\sin x)^2 \cos x dx$$

This method can be streamlined as follows ...

3. Use known differentiation formulas “backwards”

The *substitution method* discussed in Section 7.1 is based on using the *chain rule* backwards

$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

Example

$$\int 3(\sin x)^2 \cos x dx$$

We chose $g(x) = \sin x$ and $f'(w) = 3w^2$

and *substituted*

$$w = g(x)$$

3. Use known differentiation formulas “backwards”

The *substitution method* discussed in Section 7.1 is based on using the *chain rule* backwards

$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

Example

$$\int 3(\sin x)^2 \cos x dx$$

Or ... we changed variables from x to w using

$$w = g(x)$$

3. Use known differentiation formulas “backwards”

The *substitution method* discussed in Section 7.1 is based on using the *chain rule* backwards

$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

Example

$$\int 3(w)^2 \cos x dx$$

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The *substitution method* discussed in Section 7.1 is based on using the *chain rule* backwards

$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

Example

$$\int 3(w)^2 \frac{dw}{dx} dx$$

Or ... we changed variables from x to w using

$$w = g(x)$$

3. Use known differentiation formulas “backwards”

The *substitution method* discussed in Section 7.1 is based on using the *chain rule* backwards

$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

Example

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$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

Example

$$\int 3(w)^2 dw \quad (\text{notational shorthand})$$

Or ... we changed variables from x to w using

$$w = g(x)$$

3. Use known differentiation formulas “backwards”

The *substitution method* discussed in Section 7.1 is based on using the *chain rule* backwards

$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

Example

$$\int 3(w)^2 dw = w^3 + c$$

Or ... we changed variables from x to w using

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3. Use known differentiation formulas “backwards”

The *substitution method* discussed in Section 7.1 is based on using the *chain rule* backwards

$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

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Example

$$\int 3(w)^2 dw = w^3 + c$$

Or ... we changed variables from x to w using

$$w = g(x)$$

$$w = \sin x$$

3. Use known differentiation formulas “backwards”

The *substitution method* discussed in Section 7.1 is based on using the *chain rule* backwards

$$\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$$

$$\int f'(g(x)) g'(x) dx = f(g(x)) + c$$

Example

$$\int 3(w)^2 dw = \sin^3 x + c$$

Or ... we changed variables from x to w using

$$w = g(x)$$

$$w = \sin x$$

3. Use known differentiation formulas “backwards”

Method of Substitution

Change variables from x to w using

$$w = g(x), \quad dw = g'(x)dx$$

3. Use known differentiation formulas “backwards”

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Change variables from x to w using

$$w = g(x), \quad dw = g'(x)dx$$

Example

$$\int e^{-x^2} x dx :$$

3. Use known differentiation formulas “backwards”

Method of Substitution

Change variables from x to w using

$$w = g(x), \quad dw = g'(x)dx$$

Example

$$\int e^{-x^2} x dx :$$

$$w = -x^2$$

3. Use known differentiation formulas “backwards”

Method of Substitution

Change variables from x to w using

$$w = g(x), \quad dw = g'(x)dx$$

Example

$$\int e^{-x^2} x dx :$$

$$w = -x^2$$

$$\frac{dw}{dx} = -2x$$

3. Use known differentiation formulas “backwards”

Method of Substitution

Change variables from x to w using

$$w = g(x), \quad dw = g'(x)dx$$

Example

$$\int e^{-x^2} x dx :$$

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3. Use known differentiation formulas “backwards”

