

MATH 250a

Fall Semester 2007

Section 2 (J. M. Cushing)

Tuesday, September 11

<http://math.arizona.edu/~cushing/250a.html>

An Application

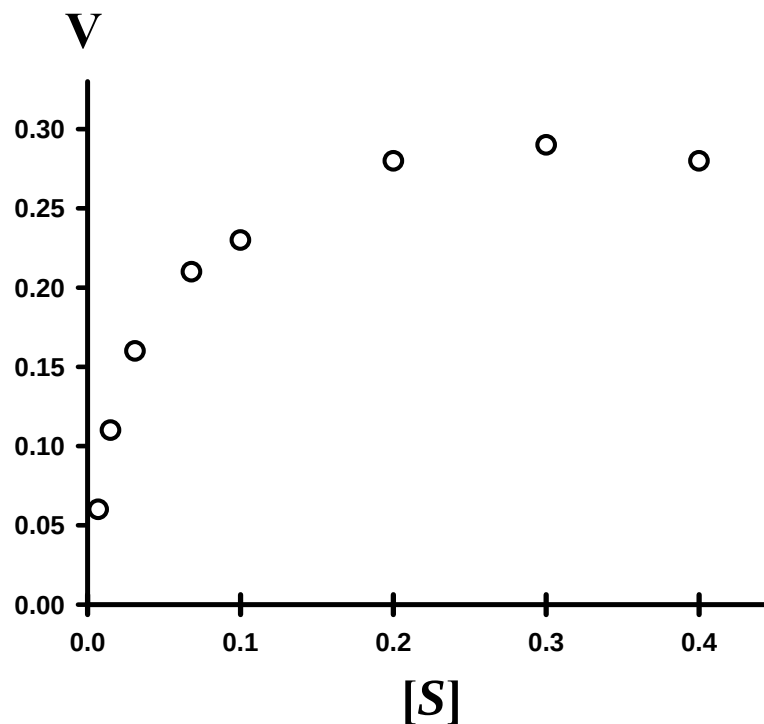
[S] (Mol/L)	V (Absorbance/min)
0.0070	0.0600
0.0150	0.1100
0.0310	0.1600
0.0680	0.2100
0.1000	0.2300
0.2000	0.2800
0.3000	0.2900
0.4000	0.2800

$$V = V_{\max} \frac{[S]}{K_m + [S]}$$

substrate = ethanol

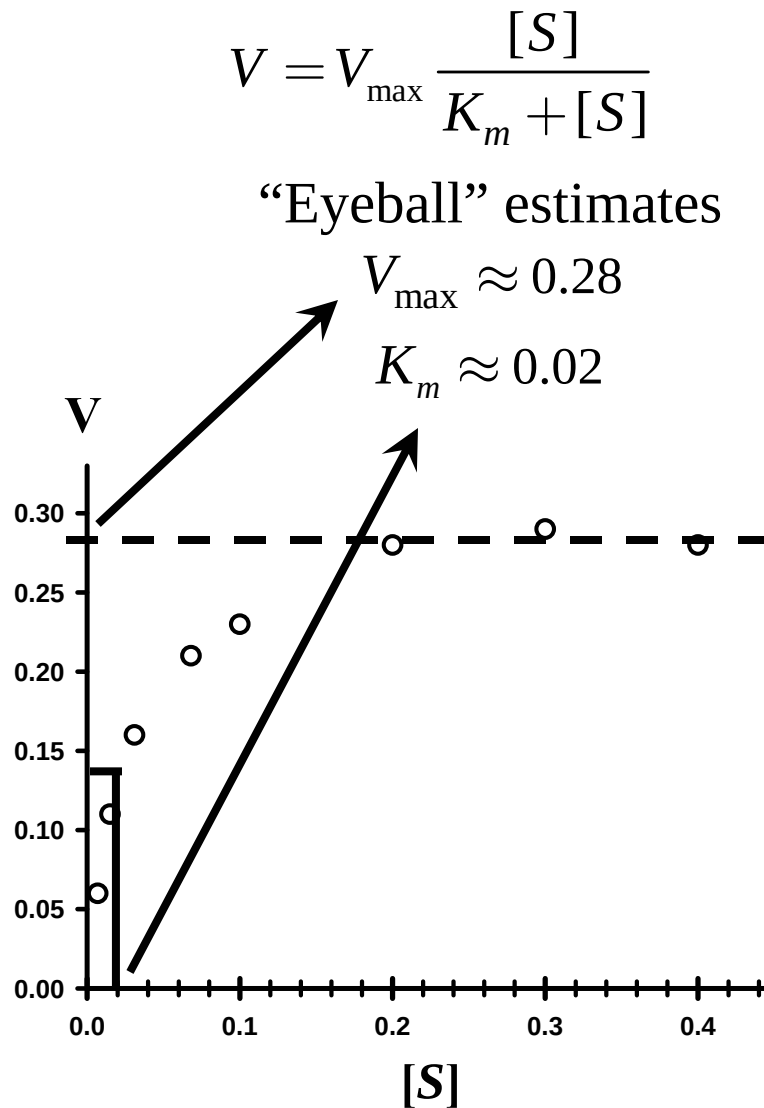
enzyme = alcohol dehydrogenase (ADH)

product = acetaldehyde



An Application

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$$V = V_{\max} \frac{[S]}{K_m + [S]}$$

Lineweaver-Burk

$$y = \frac{1}{V}, \quad x = \frac{1}{[S]}$$

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Lineweaver-Burk

$$y = \frac{1}{V}, \quad x = \frac{1}{[S]} \quad \Rightarrow \quad \frac{1}{y} = V_{\max} \frac{\frac{1}{x}}{K_m + \frac{1}{x}}$$

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Lineweaver-Burk

$$y = \frac{1}{V}, \quad x = \frac{1}{[S]} \quad \Rightarrow \quad \frac{1}{y} = V_{\max} \frac{1}{K_m x + 1}$$

An Application

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Lineweaver-Burk

$$y = \frac{1}{V}, \quad x = \frac{1}{[S]} \quad \Rightarrow \quad y = \frac{K_m}{V_{\max}} x + \frac{1}{V_{\max}}$$

y is a linear function of x !

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slope

intercept

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y is a linear function of x !

We can do Linear Least Squares

to estimate $\frac{K_m}{V_{\max}}$ and $\frac{1}{V_{\max}}$

An Application

$1/[S]$ (L/Mol)	$1/V$ (min/Absorbance)
142.8571	16.6667
66.6667	9.0909
32.2581	6.2500
14.7059	4.7619
10.0000	4.3478
5.0000	3.5714
3.3333	3.4483
2.5000	3.5714

$$V = V_{\max} \frac{[S]}{K_m + [S]}$$

Lineweaver-Burk

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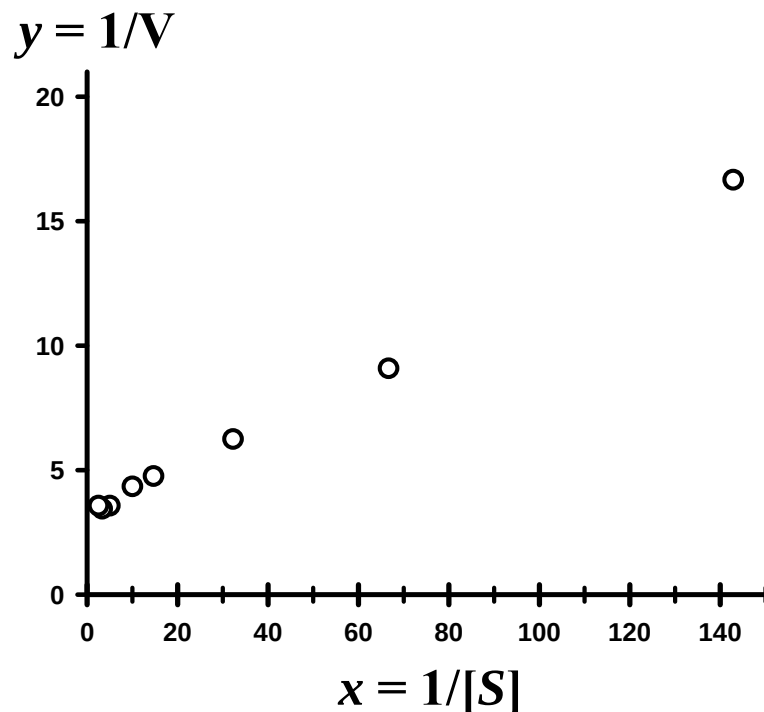
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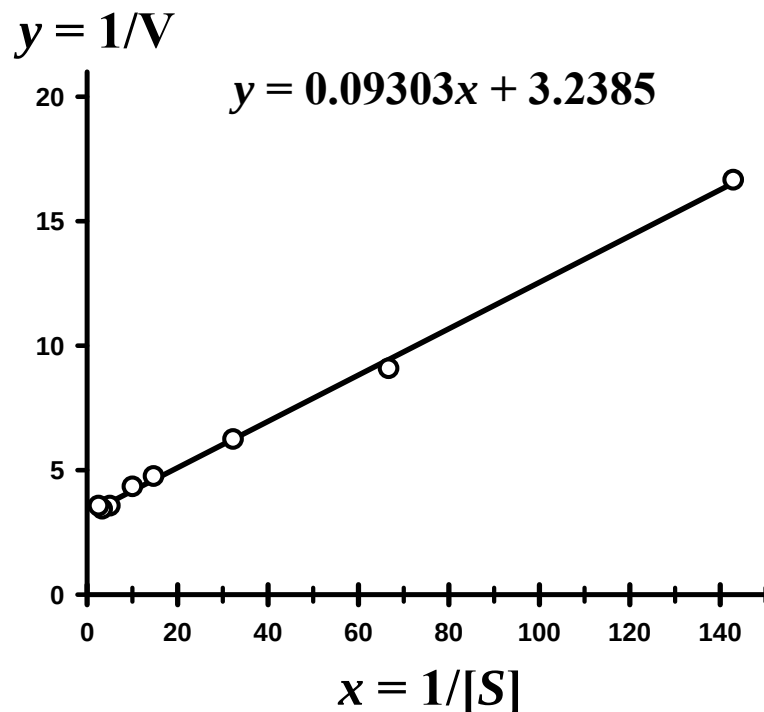


