

MATH 250a

Fall Semester 2007

Section 2 (J. M. Cushing)

Thursday, September 13

<http://math.arizona.edu/~cushing/250a.html>

Some Basic Facts About Sine and Cosine

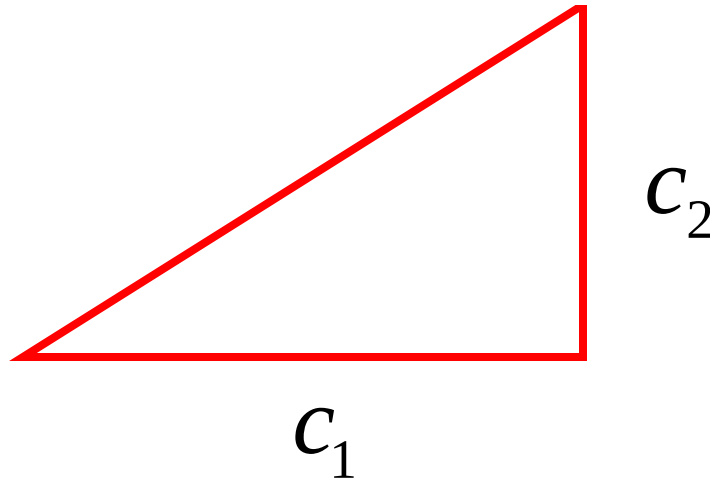
$$\sin^2 x + \cos^2 x = 1 \quad \text{for all } x$$

$$c_1 \sin \omega x + c_2 \cos \omega x = A \sin(\omega x + \beta)$$

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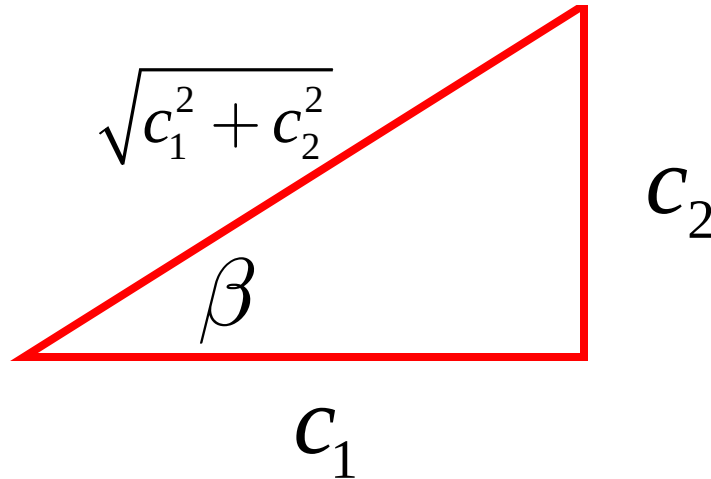
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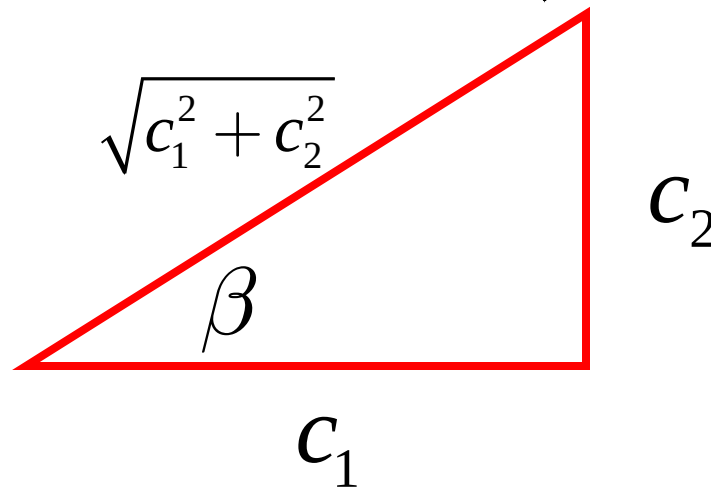


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$$A = \sqrt{c_1^2 + c_2^2}, \quad \sin \beta = \frac{c_2}{\sqrt{c_1^2 + c_2^2}}$$

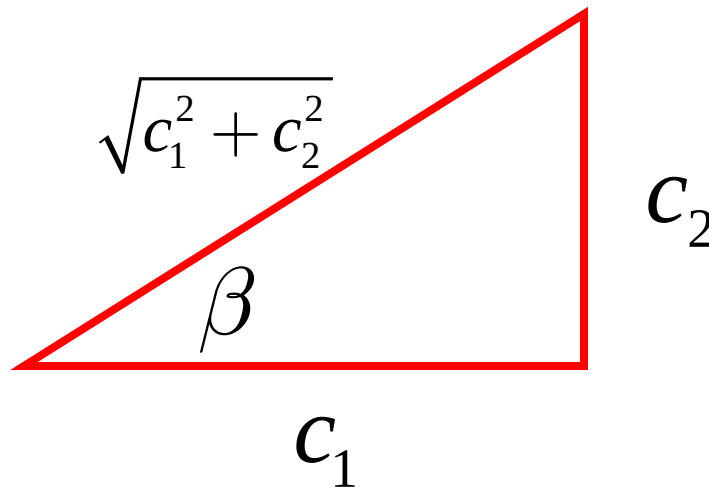


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$$A = \sqrt{c_1^2 + c_2^2}, \quad \tan \beta = \frac{c_2}{c_1} \quad \leftarrow \text{or}$$



Example

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$$\omega = 1$$

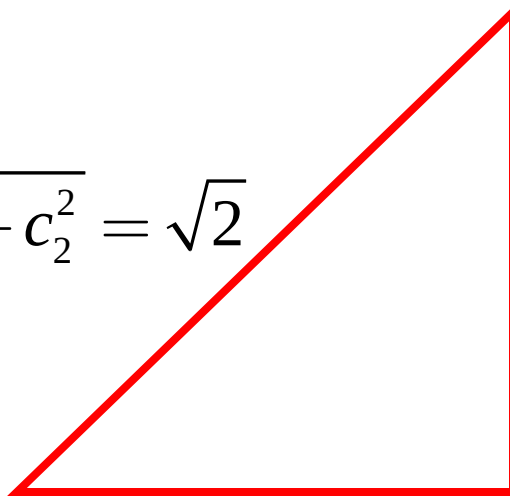
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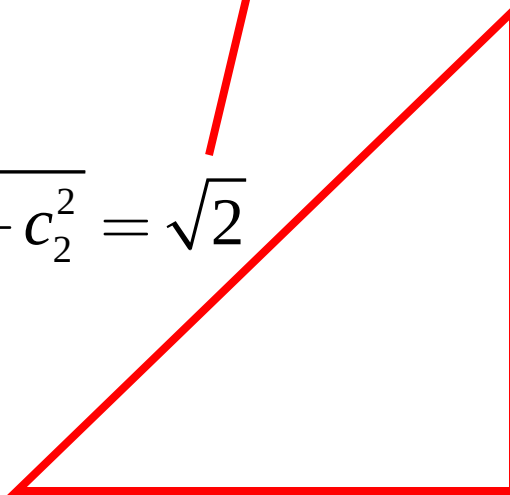
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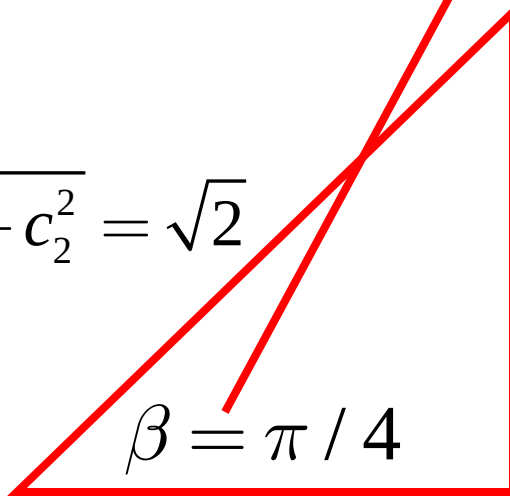
$$\sin x + \cos x = \sqrt{2} \sin \left(x + \frac{\pi}{4} \right)$$

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$$\beta = \pi / 4$$

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Example

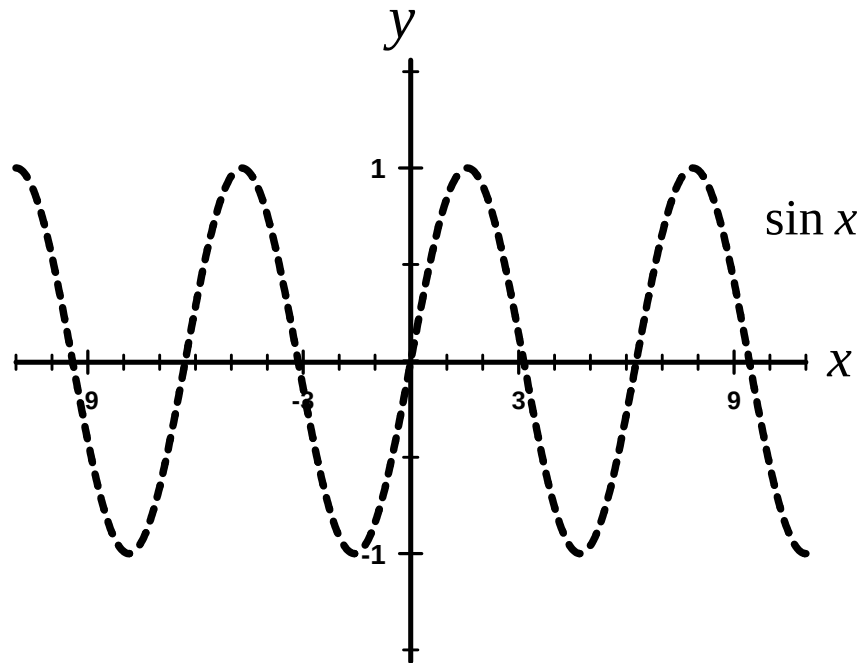
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$$\text{period} = 2\pi, \text{ amplitude} = \sqrt{2}, \text{ phase} = -\frac{\pi}{4}$$

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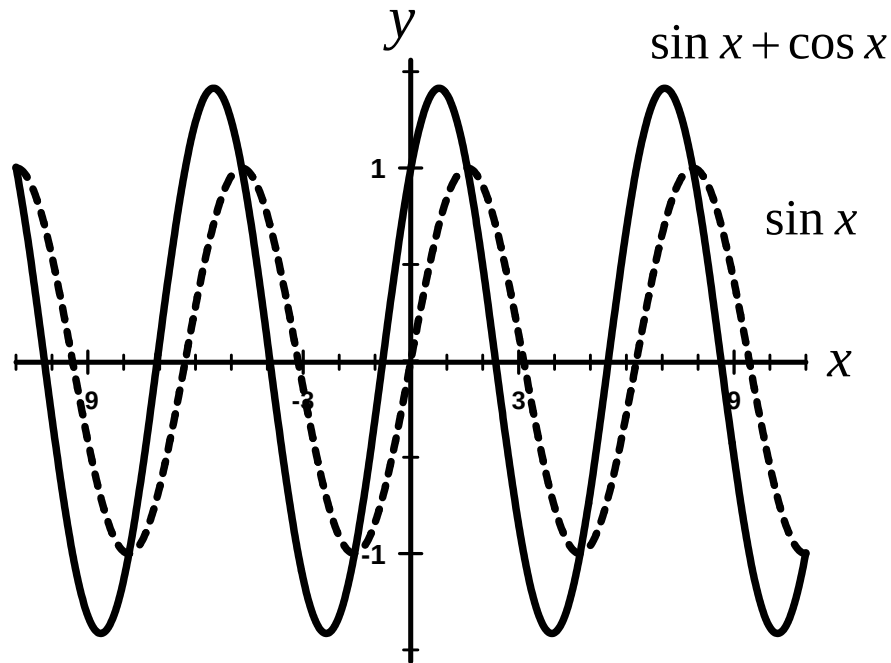
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Other Trig Functions