Method of Substitution

Change variables from x to w using

$$w = g(x), \quad dw = g'(x)dx$$

Example

$$\int e^{-x^2} x dx$$

$$w = -x^2$$

$$dw = -2x dx \quad \Rightarrow \quad x dx = -\frac{1}{2} dw$$

3. Use known differentiation formulas “backwards”

Method of Substitution

Change variables from x to w using

$$w = g(x), \quad dw = g'(x)dx$$

Example

$$\int e^{-x^2} x dx = \int e^w \left(-\frac{1}{2} dw \right)$$

$$w = -x^2$$

$$dw = -2x dx \quad \Rightarrow \quad x dx = -\frac{1}{2} dw$$

3. Use known differentiation formulas “backwards”

Method of Substitution

Change variables from x to w using

$$w = g(x), \quad dw = g'(x)dx$$

Example

$$\int e^{-x^2} x dx = -\frac{1}{2} \int e^w dw$$

$$w = -x^2$$

$$dw = -2x dx \quad \Rightarrow \quad x dx = -\frac{1}{2} dw$$

3. Use known differentiation formulas “backwards”

Method of Substitution

Change variables from x to w using

$$w = g(x), \quad dw = g'(x)dx$$

Example

$$\int e^{-x^2} x dx = -\frac{1}{2} \int e^w dw = -\frac{1}{2} e^w + c$$

$$w = -x^2$$

$$dw = -2x dx \quad \Rightarrow \quad x dx = -\frac{1}{2} dw$$

3. Use known differentiation formulas “backwards”

Method of Substitution

Change variables from x to w using

$$w = g(x), \quad dw = g'(x)dx$$

Example

$$\int e^{-x^2} x dx = -\frac{1}{2} \int e^w dw = -\frac{1}{2} e^{-x^2} + c$$

$$w = -x^2$$


$$\frac{dw}{dx} = -2x \quad \Rightarrow \quad xdx = -\frac{1}{2}dw$$

3. Use known differentiation formulas “backwards”

Method of Substitution

Change variables from x to w using

$$w = g(x), \quad dw = g'(x)dx$$

Note:

Integral doesn't necessarily have to be in
the form for the reverse chain rule

3. Use known differentiation formulas “backwards”

Example

$$\int z(z+1)^{1/3} dz = ?$$

3. Use known differentiation formulas “backwards”

Example

$$\int z(z+1)^{1/3} dz = ?$$

$$w = (z+1)^{1/3}$$

3. Use known differentiation formulas “backwards”

Example

$$\int z(z+1)^{1/3} dz = ?$$

$$w = (z+1)^{1/3} \Rightarrow z = w^3 - 1$$

3. Use known differentiation formulas “backwards”

Example

$$\int z(z+1)^{1/3} dz = \int (w^3 - 1)w dz$$

$$w = (z+1)^{1/3} \Rightarrow z = w^3 - 1$$

3. Use known differentiation formulas “backwards”

Example

$$\int z(z+1)^{1/3} dz = \int (w^3 - 1) \underbrace{w}_{?} dz$$

$$w = (z+1)^{1/3} \Rightarrow z = w^3 - 1$$

3. Use known differentiation formulas “backwards”

Example

$$\int z(z+1)^{1/3} dz = \int (w^3 - 1) \underbrace{w}_{?} dz$$

$$w = (z+1)^{1/3} \Rightarrow z = w^3 - 1$$

$$\frac{dz}{dw} = 3w^2$$

3. Use known differentiation formulas “backwards”

Example

$$\int z(z+1)^{1/3} dz = \int (w^3 - 1) w dz \quad ?$$

$$w = (z+1)^{1/3} \Rightarrow z = w^3 - 1$$

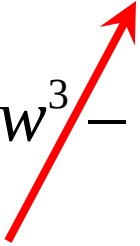
$$\frac{dz}{dw} = 3w^2 \Rightarrow dz = 3w^2 dw$$

3. Use known differentiation formulas “backwards”

Example

$$\int z(z+1)^{1/3} dz = \int (w^3 - 1) w 3w^2 dw$$

$$w = (z+1)^{1/3} \Rightarrow z = w^3 - 1$$

$$\frac{dz}{dw} = 3w^2 \Rightarrow dz = 3w^2 dw$$


3. Use known differentiation formulas “backwards”

Example

$$\int z(z+1)^{1/3} dz = \int (w^3 - 1) w 3w^2 dw$$

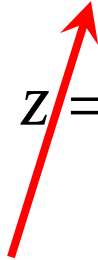
$$w = (z+1)^{1/3} \Rightarrow z = w^3 - 1$$


a polynomial in w !

3. Use known differentiation formulas “backwards”

Example

$$\int z(z+1)^{1/3} dz = 3 \int (w^6 - w^3) dw$$

$$w = (z+1)^{1/3} \Rightarrow z = w^3 - 1$$


a polynomial in w !

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Example

$$\int z(z+1)^{1/3} dz = 3 \left(\frac{1}{7} w^7 - \frac{1}{4} w^4 \right) + c$$

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Example

$$\int z(z+1)^{1/3} dz = 3 \left(\frac{1}{7} (z+1)^{7/3} - \frac{1}{4} (z+1)^{4/3} \right) + c$$