K. Dendinkas et al., *J. Chemical Education* 82(7), 1068 (2005)

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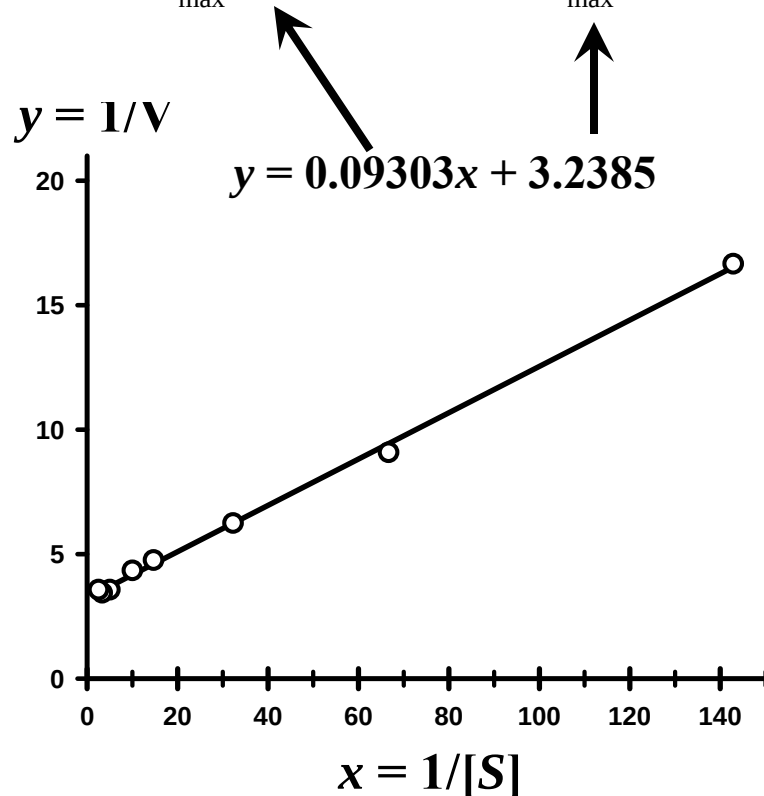
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$$V = V_{\max} \frac{[S]}{K_m + [S]}$$

$$\frac{K_m}{V_{\max}} = 0.09303, \quad \frac{1}{V_{\max}} = 3.2385$$

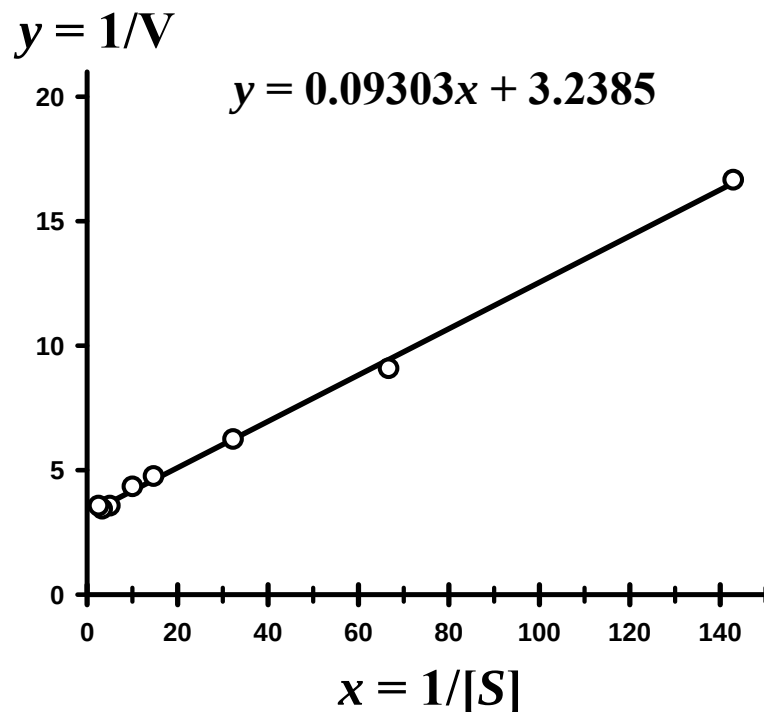


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$$V = V_{\max} \frac{[S]}{K_m + [S]}$$

$$\frac{K_m}{V_{\max}} = 0.09303, \quad V_{\max} = 0.3088$$

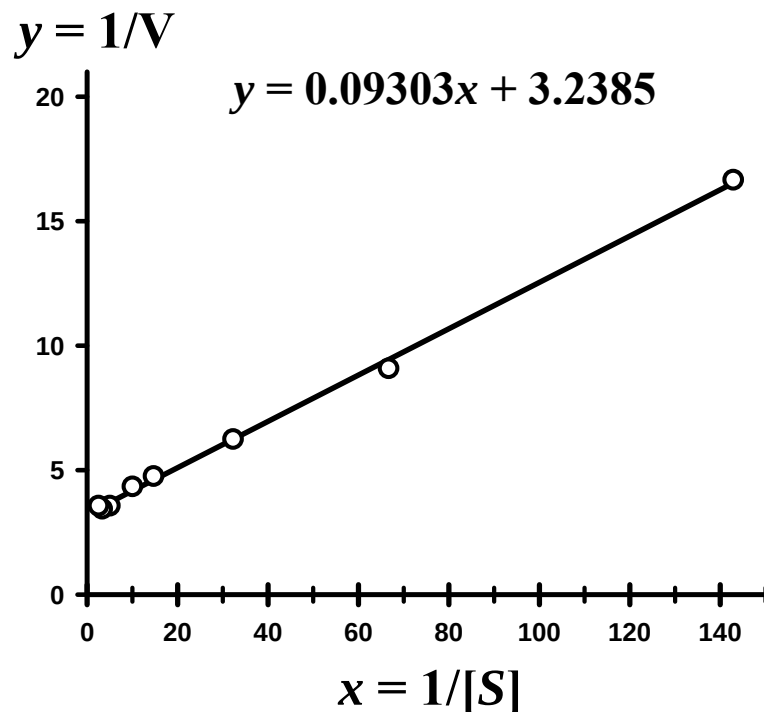


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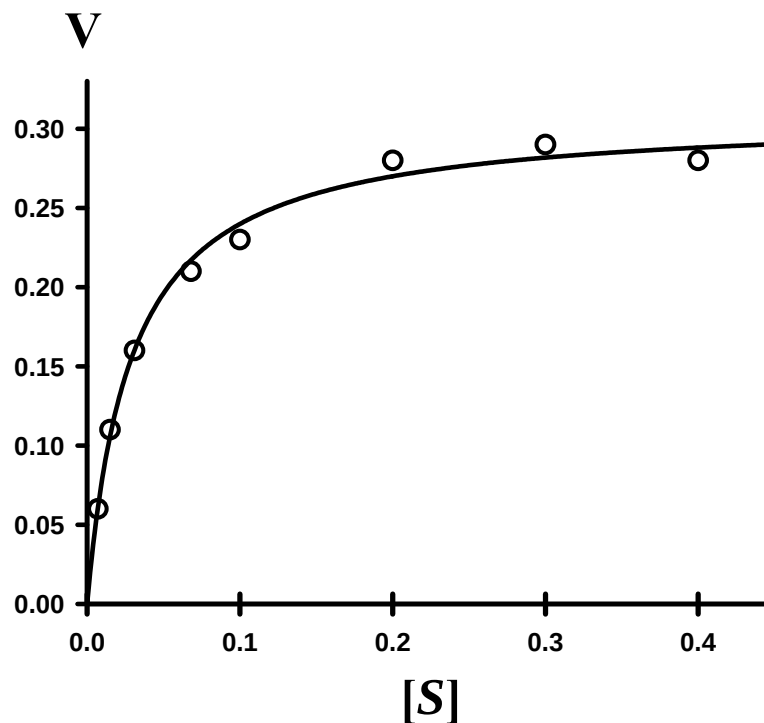
$$V_{\max} = 0.02873, \quad V_{\max} = 0.3088$$



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$$V = 0.3088 \frac{[S]}{0.02873 + [S]}$$



K. Dendinskas et al., *J. Chemical Education* 82(7), 1068 (2005)

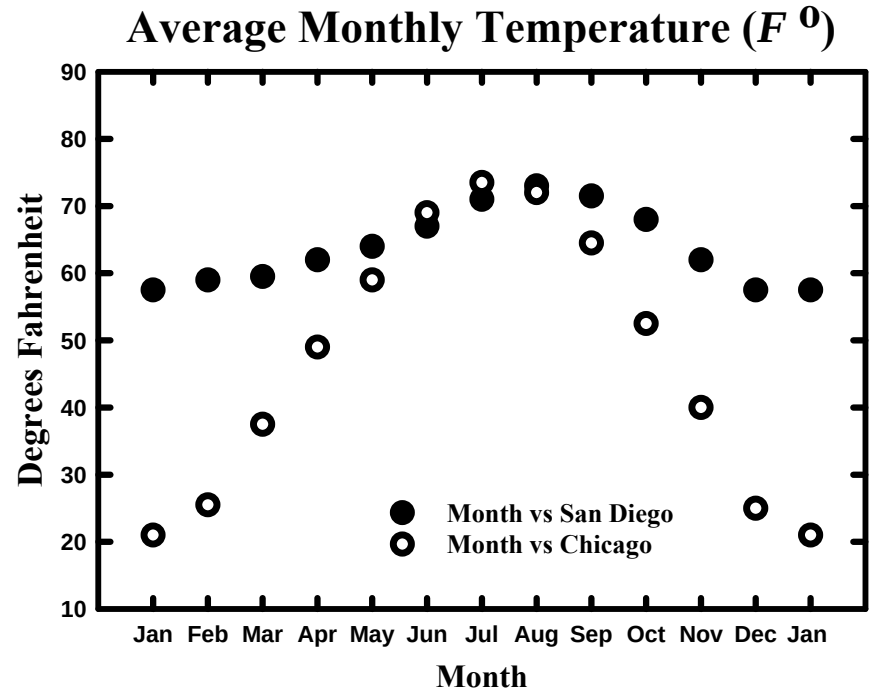
Chapters 1.7

Trigonometric Functions

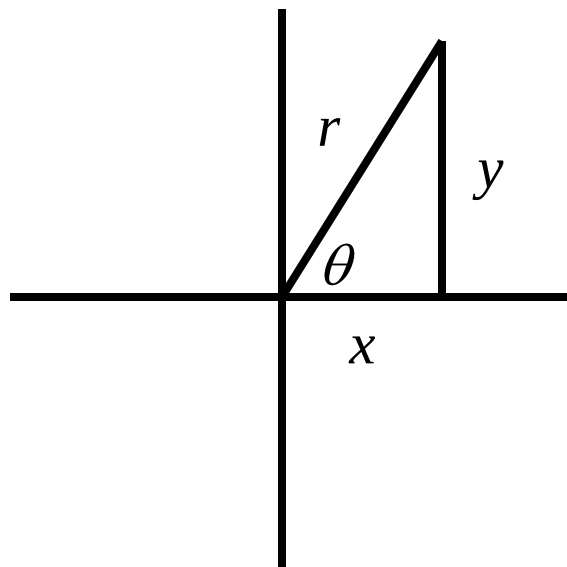
An Application

Average Monthly Temperatures (F°)

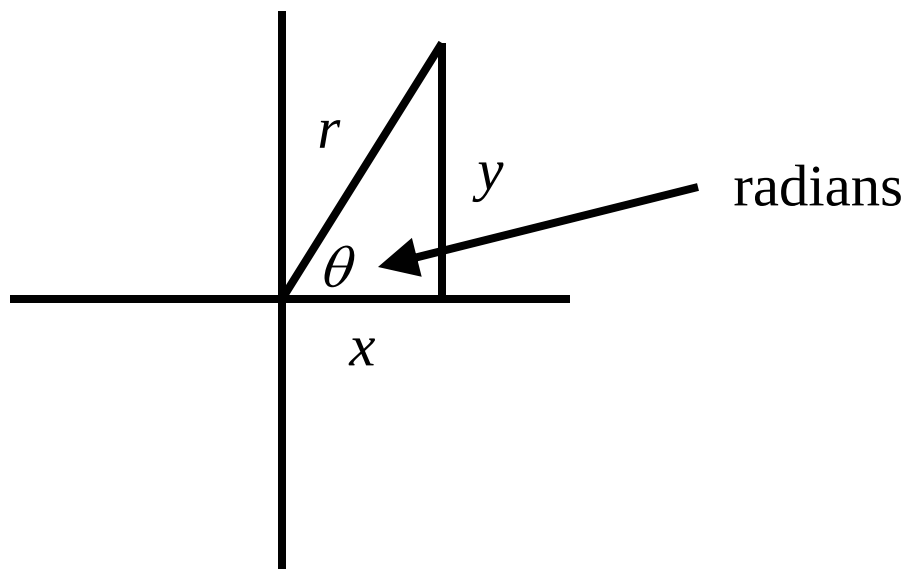
Month	San Diego	Chicago
Jan	57.5	21.0
Feb	59.0	25.5
March	59.5	37.5
Apr	62.0	49.0
May	64.0	59.0
June	67.0	69.0
July	71.0	73.5
August	73.0	72.0
Sept	71.5	64.5
Nov	68.0	52.5
Dec	62.0	40.0