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}$$

have singularities at roots of denominators

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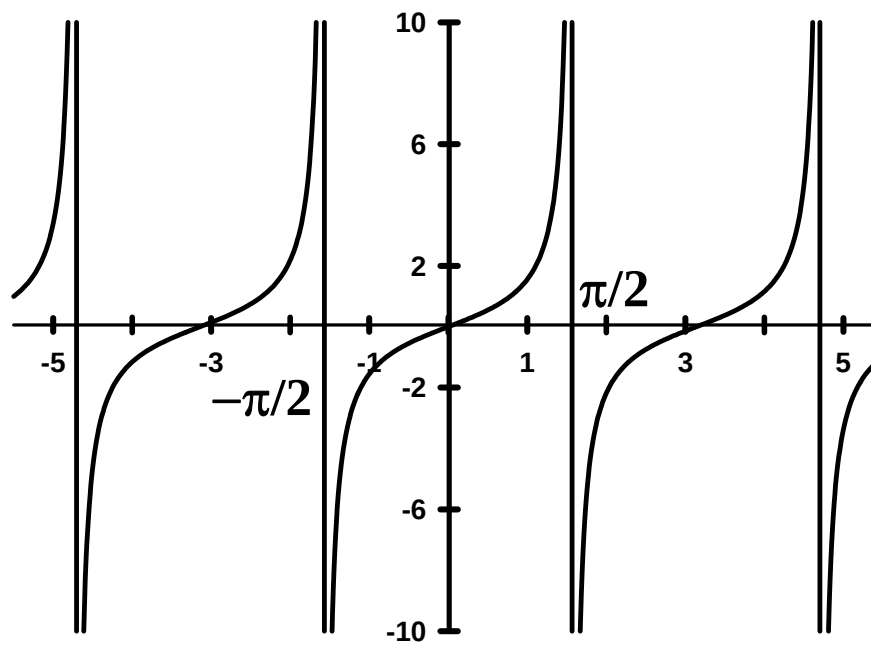
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$\tan x$, $\cot x$ are periodic with period π

$\sec x$, $\csc x$ are periodic with period 2π

Other Trig Functions

$$y = \tan x$$



Basic Facts About Trig Functions

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$$\frac{d \tan x}{dx} = \frac{\cos x \frac{d \sin x}{dx} - \sin x \frac{d \cos x}{dx}}{\cos^2 x}$$

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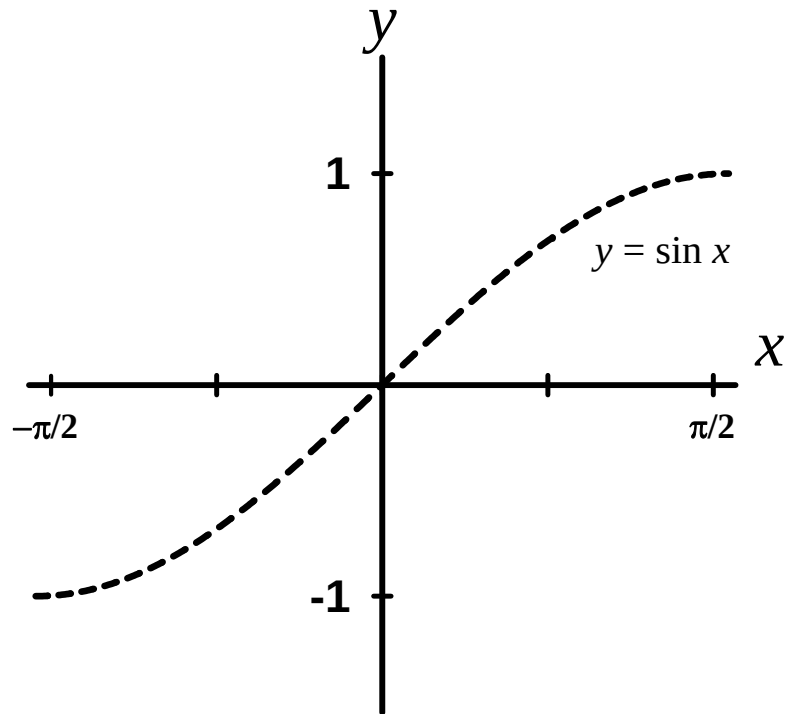
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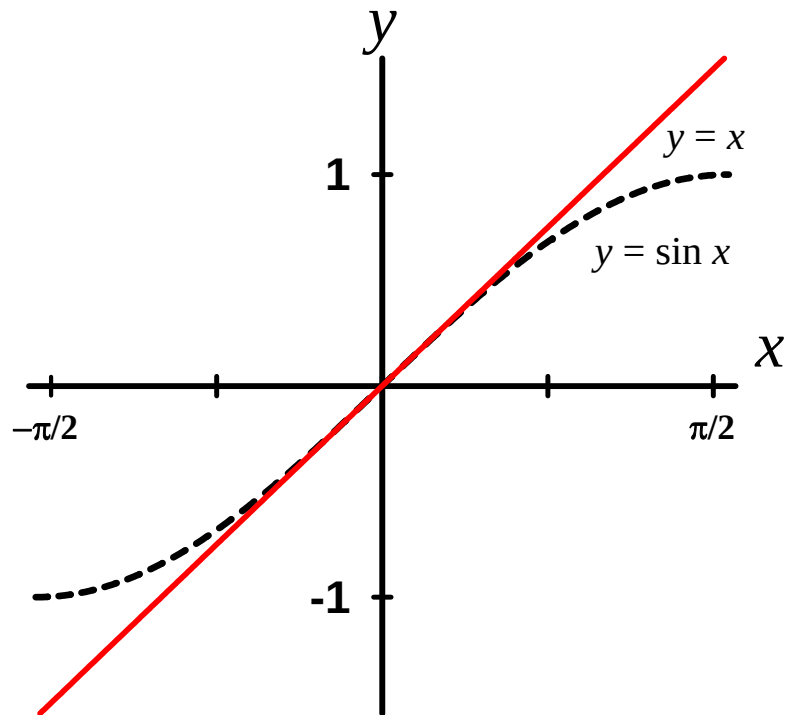
with range $-\pi/2 \leq x \leq \pi/2$

the *principle branch* of the inverse

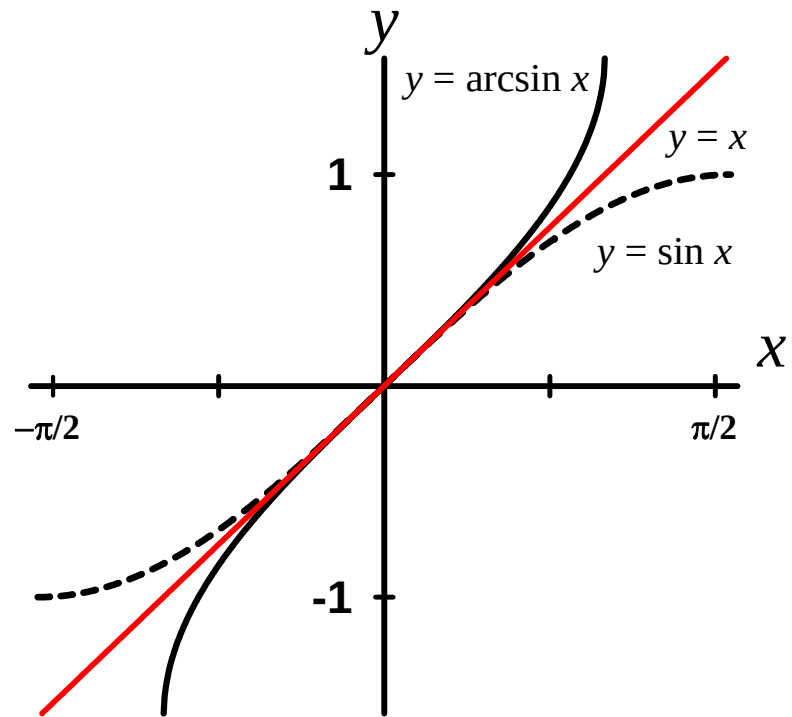
Inverse Trig Functions



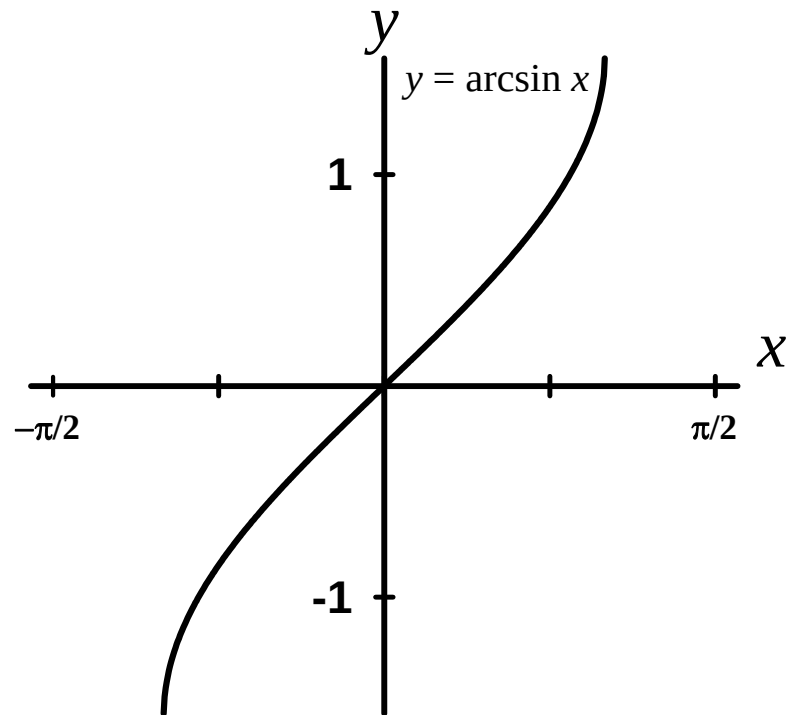
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Inverse Trig Functions

Review inverses of other trig functions on page 33-34.

Inverse Trig Functions

Differentiation of inverse functions ?

Apply the chain rule to (page 140)

$$f(f^{-1}(x)) = x$$

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$$f(x) = \sin x, \quad f^{-1}(x) = \arcsin x$$

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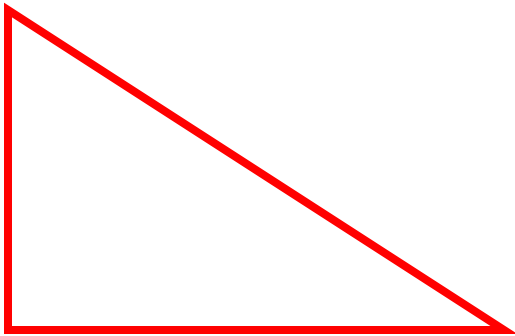
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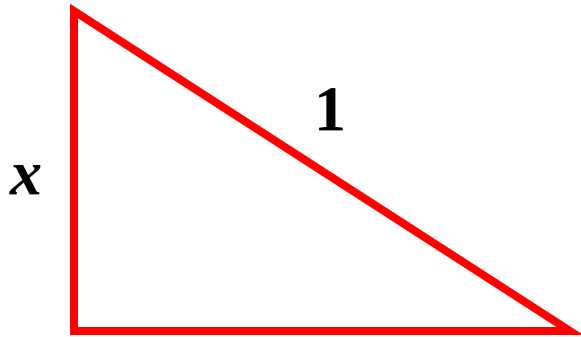
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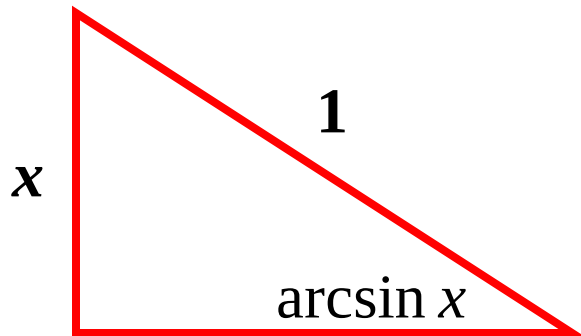
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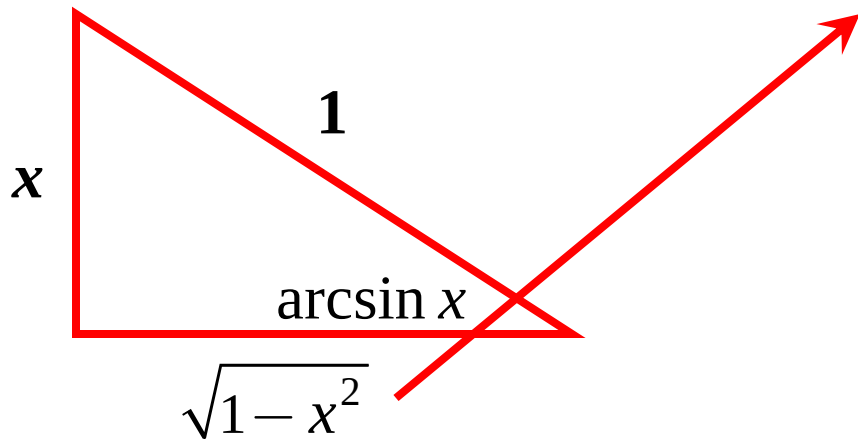
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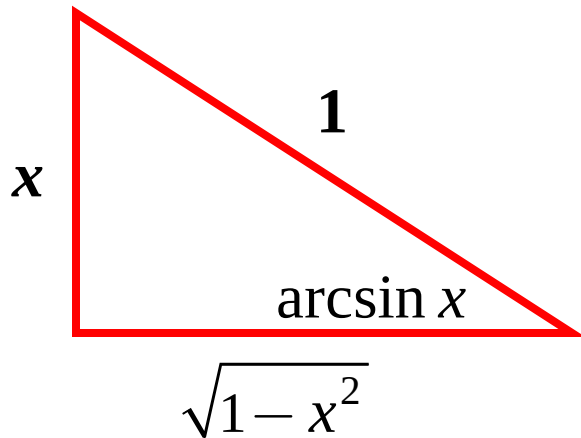
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$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