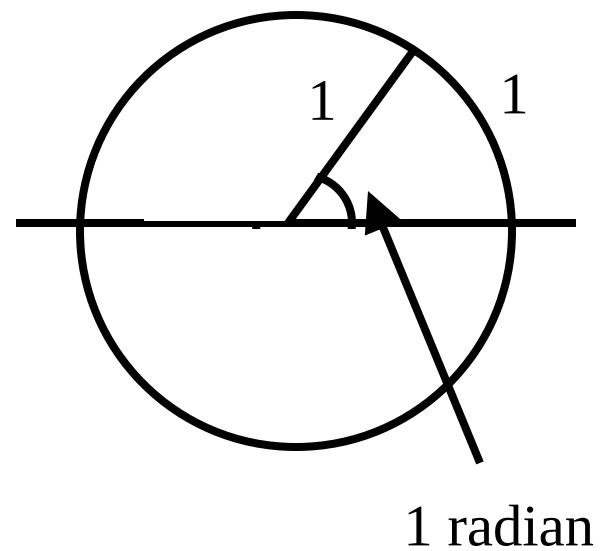
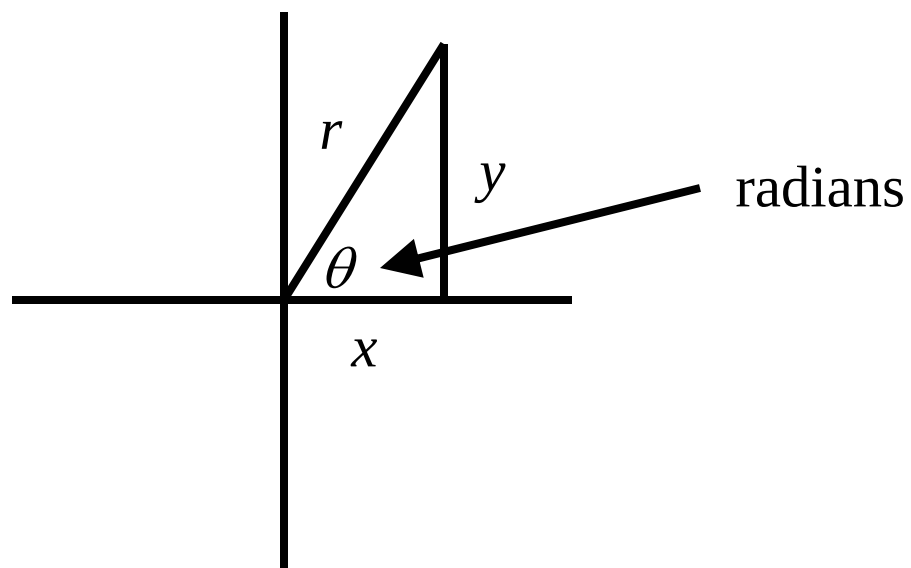
Recall $\sin\theta$ and $\cos\theta$



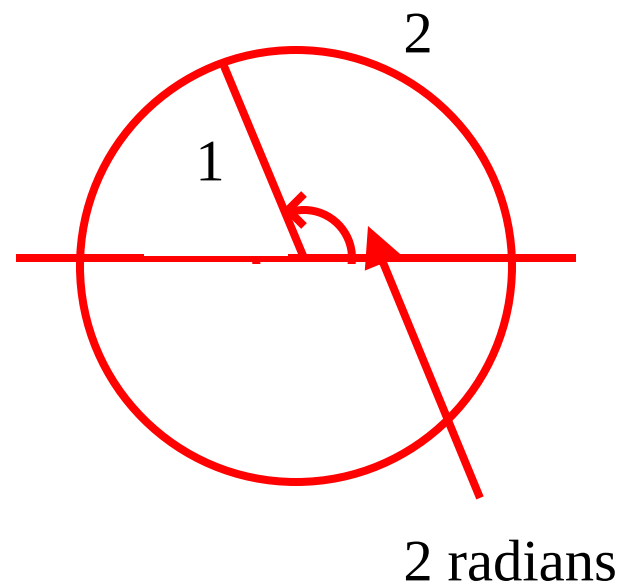
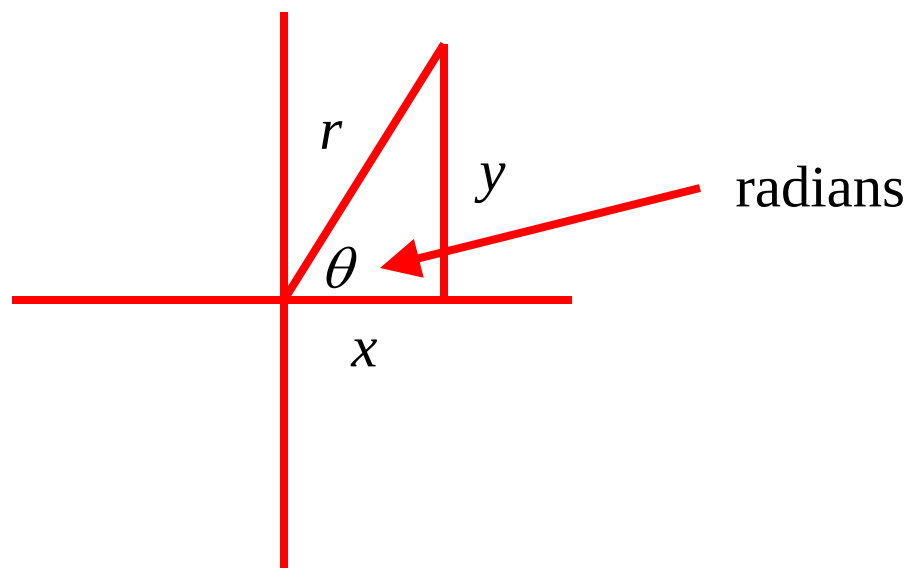
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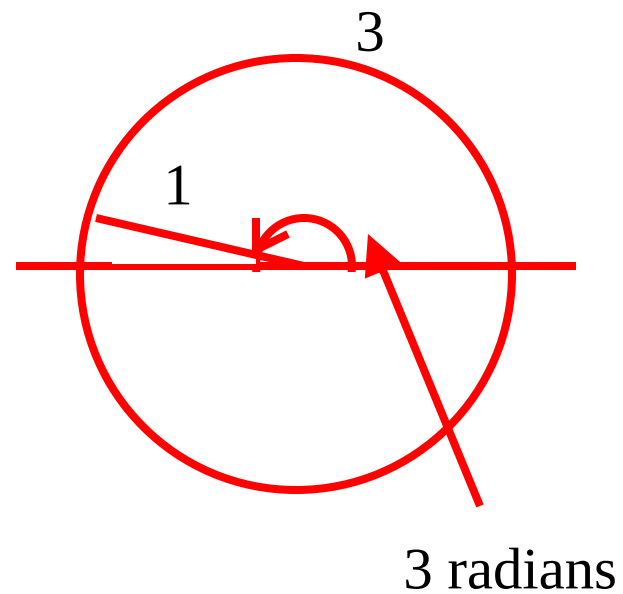
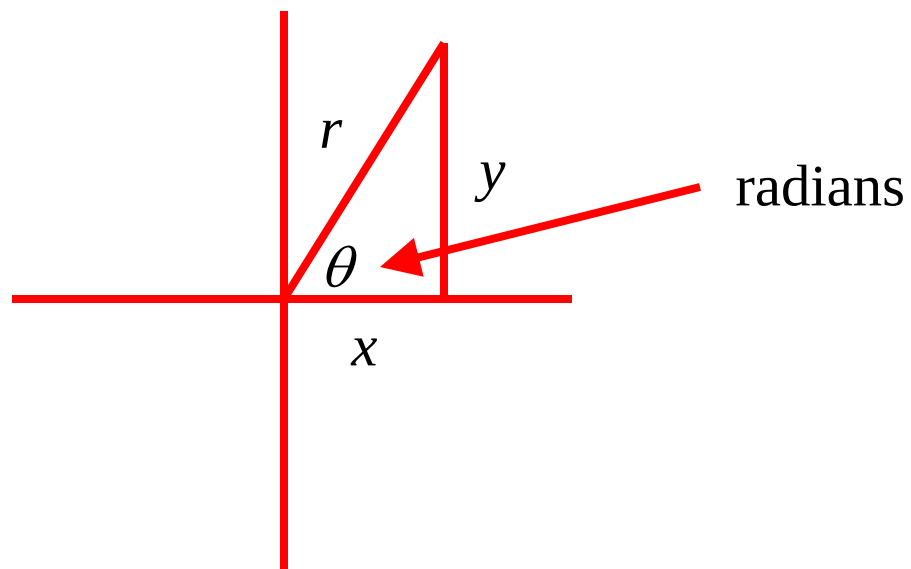
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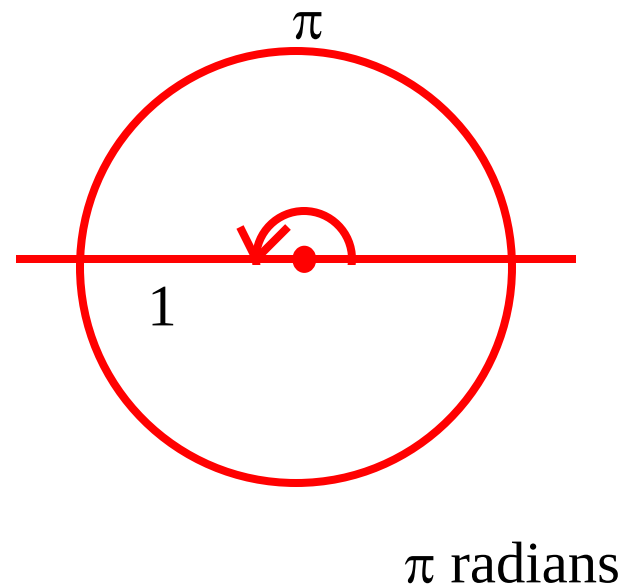
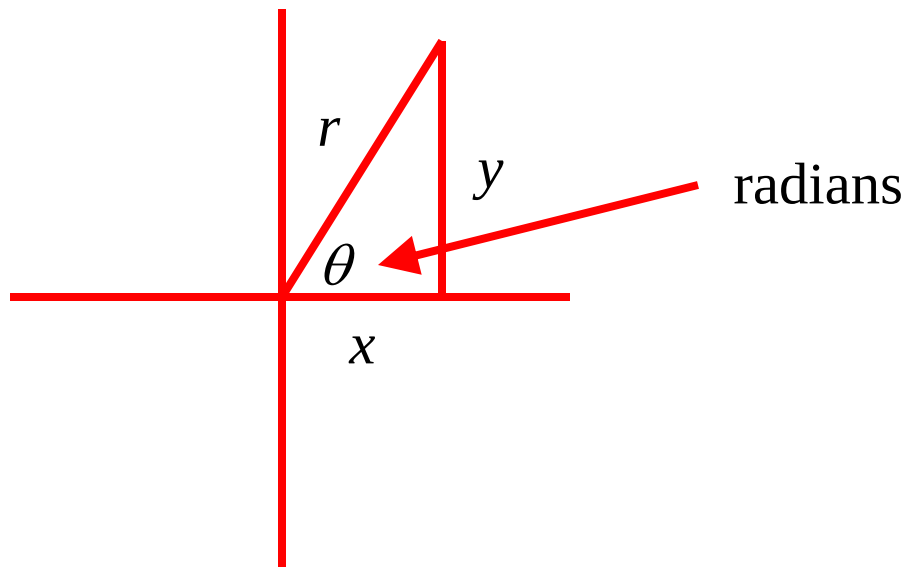
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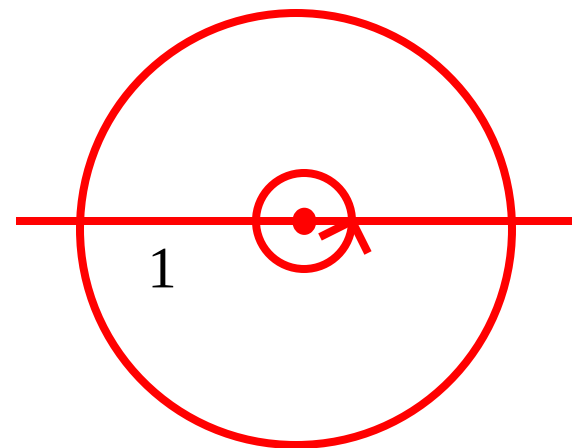
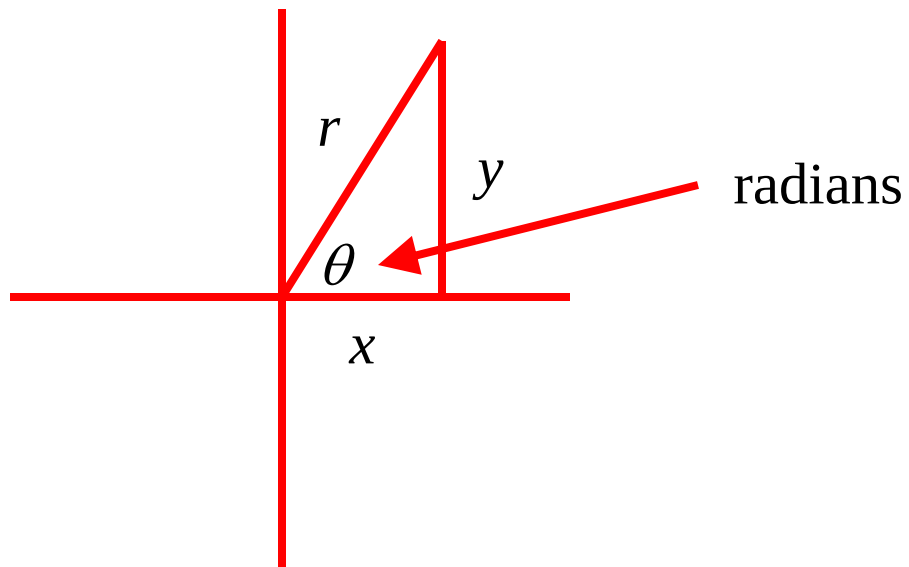
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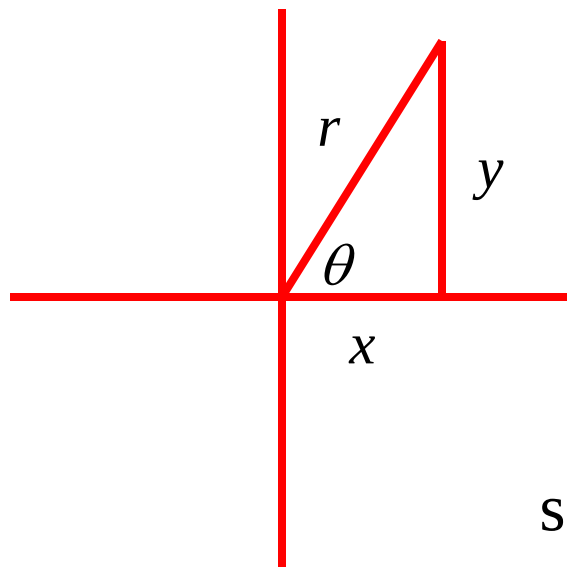