Chapter 6: Integration Techniques

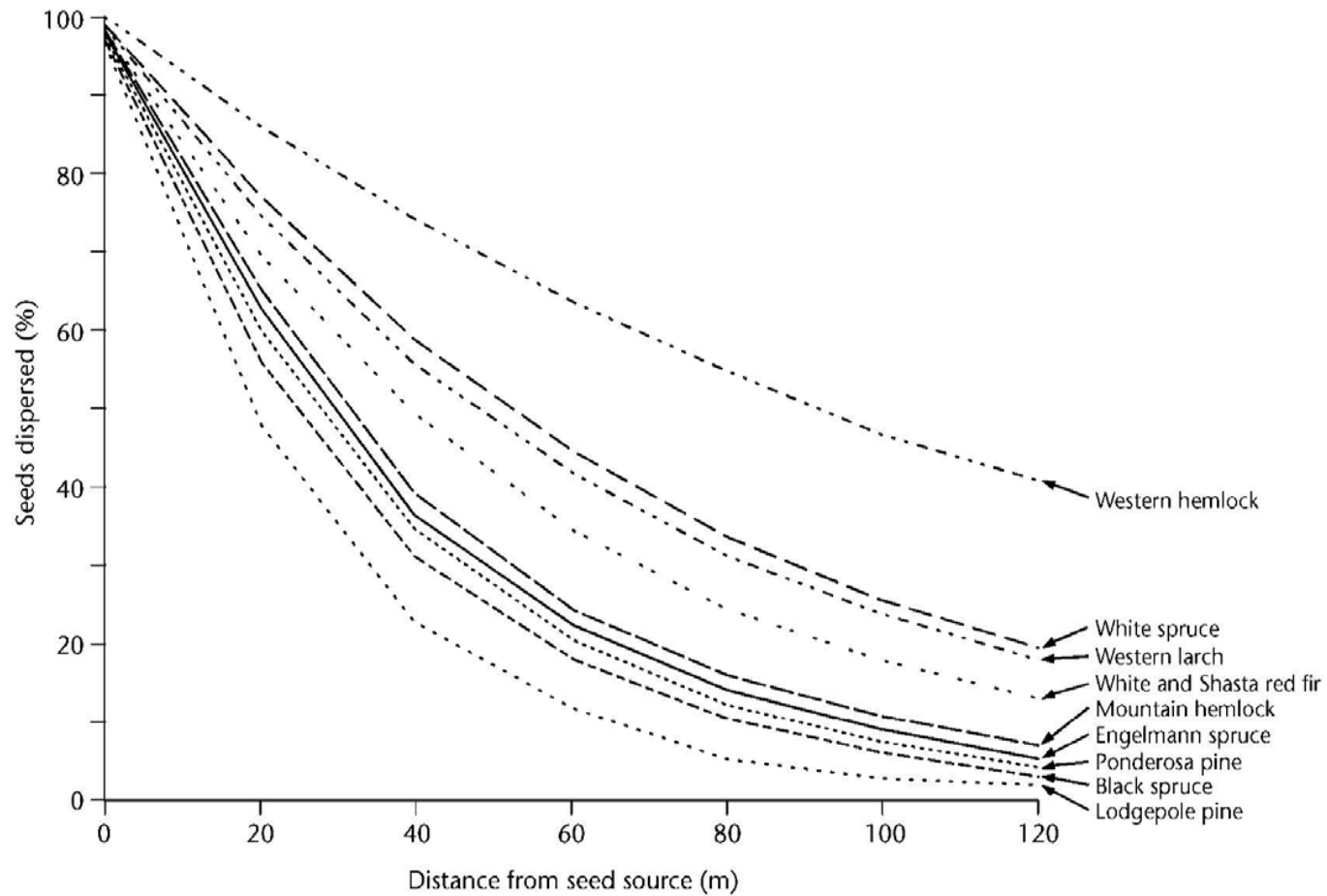
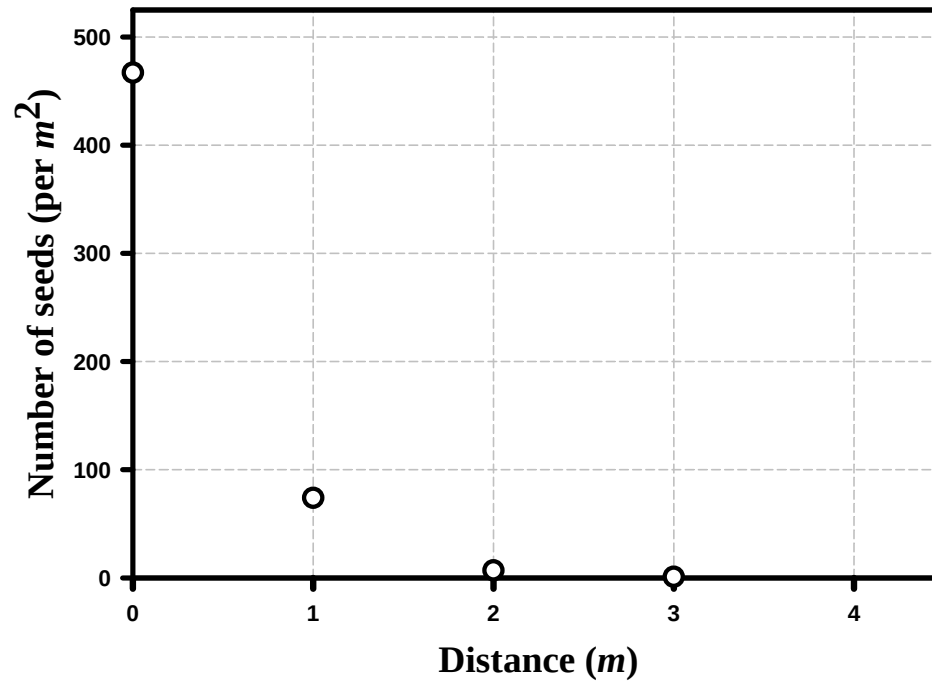


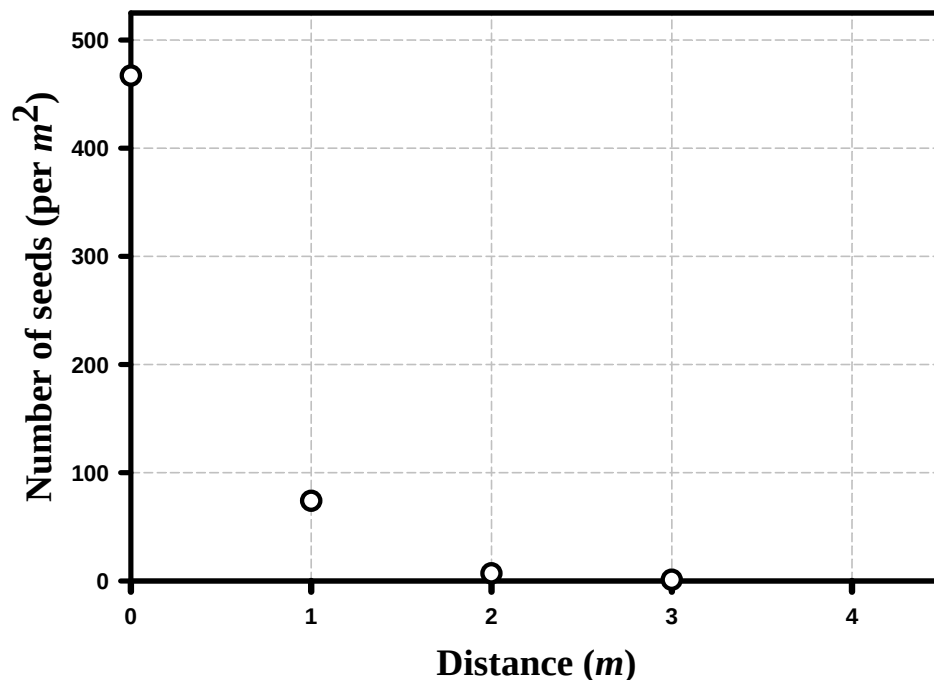
FIGURE 4.2 Seed dispersal curves for nine conifers of the Inland Mountain West (McCaughey et al. 1986). Dispersal characteristics of different species can be compared by showing the relative number (percentage) of seeds found at different distances from the seed source.

Seed Dispersal *Eriogonum fasciculatum*



Distance (m)	Number of Seeds (per m)
0	467
1	74
2	7
3	1

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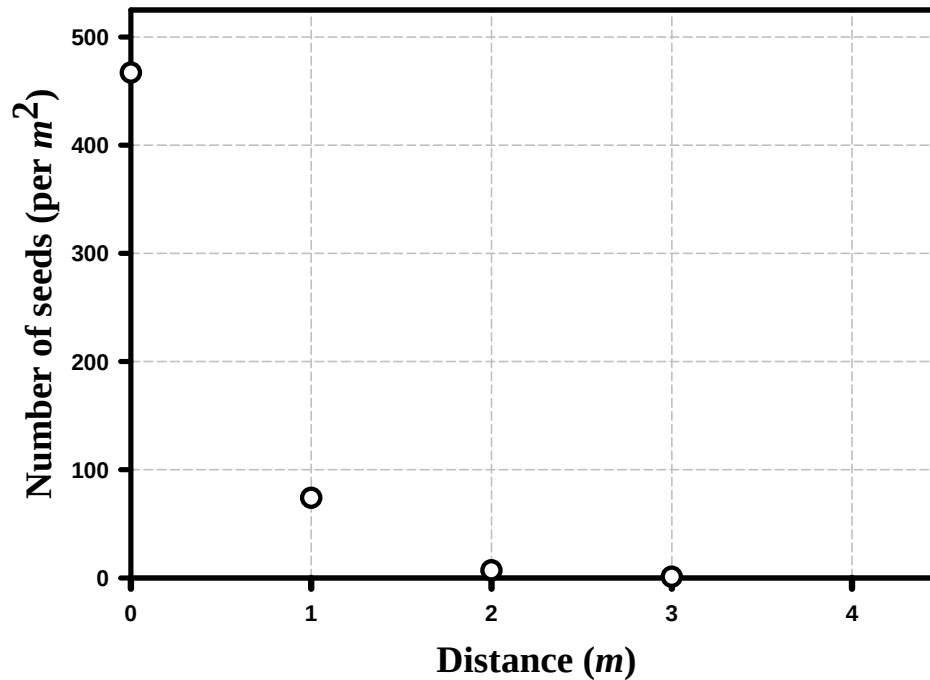


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Some Typical Questions

- What is the proportion of seeds between a and b meters away?
- What's the probability a seed will fall between a and b meters away?
- What is the mean dispersal distance?

Seed Dispersal *Eriogonum fasciculatum*



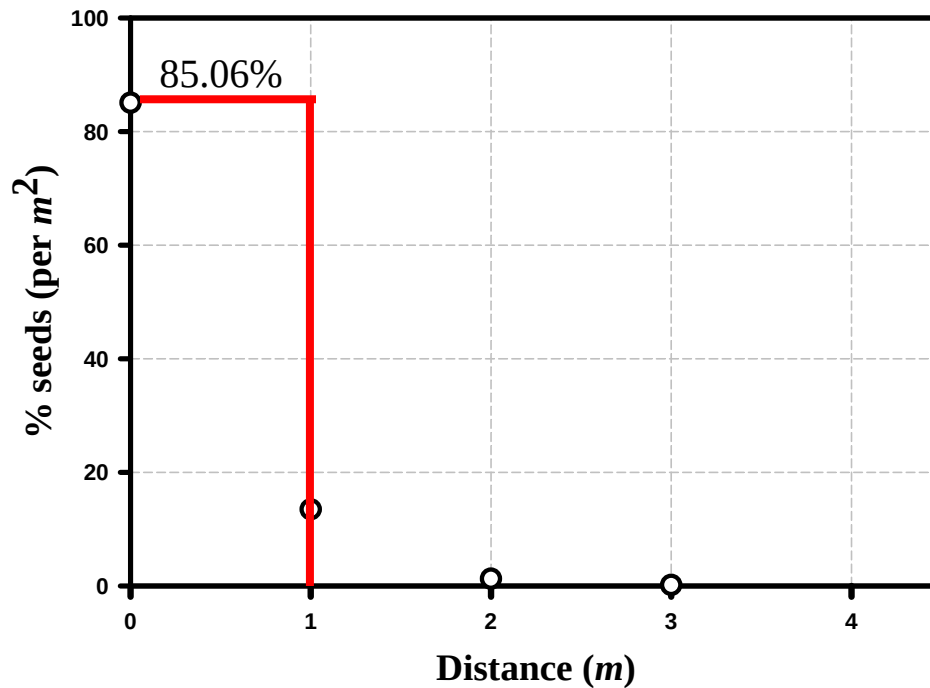
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Total number of seeds = 549

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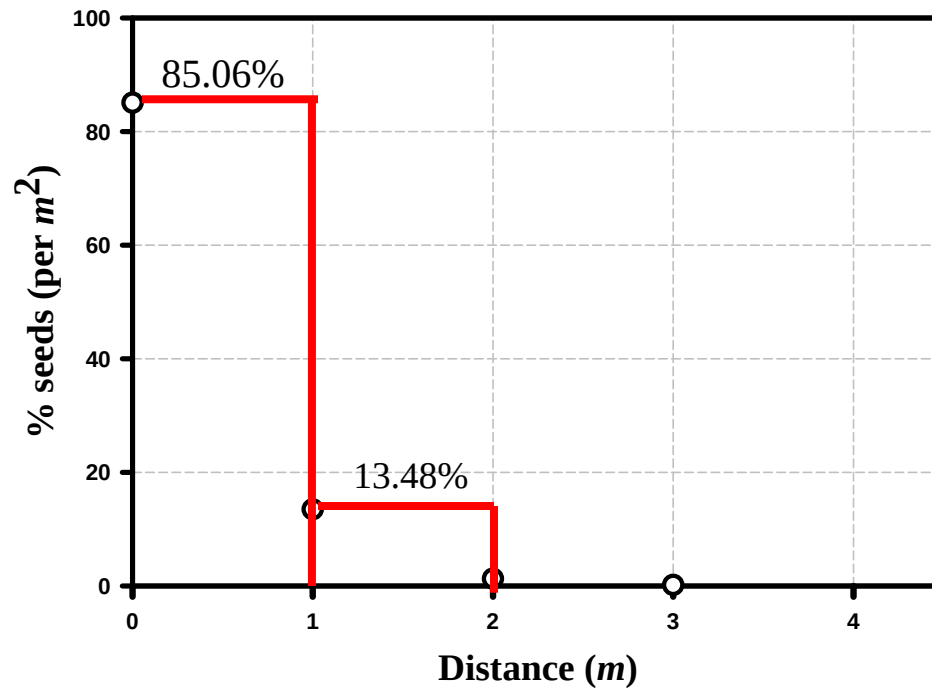


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- What is the proportion of seeds between 0 and 2 meters away?

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1	13.48
2	1.275
3	0.1821

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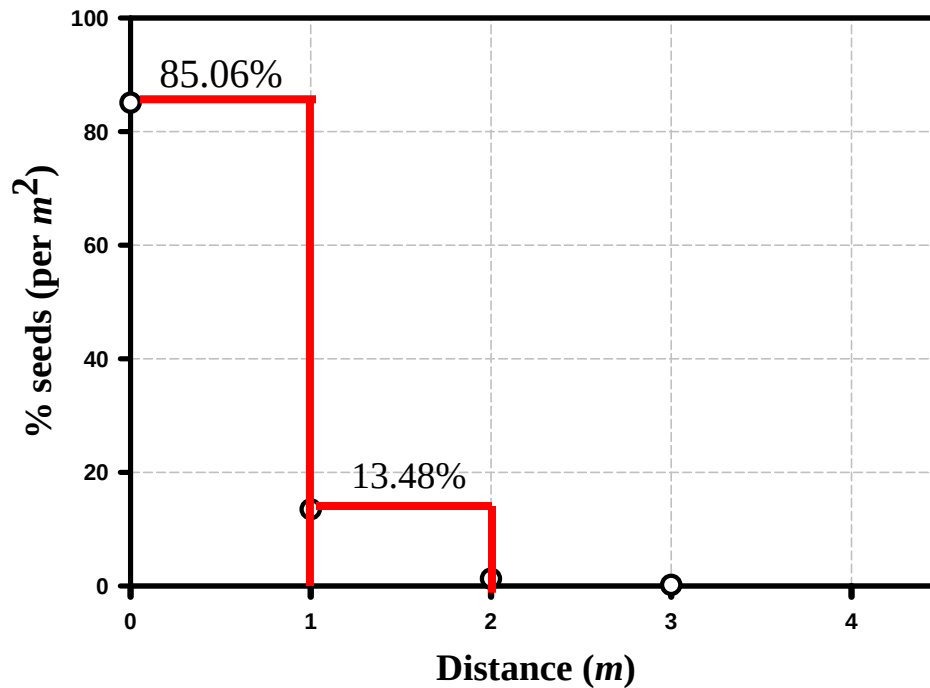


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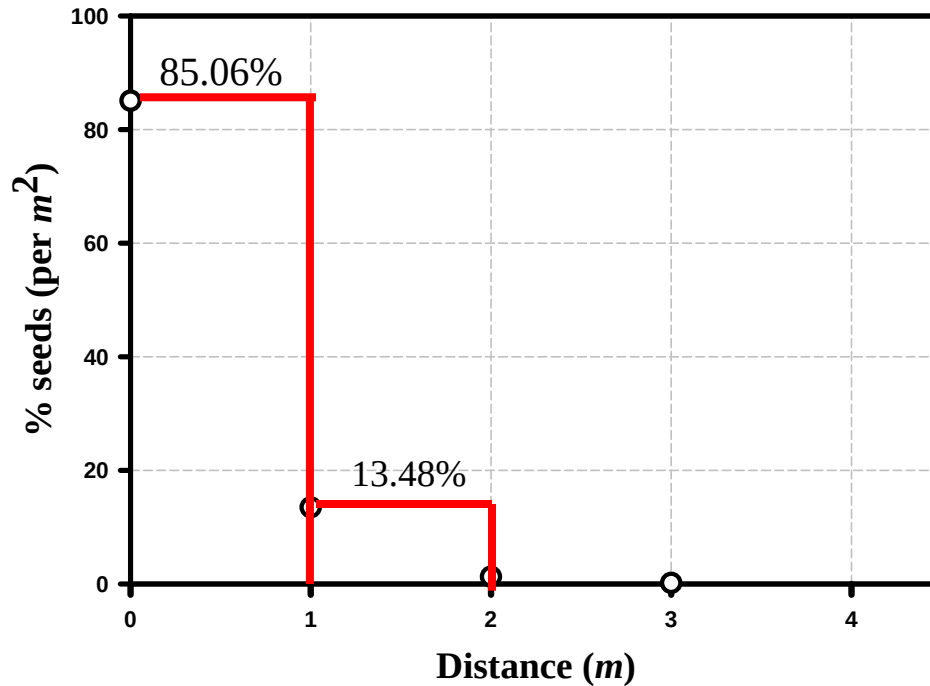
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$$85.06\% + 13.48\% = 98.54\%$$

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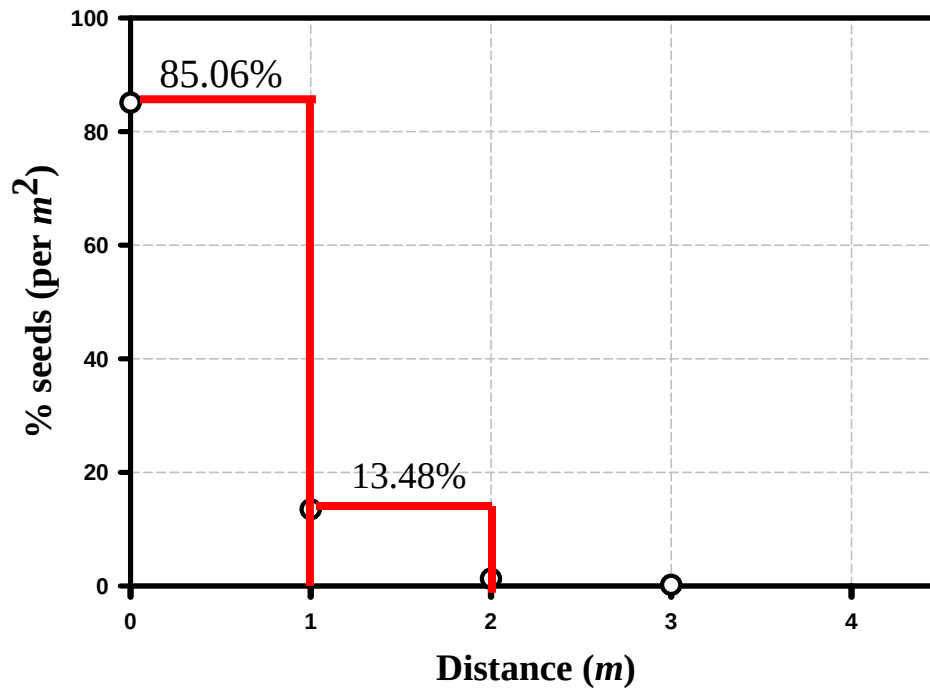
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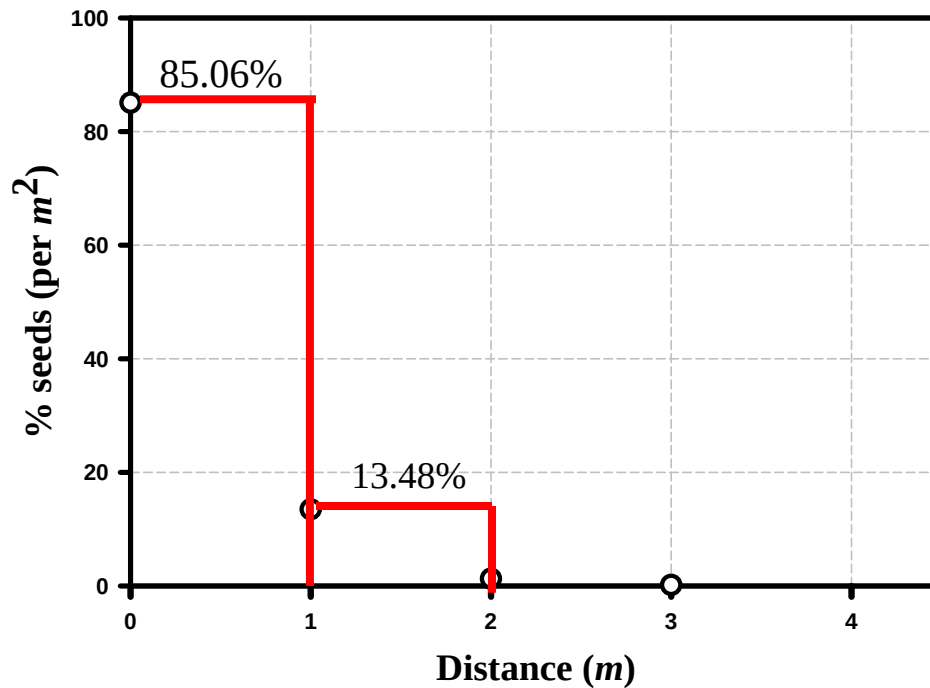
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It is unlikely that the density remains constant from 0m to 1m and instantaneously drops at 2m.