Recall $\sin\theta$ and $\cos\theta$



2π radians

Recall $\sin\theta$ and $\cos\theta$

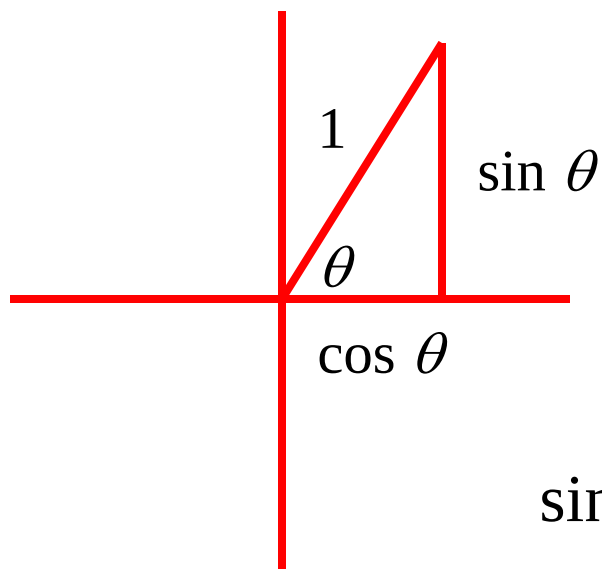


$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$r = \sqrt{x^2 + y^2}$$

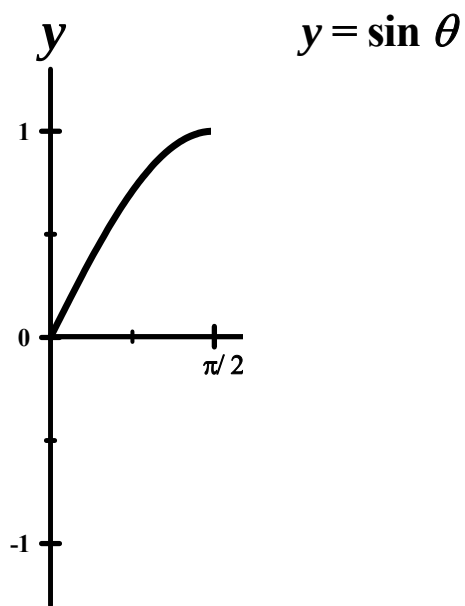
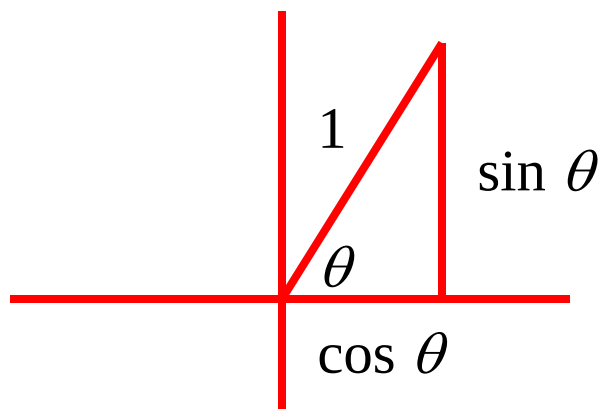
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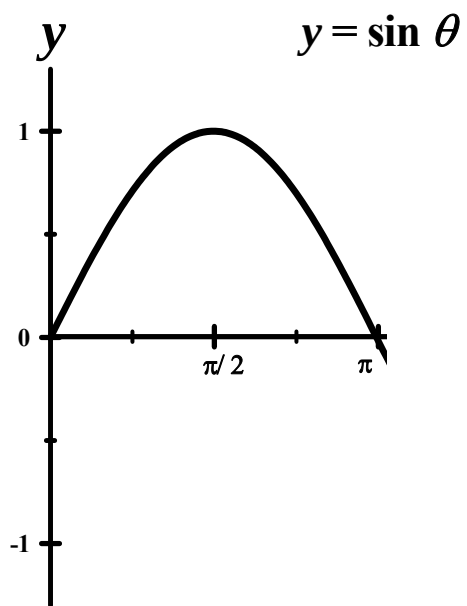
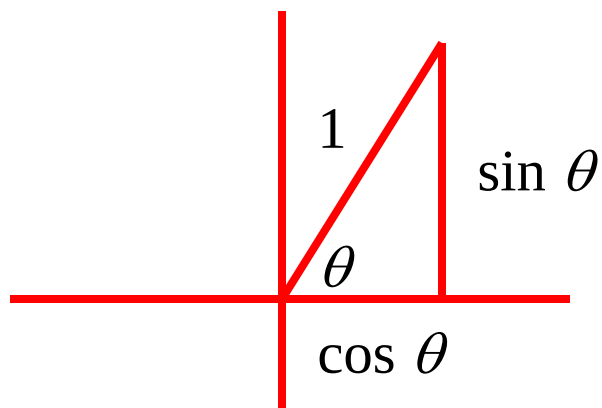
$$\sin \theta = y$$