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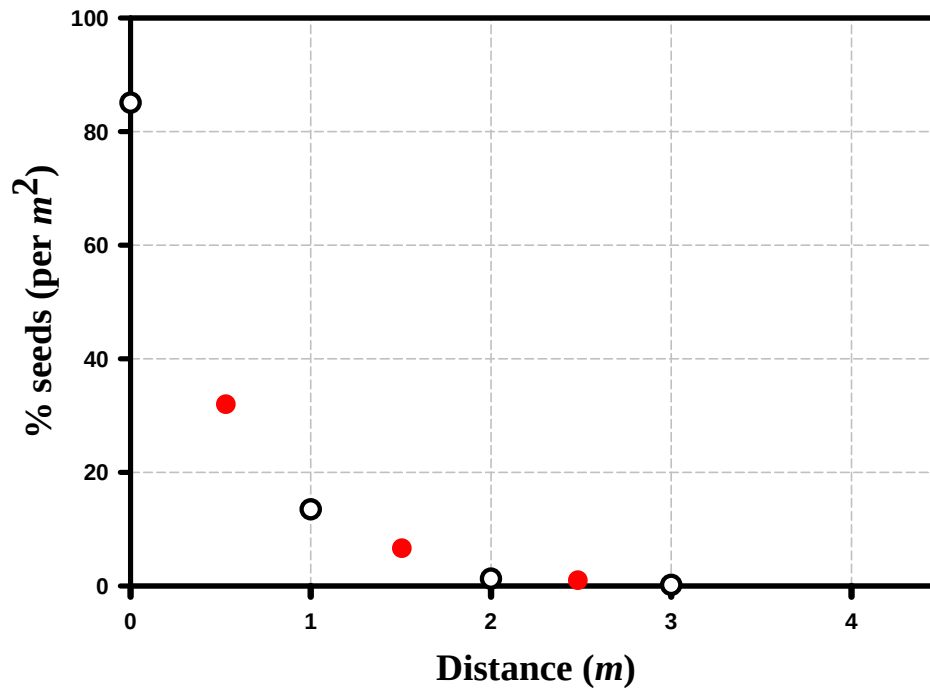
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Seed Dispersal *Eriogonum fasciculatum*



Suppose we had more data.

Some Typical Questions

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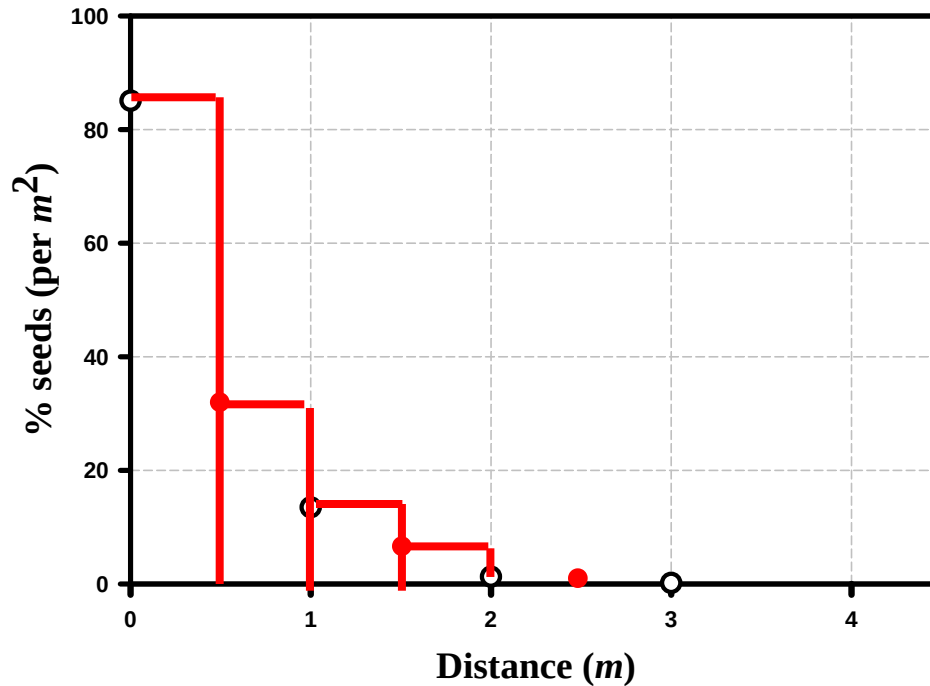
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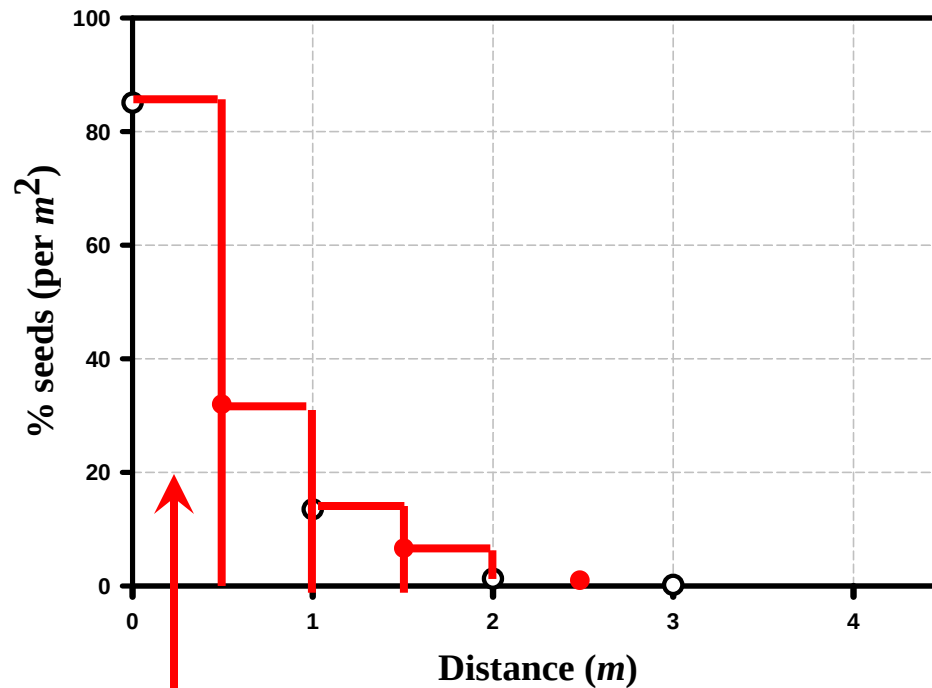
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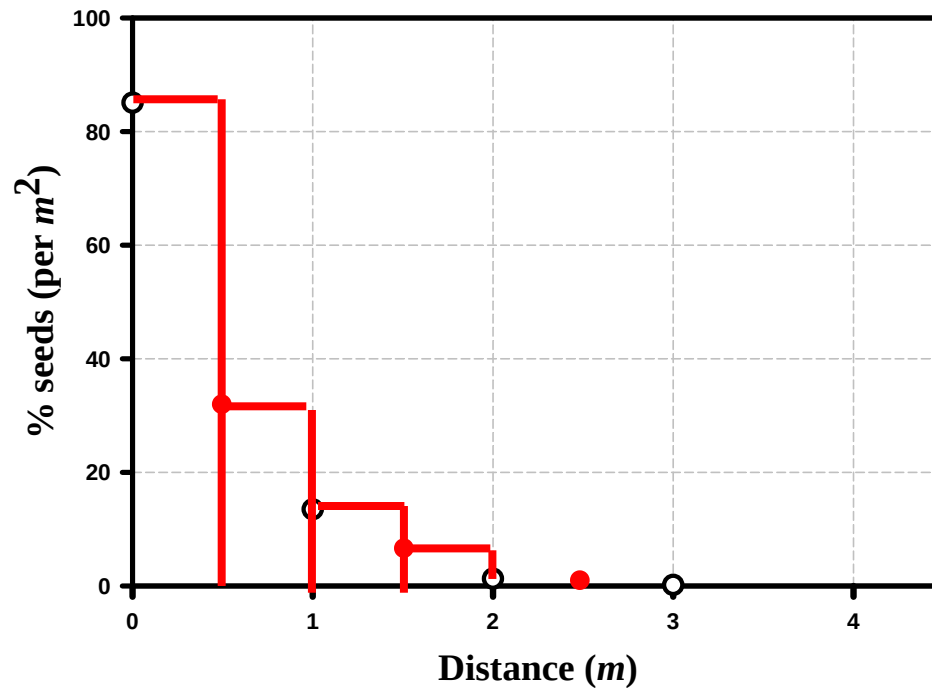


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$\frac{1}{2} \times 85.06\%$, etc.

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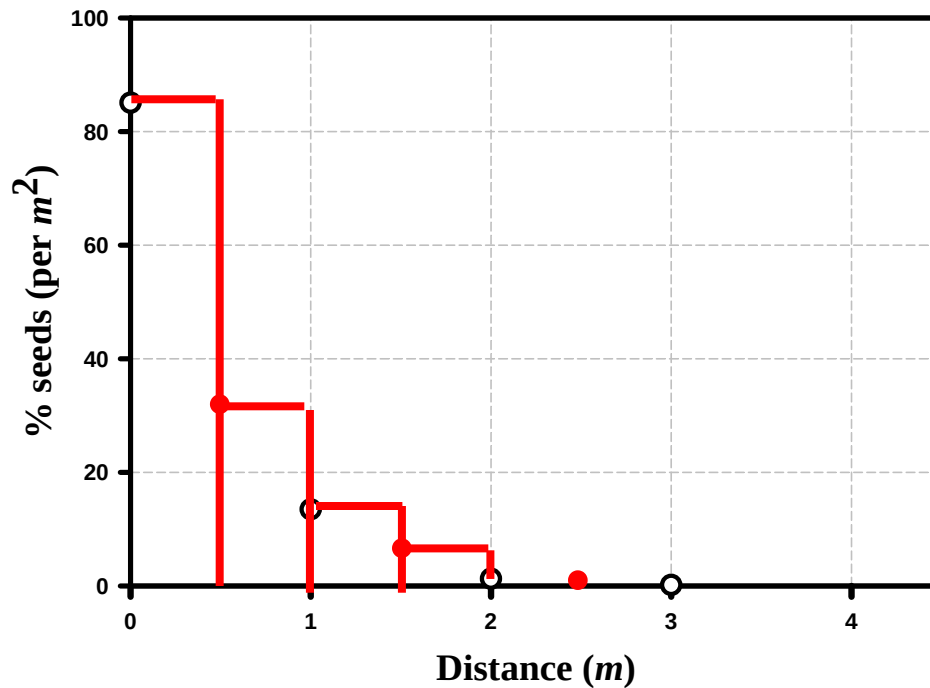


Some Typical Questions

- What is the proportion of seeds between 0 and 2 meters away?

Answer = sum of four rectangular areas

Seed Dispersal *Eriogonum fasciculatum*



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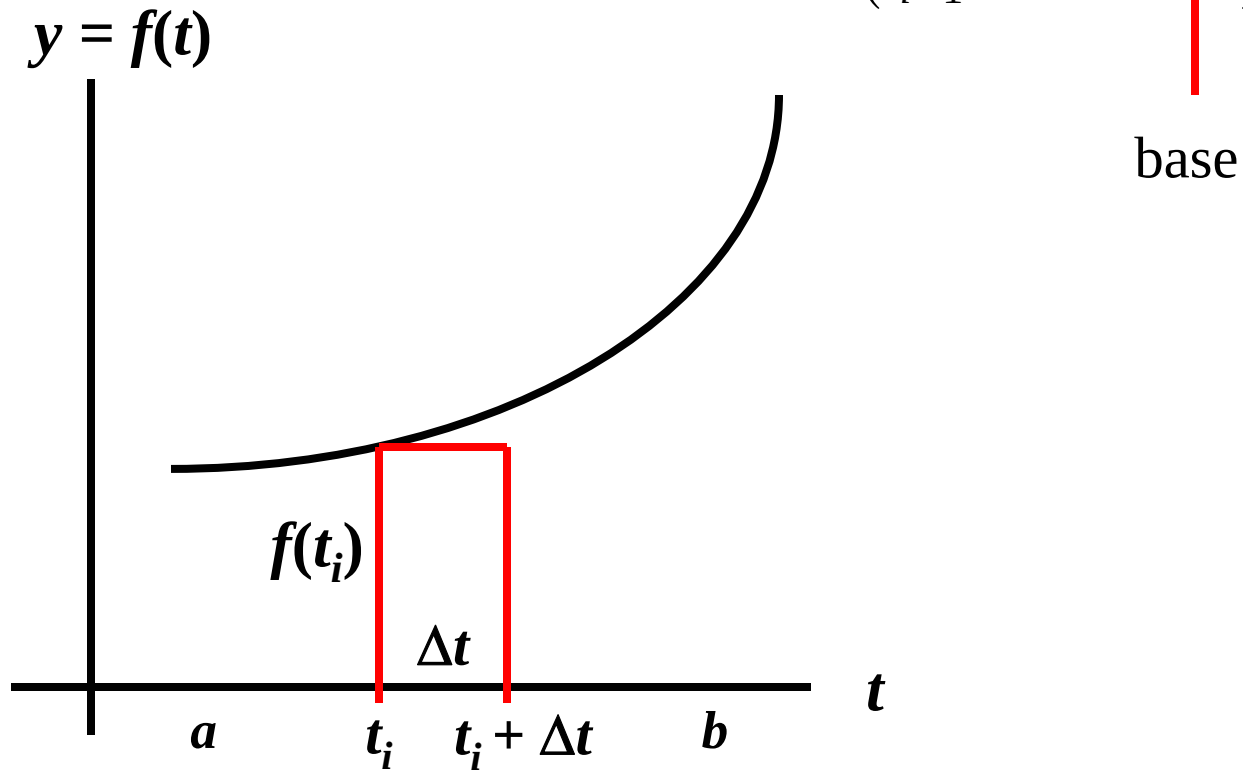
This procedure suggests that we are approximating
the area under a continuous density function
(that the data is a sampling).