$$\cos \theta = x$$

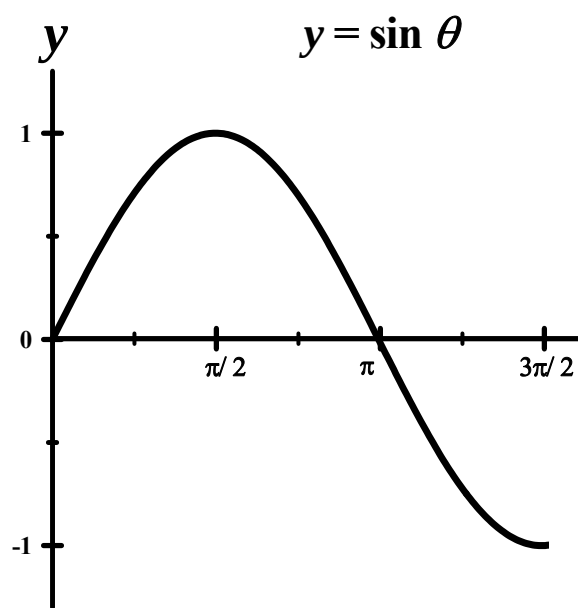
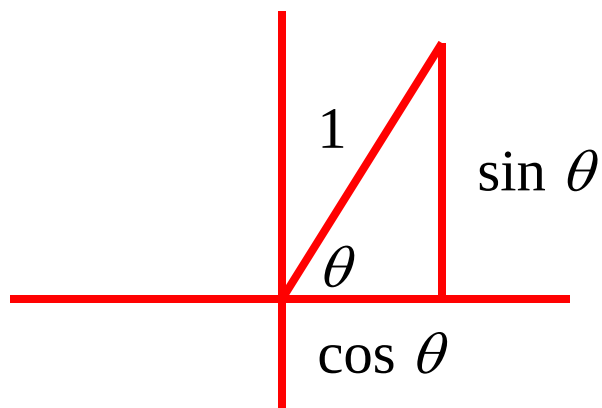
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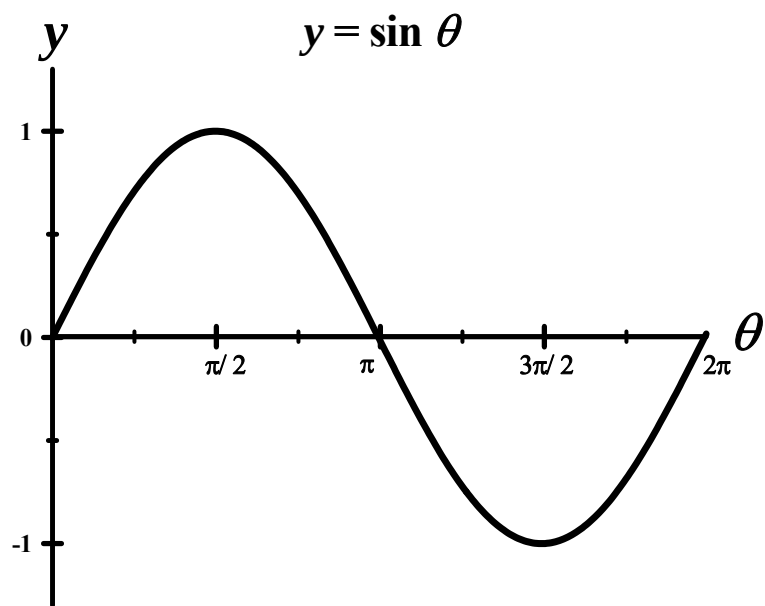
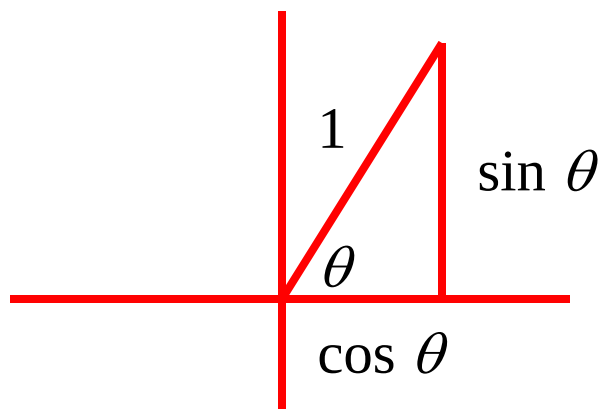
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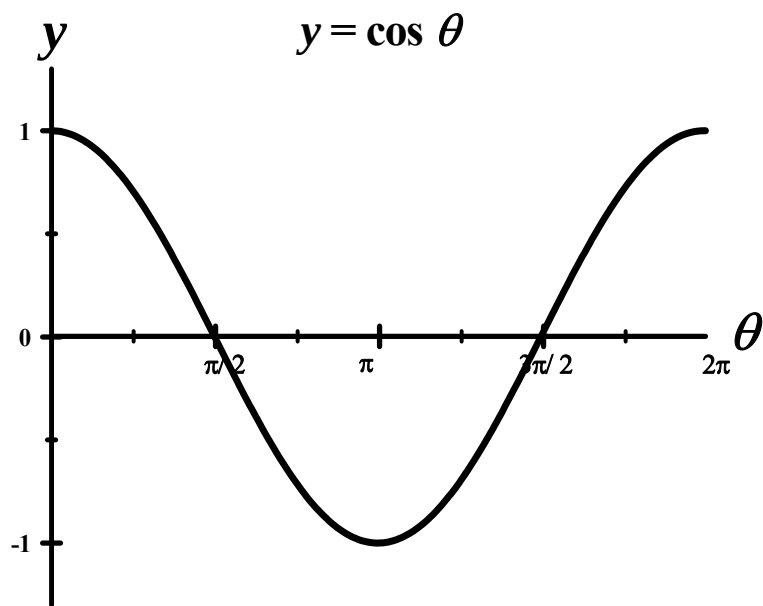
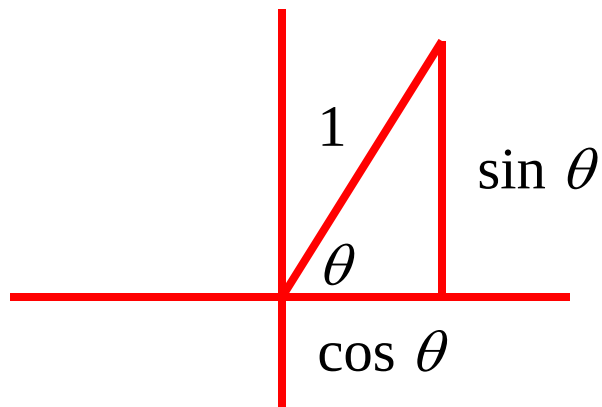
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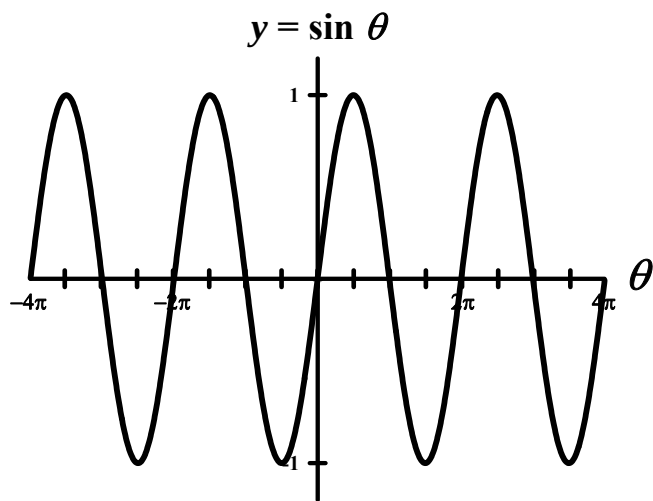


Recall $\sin \theta$ and $\cos \theta$



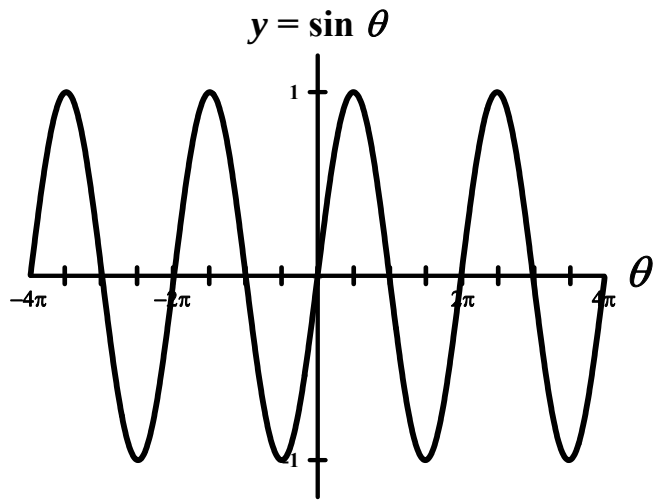
Recall $\sin \theta$ and $\cos \theta$





Other periods
(stretch/shrink the θ -axis)

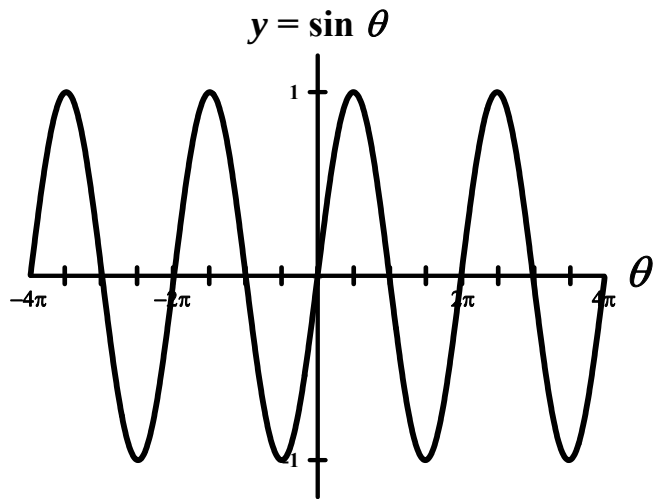
$$y = \sin \theta$$



Other periods
(stretch/shrink the θ -axis)

$$y = \sin \theta$$

$$y = \sin 2\theta$$

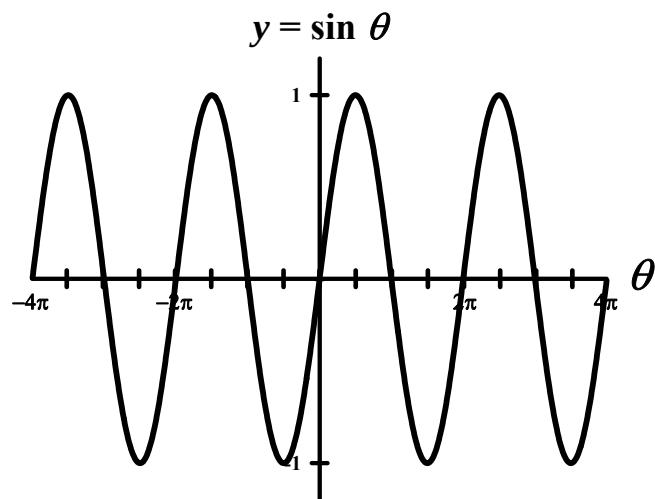


Other periods
(stretch/shrink the θ -axis)

$$y = \sin \theta$$

$$y = \sin 2\theta$$

$$\text{period} = \pi$$

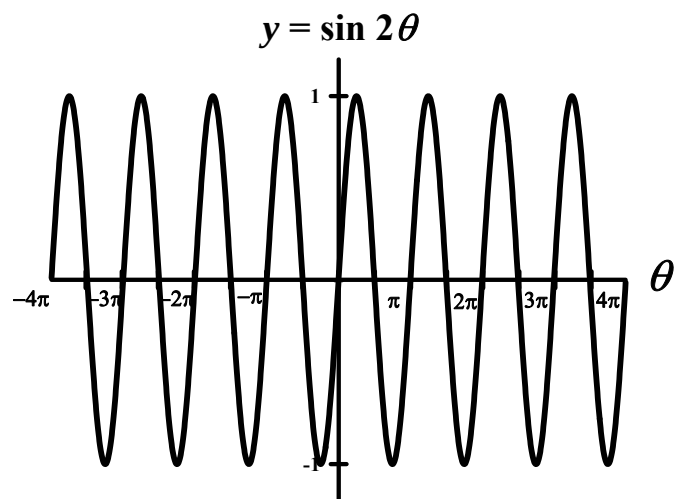


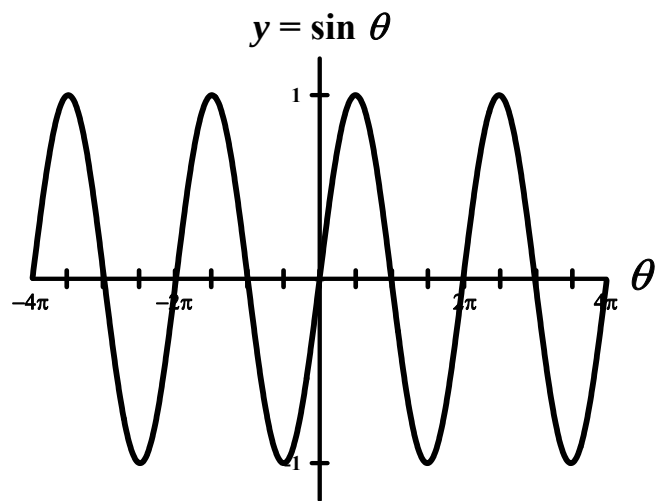
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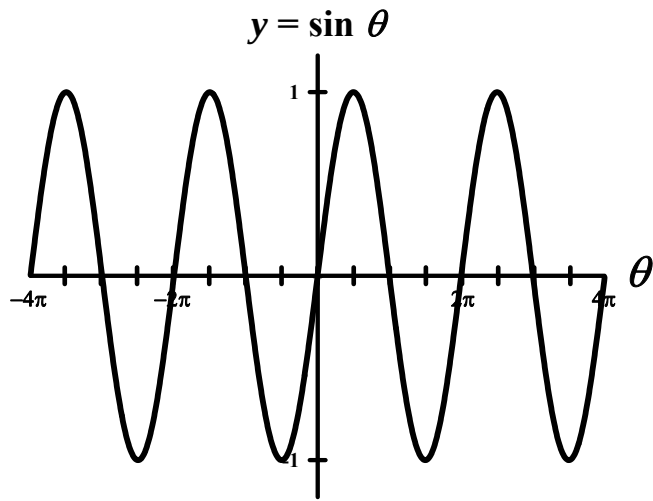