Review Of Definite Integrals

$$\int_a^b f(t)dt = \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(t_i) \Delta t \right)$$

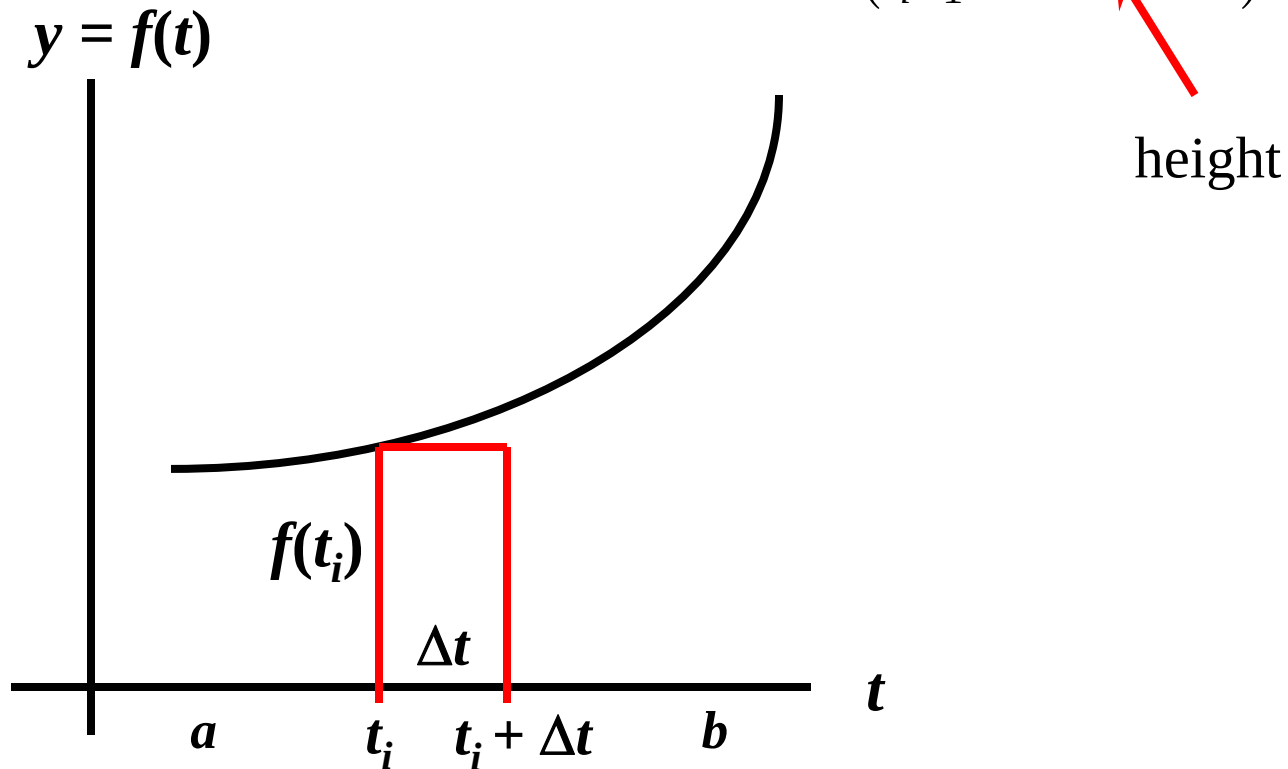
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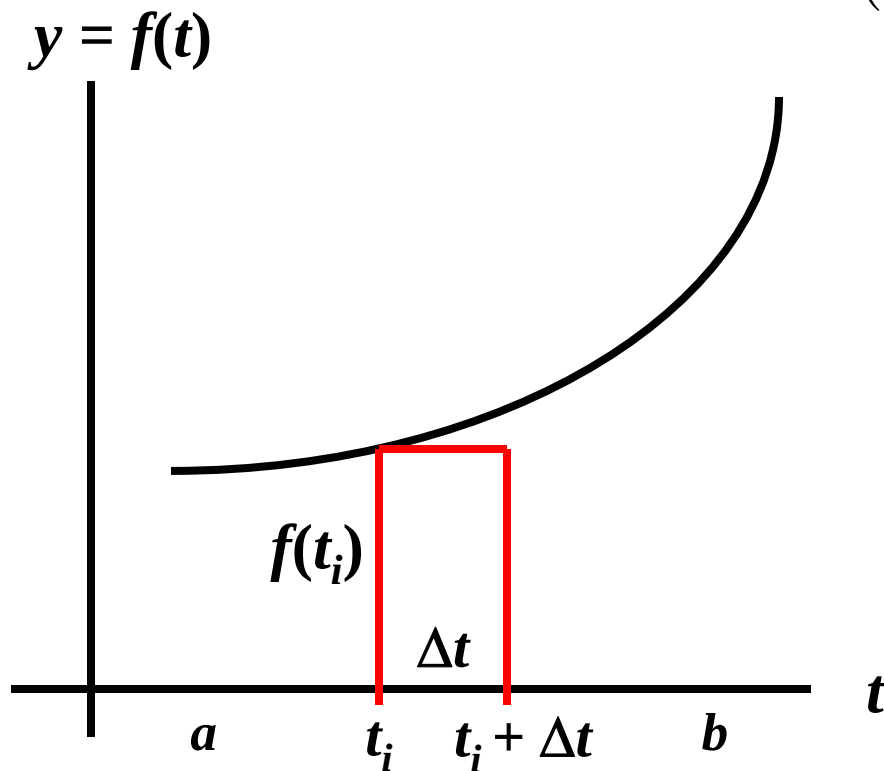
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Review Of Definite Integrals

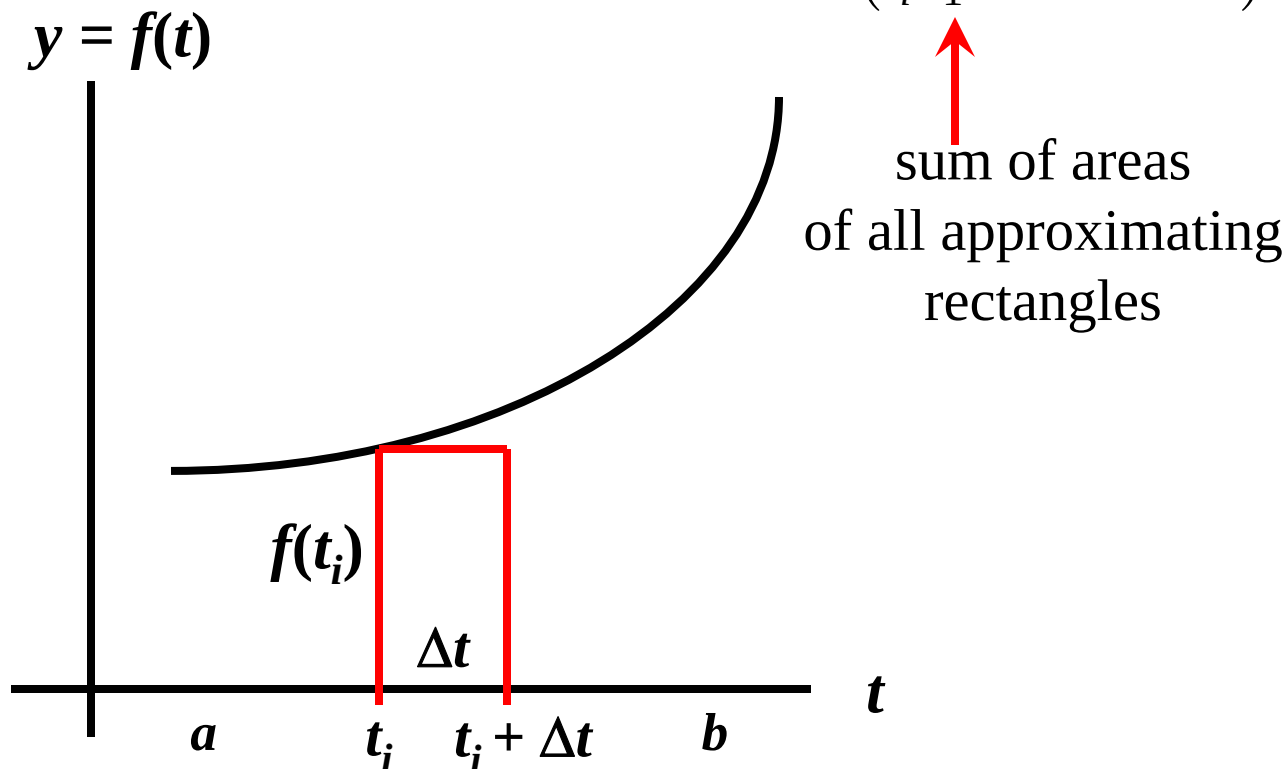
$$\int_a^b f(t) dt = \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(t_i) \Delta t \right)$$



↑
area of
approximating
rectangle

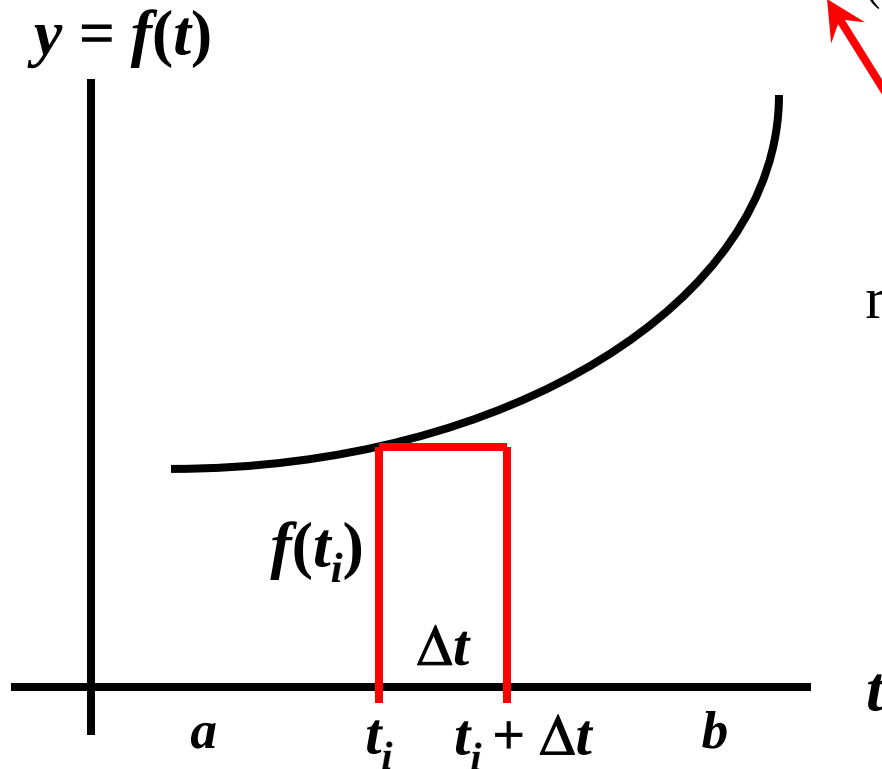
Review Of Definite Integrals

$$\int_a^b f(t) dt = \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(t_i) \Delta t \right)$$