Other periods
(stretch/shrink the θ -axis)

$$y = \sin \theta$$

$$y = \sin \frac{1}{2} \theta$$

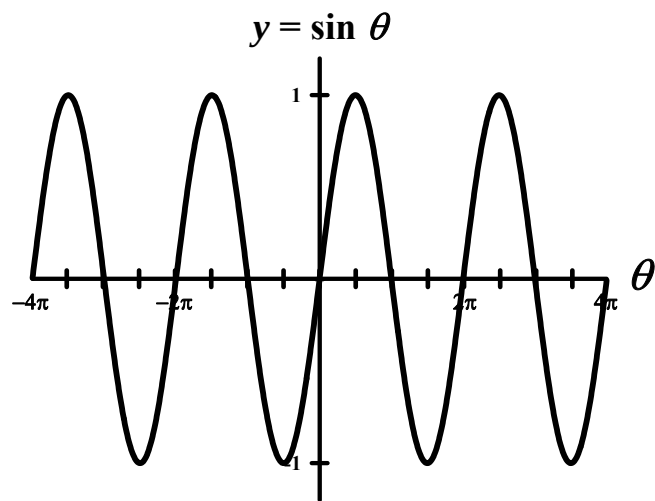


Other periods
(stretch/shrink the θ -axis)

$$y = \sin \theta$$

$$y = \sin \frac{1}{2} \theta$$

$$\text{period} = 4\pi$$

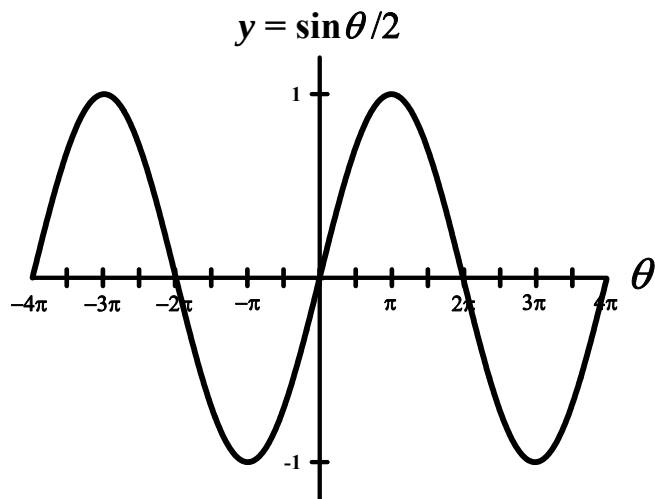


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(stretch/shrink the θ -axis)

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In General

$$y = \sin \omega \theta$$

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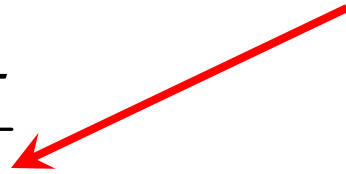
$$\text{period} = \frac{2\pi}{\omega}$$

In General

$$y = \sin \omega \theta$$

$$\text{period} = \frac{2\pi}{\omega}$$

frequency

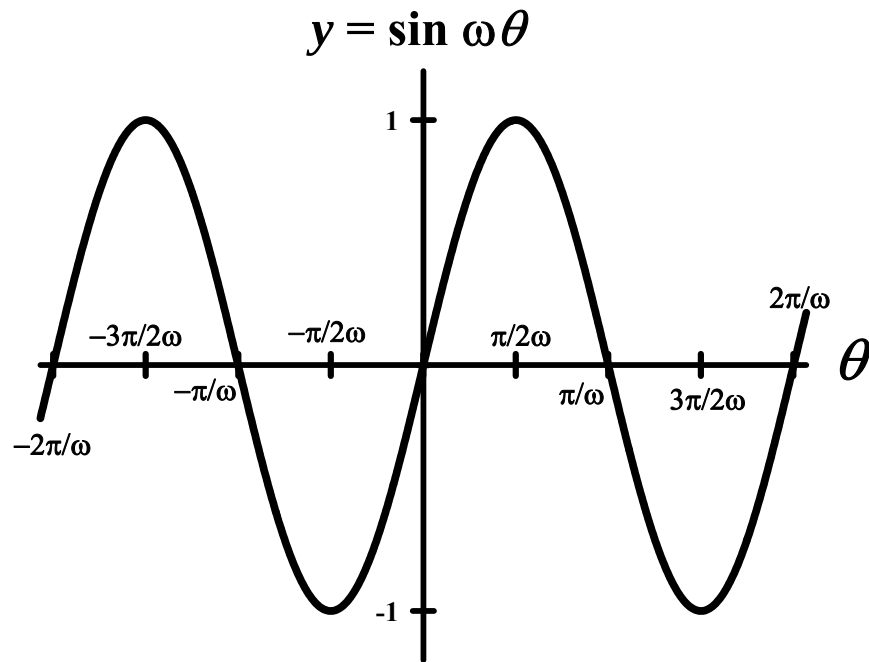


In General

$$y = \sin \omega \theta$$

$$\text{period} = \frac{2\pi}{\omega}$$

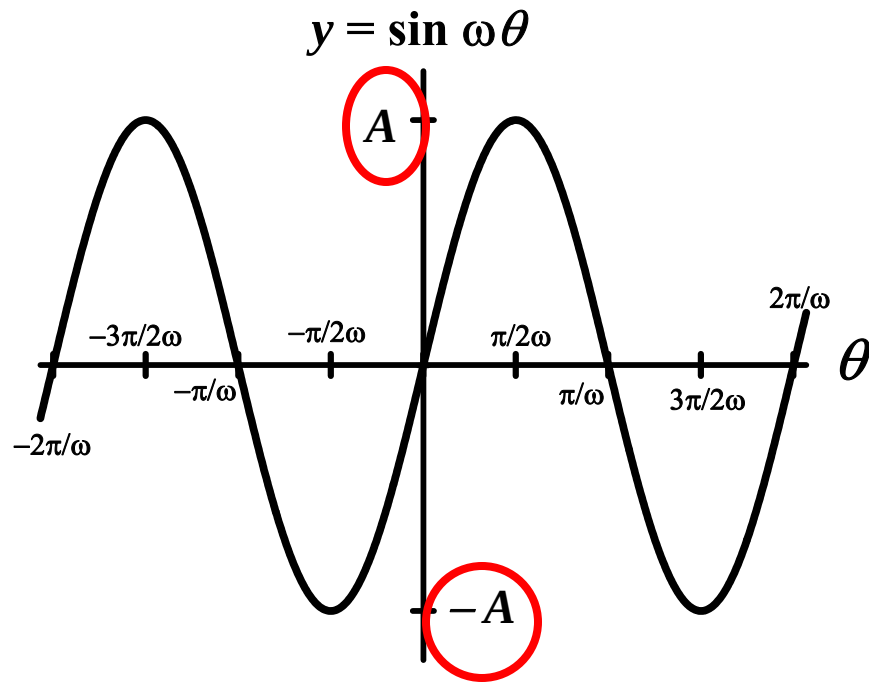
frequency



Other Amplitudes

$$y = A \sin \omega \theta$$

$$\text{period} = \frac{2\pi}{\omega}$$



Phase Shifts

Graph of $y = f(x - 1)$

Phase Shifts

Graph of $y = f(x-1)$
is the shift of graph of $y = f(x)$
1 unit to the *right*

Phase Shifts

Graph of $y = f(x + 1)$

Phase Shifts

Graph of $y = f(x+1)$
is the shift of graph of $y = f(x)$
1 unit to the *left*

Phase Shifts

Graph of $y = f(x - \varphi)$, $\varphi > 0$

Phase Shifts

Graph of $y = f(x - \varphi)$, $\varphi > 0$
is the shift of graph of $y = f(x)$
 φ units to the *right*

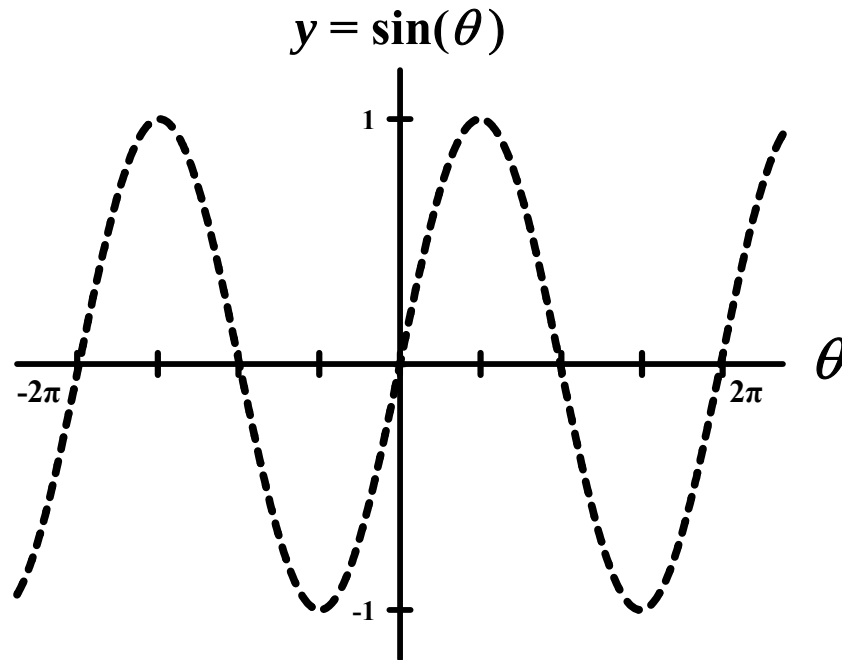
Phase Shifts

Graph of $y = f(x - \varphi)$, $\varphi > 0$
is the shift of graph of $y = f(x)$
 φ units to the *right*

Graph of $y = f(x + \varphi)$, $\varphi > 0$
is the shift of graph of $y = f(x)$
 φ units to the *left*

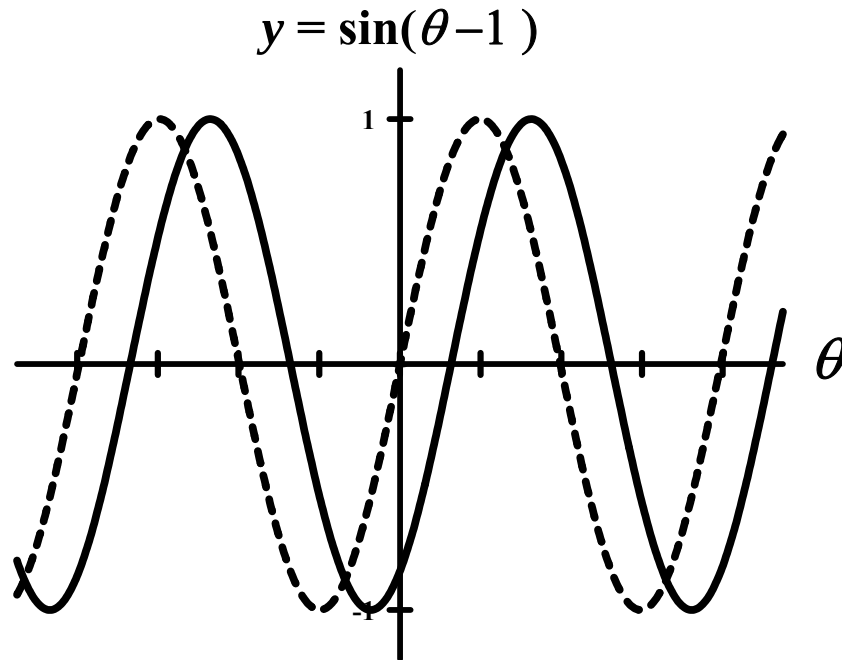
Phase Shifts

Graph of $y = \sin(\theta - 1)$
is the shift of graph of $y = \sin \theta$
1 unit to the *right*