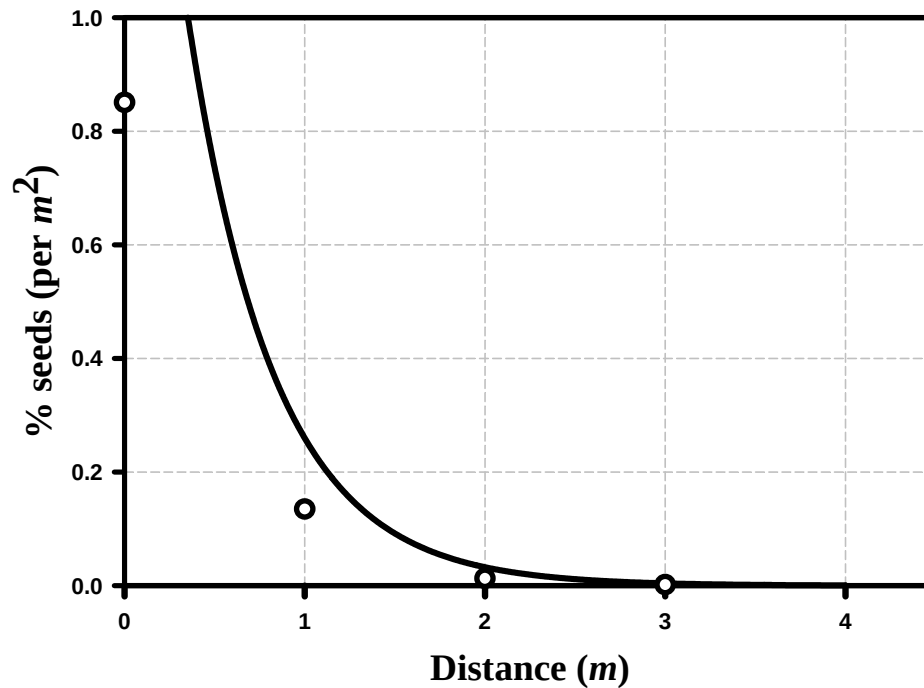
Review Of Definite Integrals

$$\int_a^b f(t) dt = \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(t_i) \Delta t \right)$$



limit as number of
approximating
rectangles increases
without bound

Seed Dispersal *Eriogonum fasciculatum*

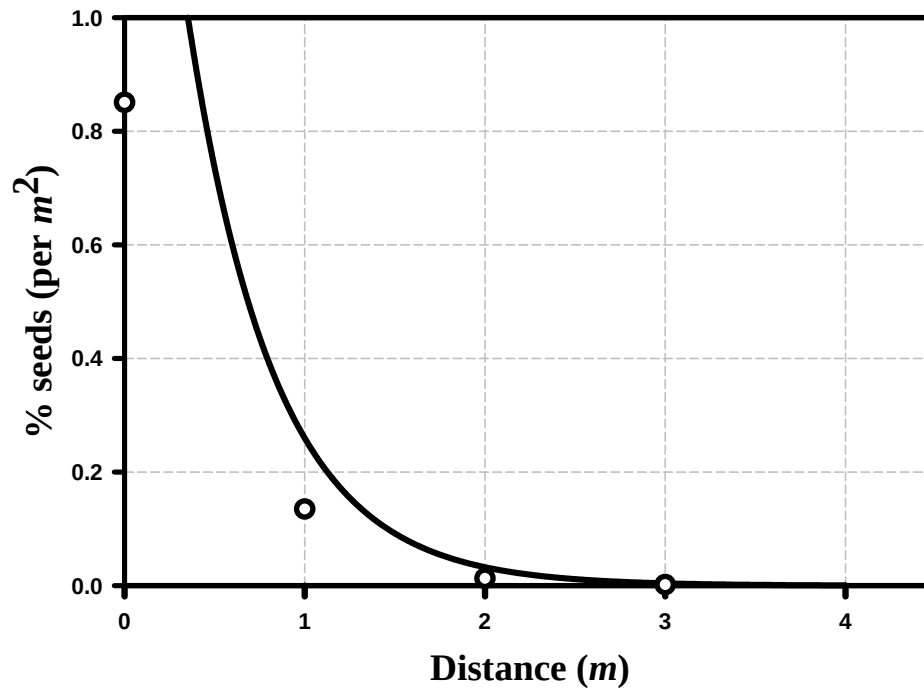


Some Typical Questions

- What is the proportion of seeds between 0 and 2 meters away?

For example, $f(x) = 2.08e^{-2.08x}$.

Seed Dispersal *Eriogonum fasciculatum*



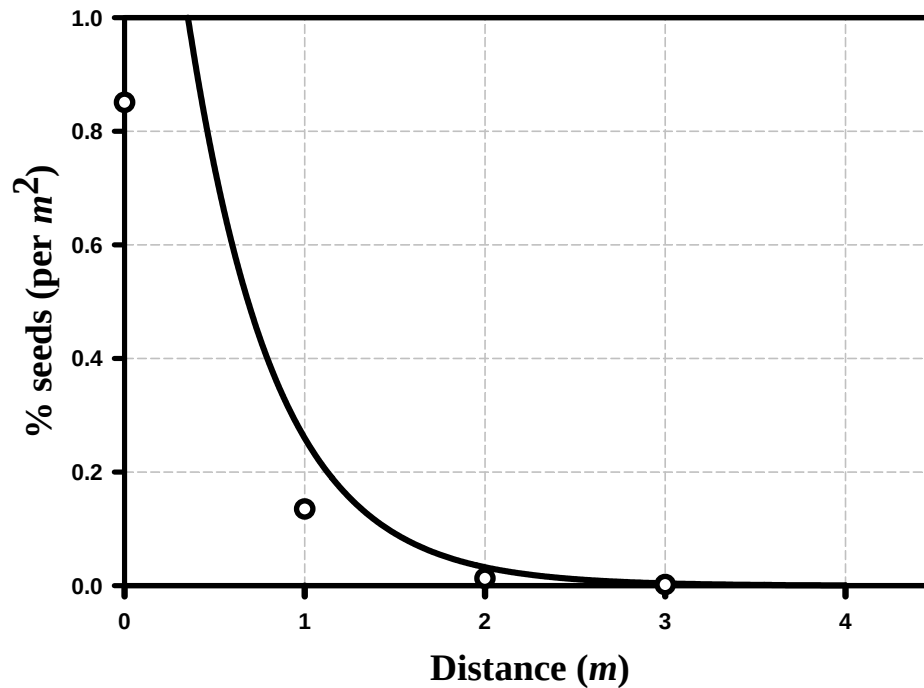
Some Typical Questions

- What is the proportion of seeds between 0 and 2 meters away?

$$\int_0^2 f(x)dx = \int_0^2 2.08e^{-2.08x}dx$$

For example, $f(x) = 2.08e^{-2.08x}$.

Seed Dispersal *Eriogonum fasciculatum*



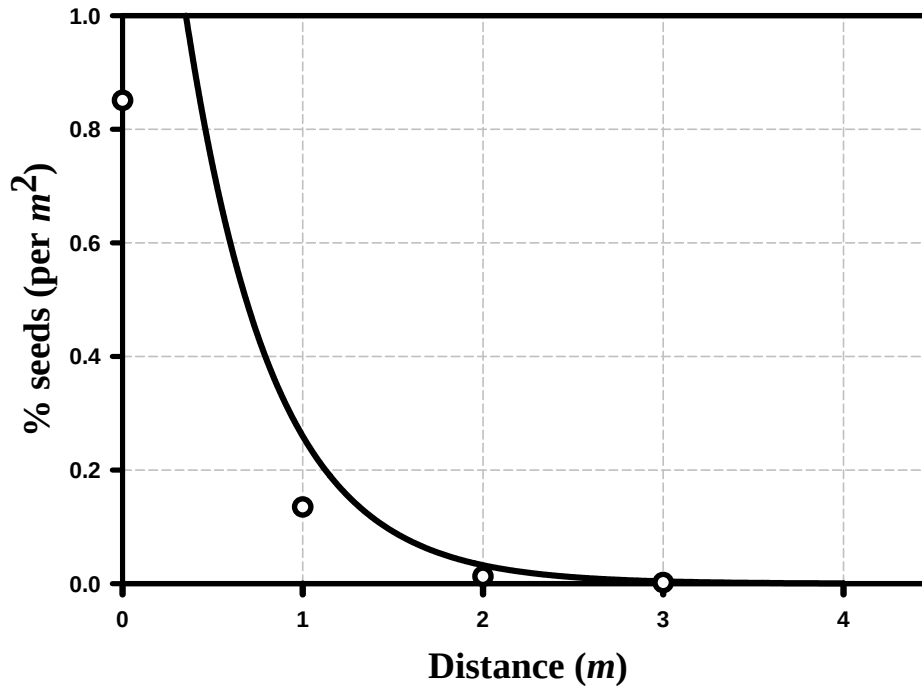
Some Typical Questions

- What is the proportion of seeds between 0 and 2 meters away?

$$\int_0^2 f(x)dx = \int_0^2 2.08e^{-2.08x}dx = ?$$

For example, $f(x) = 2.08e^{-2.08x}$.

Seed Dispersal *Eriogonum fasciculatum*



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Some Typical Questions

- What is the proportion of seeds between 0 and 2 meters away?

$$\int_0^2 f(x)dx = \int_0^2 2.08e^{-2.08x}dx = ?$$

Can evaluate using the
Fundamental Theorem of
Calculus

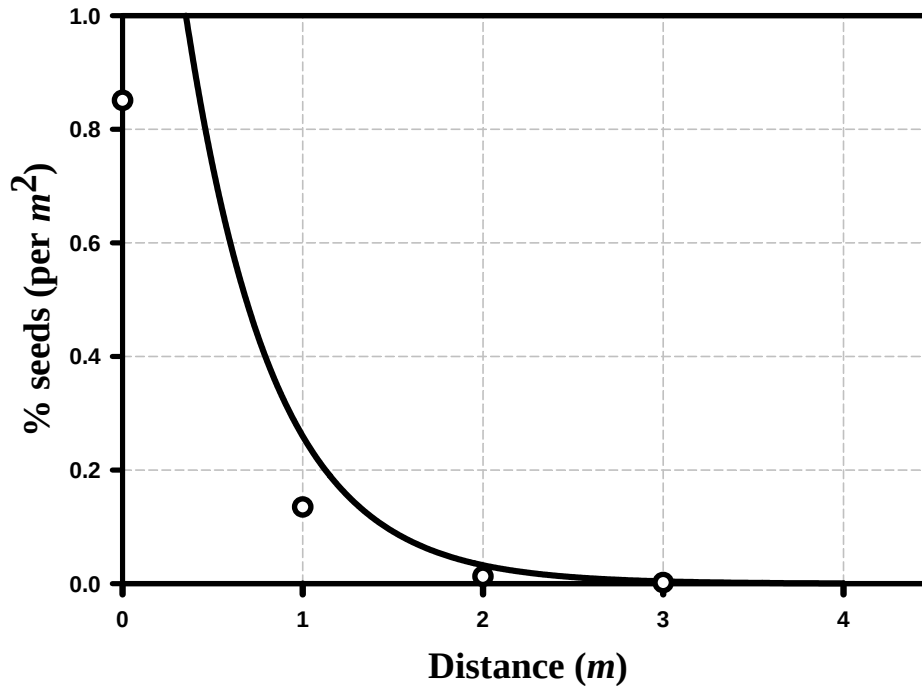
Fundamental Theorem of Calculus

$$\int_a^b f(t)dt = F(b) - F(a)$$

where

$$\frac{dF(t)}{dt} = f(t)$$

Seed Dispersal *Eriogonum fasciculatum*



For example, $f(x) = 2.08e^{-2.08x}$.

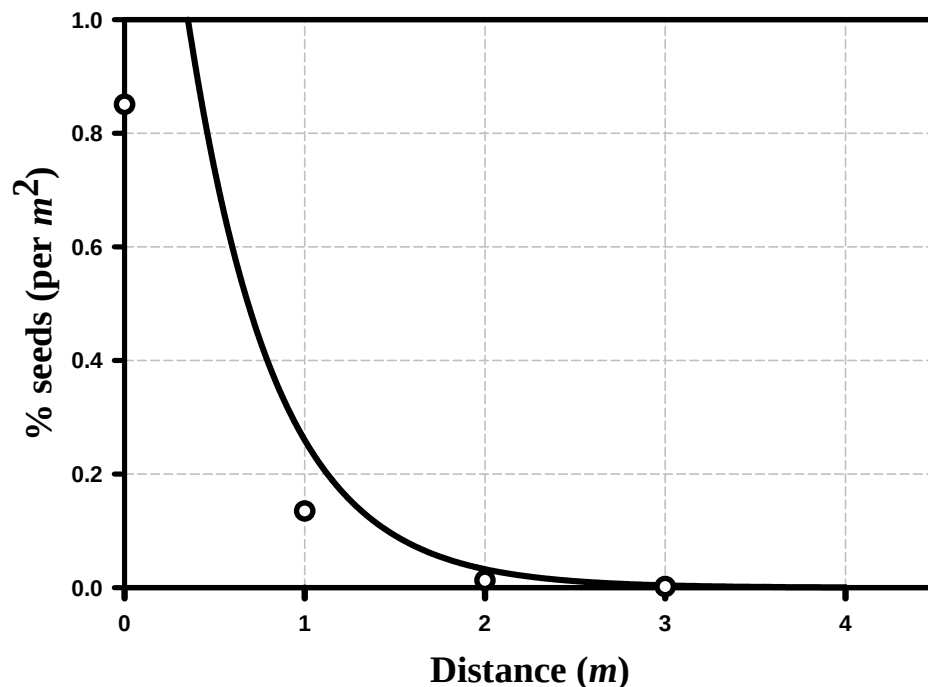
Some Typical Questions

- What is the proportion of seeds between 0 and 2 meters away?

$$\int_0^2 f(x)dx = \int_0^2 2.08e^{-2.08x}dx = ?$$

$$f(x) = 2.08e^{-2.08x} \Rightarrow F(x) = ?$$

Seed Dispersal *Eriogonum fasciculatum*



For example, $f(x) = 2.08e^{-2.08x}$.