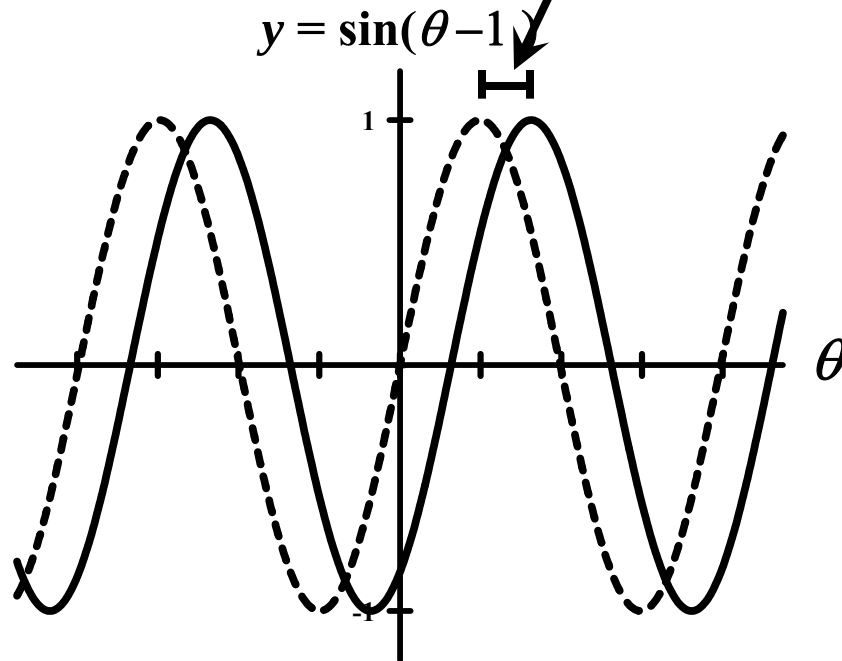
Phase Shifts

Graph of $y = \sin(\theta - 1)$
is the shift of graph of $y = \sin \theta$
1 unit to the *right*



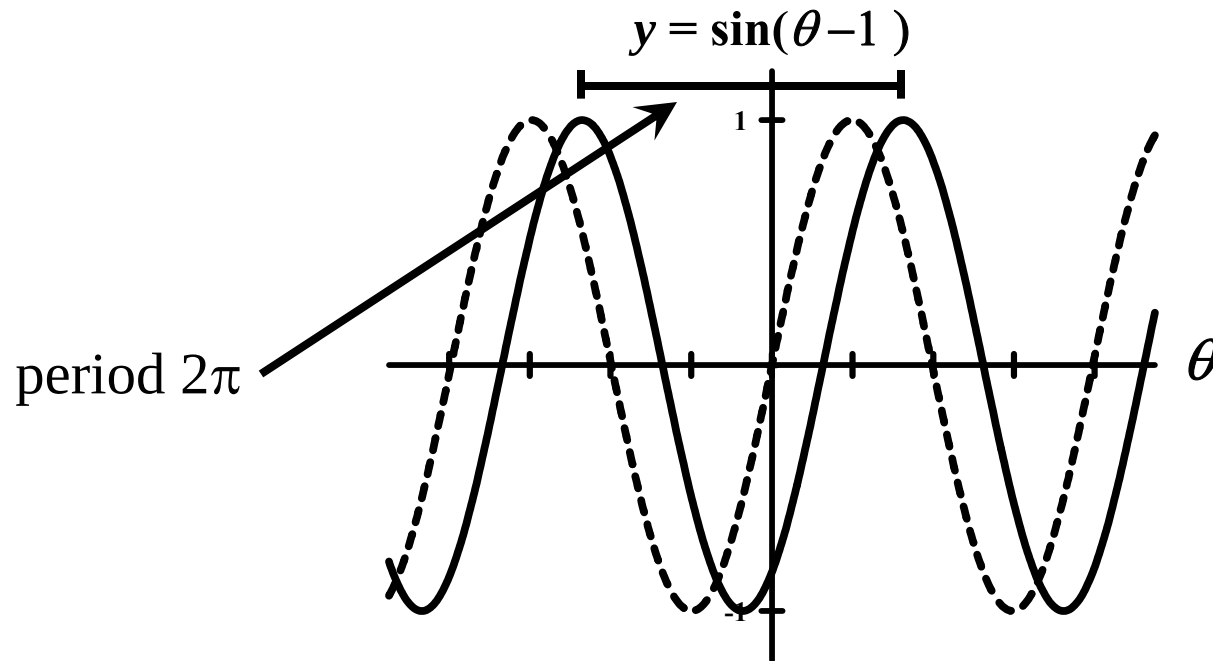
Phase Shifts

Graph of $y = \sin(\theta - 1)$
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1 unit to the *right*



Phase Shifts

Graph of $y = \sin(\theta - 1)$
is the shift of graph of $y = \sin \theta$
1 unit to the *right*



$$y = A \sin \omega (\theta - \varphi)$$

$$y = A \sin \omega (\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi$$

$$y = A \sin \omega (\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi$$

Vertical Shifts

Graph of $y = 1 + f(x)$
is the shift of graph of $y = f(x)$
1 unit up

$$y = A \sin \omega (\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi$$

Vertical Shifts

Graph of $y = B + f(x)$

is the shift of graph of $y = f(x)$

B units up if $B > 0$

B units down if $B < 0$

$$y = A \sin \omega (\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi$$

sine functions “oscillate” around 0.

... they have an average of 0.

$$y = A \sin \omega (\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi$$

sine functions “oscillate” around 0.

... they have an average of 0.

Vertical shifts change the average.

A General Sine Function

$$y = B + A \sin \omega(\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi, \quad \text{average} = B$$

A General Sine Function

$$y = B + A \sin \omega(\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi, \quad \text{average} = B$$

A General Cosine Function

$$y = B + A \cos \omega(\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi, \quad \text{average} = B$$

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$$\text{Note : } B + A \cos \omega \left(\theta - \quad \right) = B + A \sin \omega \theta$$

A General Sine Function

$$y = B + A \sin \omega(\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi, \quad \text{average} = B$$

A General Cosine Function

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$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi, \quad \text{average} = B$$

$$\text{Note : } B + A \cos \omega \left(\theta - \frac{1}{4} \frac{2\pi}{\omega} \right) = B + A \sin \omega \theta$$

A General Sine Function

$$y = B + A \sin \omega(\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi, \quad \text{average} = B$$

A General Cosine Function

$$y = B + A \cos \omega(\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi, \quad \text{average} = B$$

$$\text{Note : } B + A \cos \omega \left(\theta - \frac{\pi}{2\omega} \right) = B + A \sin \omega \theta$$

A General Sine Function

$$y = B + A \sin \omega(\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi, \quad \text{average} = B$$

A General Cosine Function

$$y = B + A \cos \omega(\theta - \varphi)$$

$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi, \quad \text{average} = B$$

$$\text{Note :} \quad B + A \cos\left(\omega\theta - \frac{\pi}{2}\right) = B + A \sin \omega\theta$$

$$y = B + A \sin \omega(\theta - \varphi)$$

$$y = B + A \cos \omega(\theta - \varphi)$$

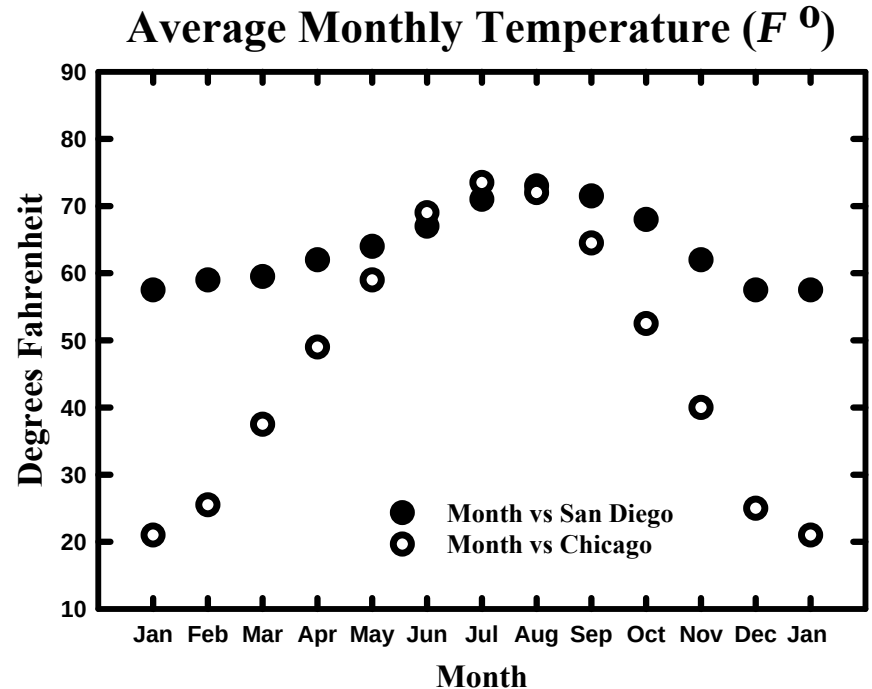
$$\text{period} = \frac{2\pi}{\omega}, \quad \text{amplitude} = A, \quad \text{phase shift} = \varphi, \quad \text{average} = B$$

To use a sine (or cosine) function as a model for data,
we have *four* parameters to estimate.

An Application

Average Monthly Temperatures (F°)

Month	San Diego	Chicago
Jan	57.5	21.0
Feb	59.0	25.5
March	59.5	37.5
Apr	62.0	49.0
May	64.0	59.0
June	67.0	69.0
July	71.0	73.5
August	73.0	72.0
Sept	71.5	64.5
Nov	68.0	52.5
Dec	62.0	40.0



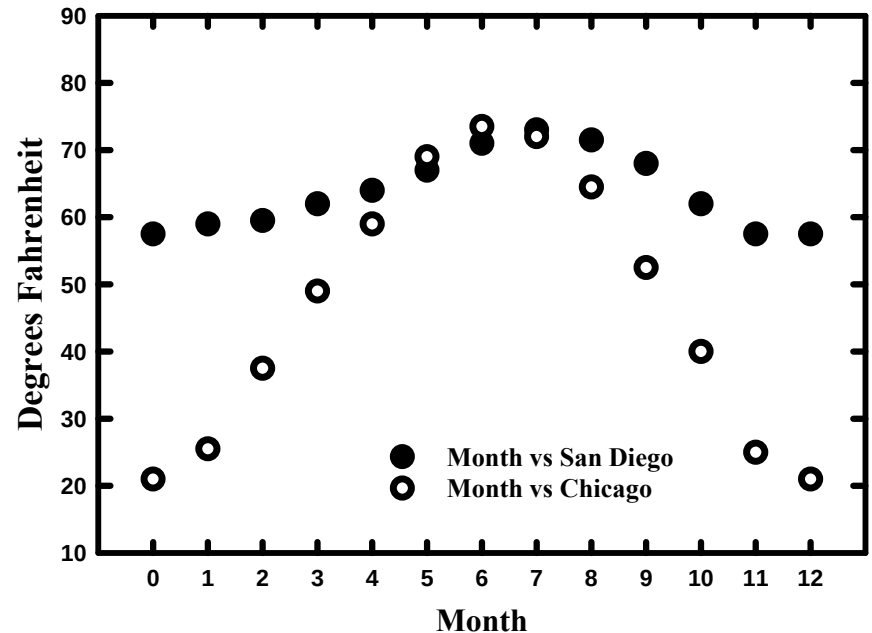
An Application

Average Monthly Temperatures (F°)

Month	San Diego	Chicago
0	57.5	21.0
1	59.0	25.5
2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

$$y = B + A \cos \omega(\theta - \varphi)$$

Average Monthly Temperature (F°)



An Application

Average Monthly Temperatures (F°)

$$y = B + A \cos \omega(\theta - \varphi)$$

$B =$ average

Month	San Diego	Chicago
0	57.5	21.0
1	59.0	25.5
2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

An Application

Average Monthly Temperatures (F°)