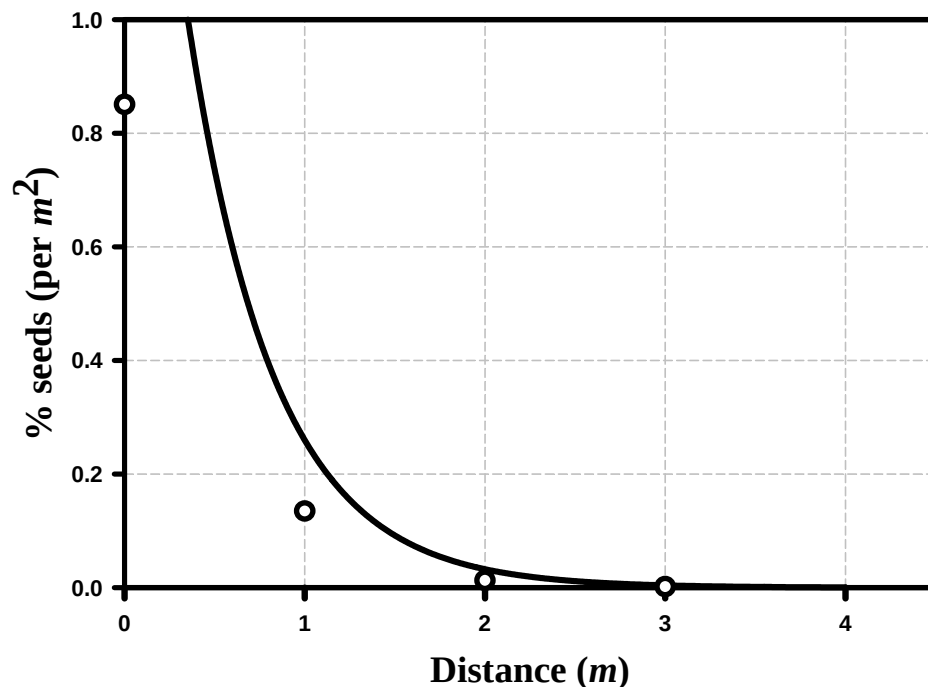
Some Typical Questions

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$$\int_0^2 f(x)dx = \int_0^2 2.08e^{-2.08x}dx = ?$$

$$f(x) = 2.08e^{-2.08x} \Rightarrow F(x) = -e^{-2.08x}$$

Seed Dispersal *Eriogonum fasciculatum*



For example, $f(x) = 2.08e^{-2.08x}$.

Some Typical Questions

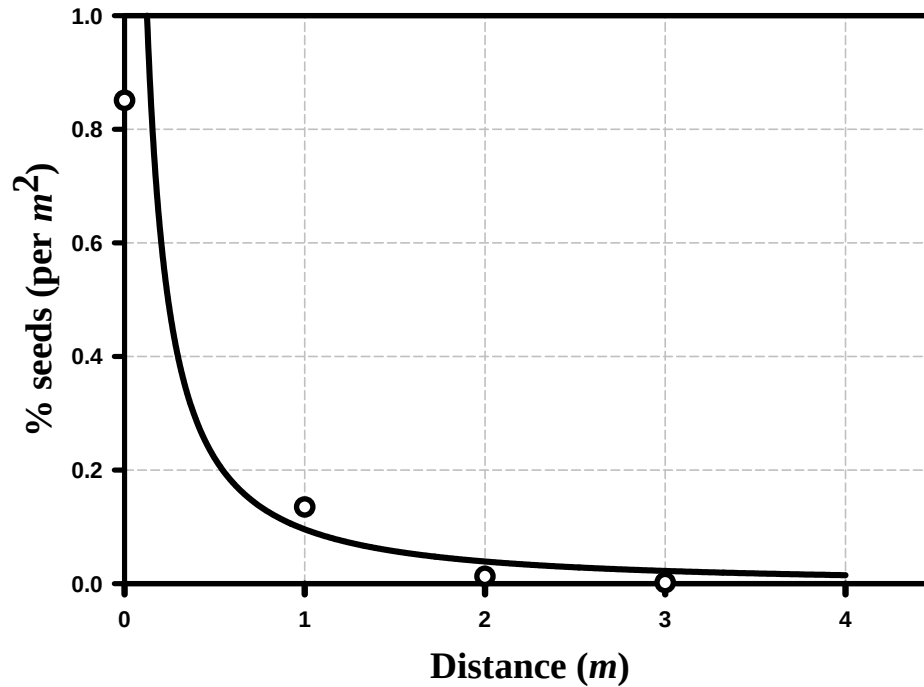
- What is the proportion of seeds between 0 and 2 meters away?

$$\int_0^2 f(x)dx = \int_0^2 2.08e^{-2.08x}dx = ?$$

$$f(x) = 2.08e^{-2.08x} \Rightarrow F(x) = -e^{-2.08x}$$

$$\begin{aligned} \int_0^2 2.08e^{-2.08x}dx &= F(2) - F(0) \\ &= e^{-4.16} - (-1) \\ &\approx 0.9844 \end{aligned}$$

Seed Dispersal *Eriogonum fasciculatum*

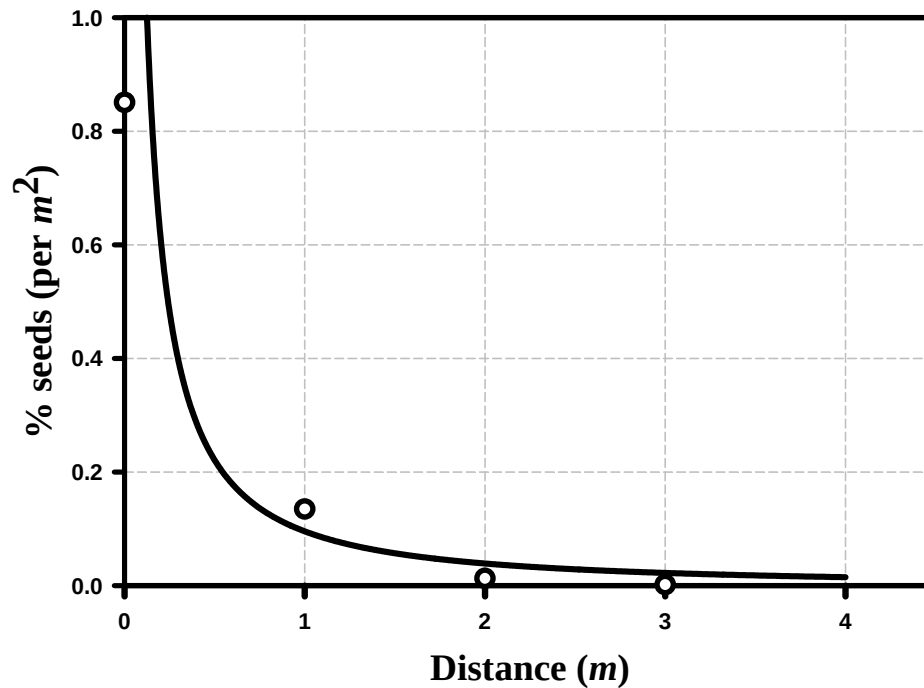


Some Typical Questions

- What is the proportion of seeds between 0 and 2 meters away?

Another example, $f(x) = 2.567 \times 10^4 e^{-12.5x^{1/10}}$.

Seed Dispersal *Eriogonum fasciculatum*



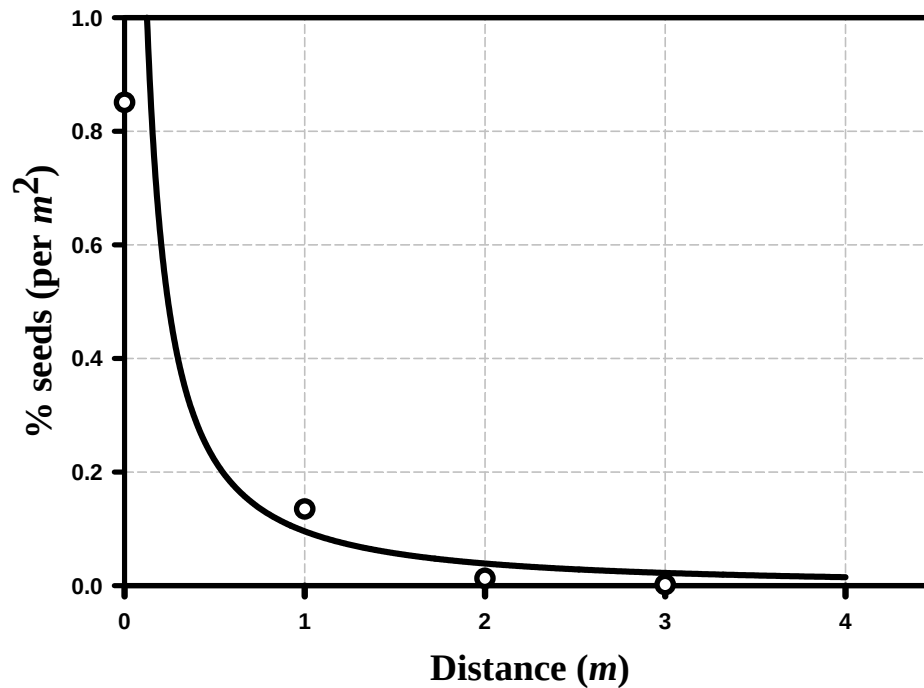
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Seed Dispersal *Eriogonum fasciculatum*



Some Typical Questions

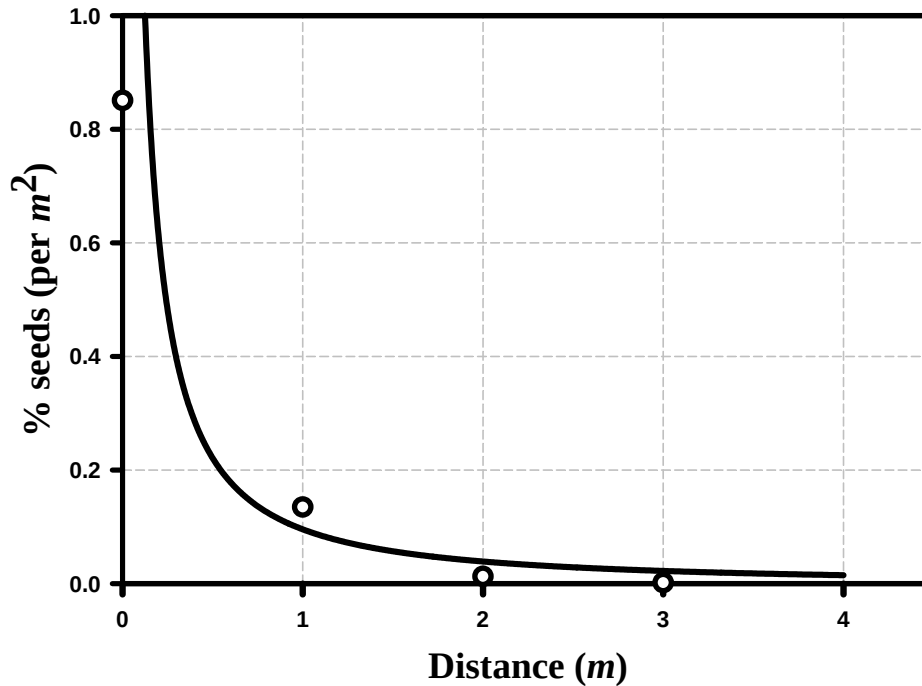
- What is the proportion of seeds between 0 and 2 meters away?

$$\int_0^2 f(x) dx = ?$$

$$F(x) = ?$$

Another example, $f(x) = 2.567 \times 10^4 e^{-12.5x^{1/10}}$.

Seed Dispersal *Eriogonum fasciculatum*



Some Typical Questions

- What is the proportion of seeds between 0 and 2 meters away?

$$\int_0^2 f(x) dx = ?$$

$$F(x) = ?$$

Turns out, it's not possible to find a formula for $F(x)$?

What to do ?

Another example, $f(x) = 2.567 \times 10^4 e^{-12.5x^{1/10}}$.