Month	San Diego	Chicago
0	57.5	21.0
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7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

$$y = B + A \cos \omega(\theta - \varphi)$$

$$B = \text{average} = \begin{cases} 64.29 & \text{for San Diego} \\ 49.17 & \text{for Chicago} \end{cases}$$

←
average numbers
in each column

An Application

Average Monthly Temperatures (F°)

Month	San Diego	Chicago
0	57.5	21.0
1	59.0	25.5
2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

$$y = B + A \cos \omega(\theta - \varphi)$$

$$B = \text{average} = \begin{cases} 64.29 & \text{for San Diego} \\ 49.17 & \text{for Chicago} \end{cases}$$

For both cities

$$\text{period} = \frac{2\pi}{\omega} = 12 \text{ (months)}$$

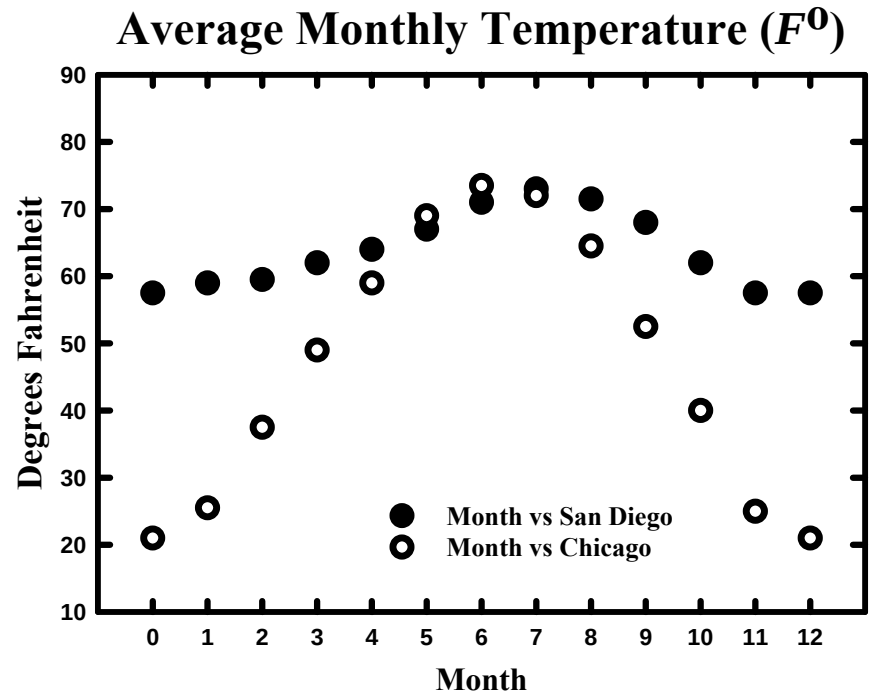
$$\omega = \frac{\pi}{6} \approx 0.5236$$

An Application

Average Monthly Temperatures (F°) $64.29 + A\cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for San Diego

$49.17 + A\cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for Chicago

Month	San Diego	Chicago
0	57.5	21.0
1	59.0	25.5
2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

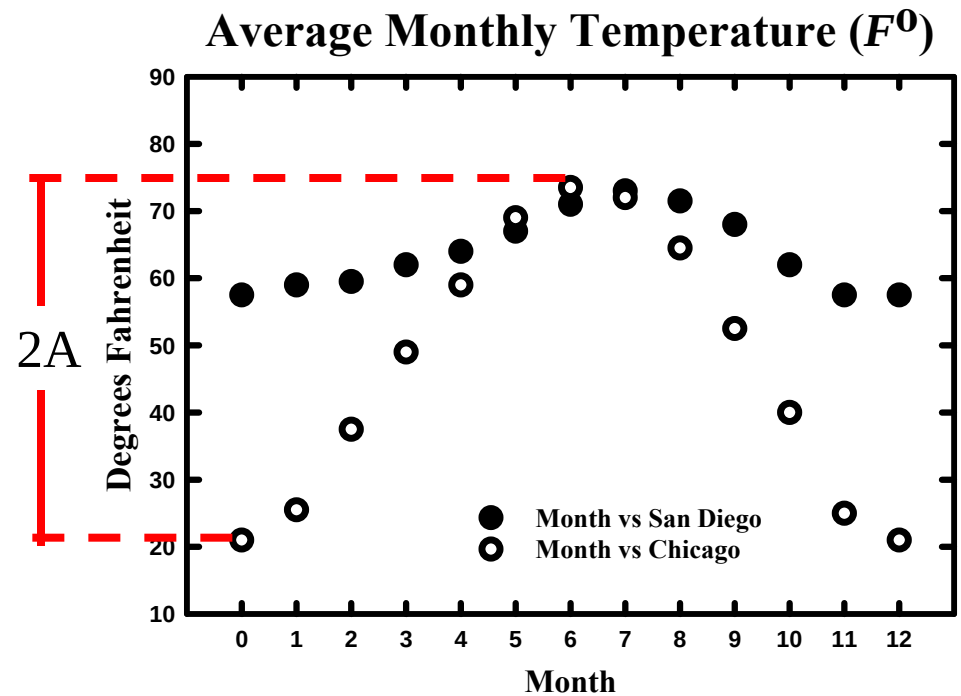


An Application

Average Monthly Temperatures (F°) $64.29 + A\cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for San Diego

$49.17 + A\cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for Chicago

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2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

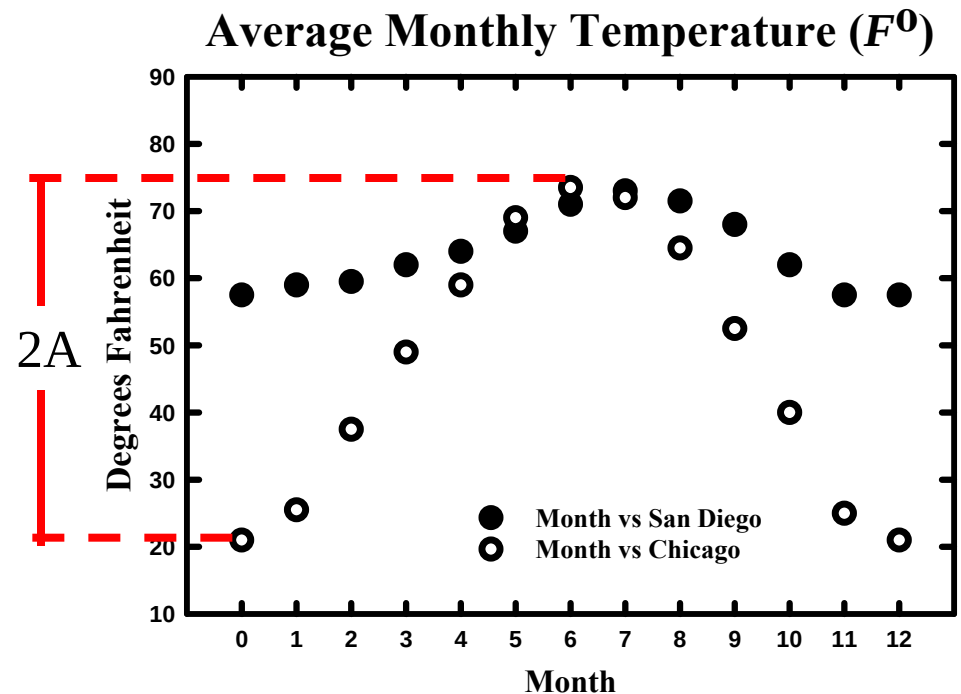


An Application

Average Monthly Temperatures (F°) $64.29 + A \cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for San Diego

$49.17 + A \cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for Chicago

Month	San Diego	Chicago
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1	59.0	25.5
2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

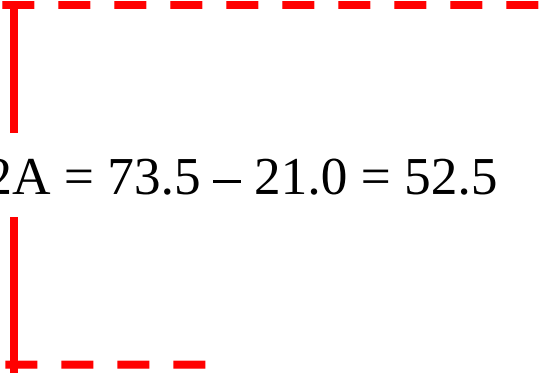


An Application

Average Monthly Temperatures (F°) $64.29 + A\cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for San Diego

$49.17 + A\cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for Chicago

Month	San Diego	Chicago
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2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

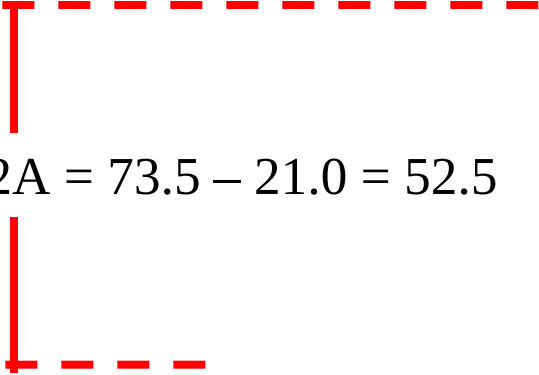

$$2A = 73.5 - 21.0 = 52.5$$

An Application

Average Monthly Temperatures (F°) $64.29 + A \cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for San Diego

Month	San Diego	Chicago
0	57.5	21.0
1	59.0	25.5
2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

$49.17 + 26.25 \cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for Chicago

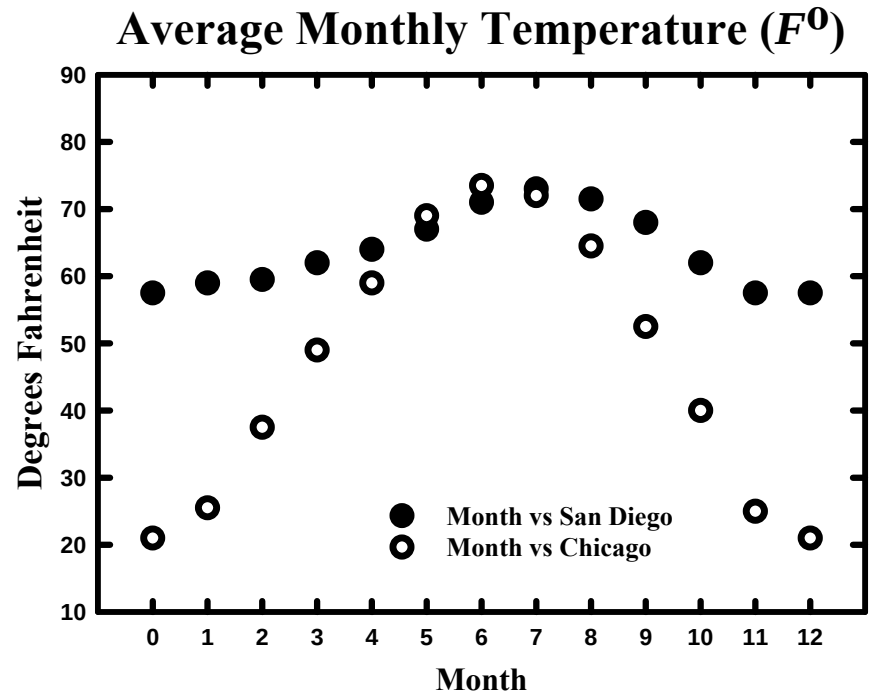

$$2A = 73.5 - 21.0 = 52.5$$

An Application

Average Monthly Temperatures (F°) $64.29 + 7.75 \cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for San Diego

$49.17 + 26.25 \cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for Chicago

Month	San Diego	Chicago
0	57.5	21.0
1	59.0	25.5
2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

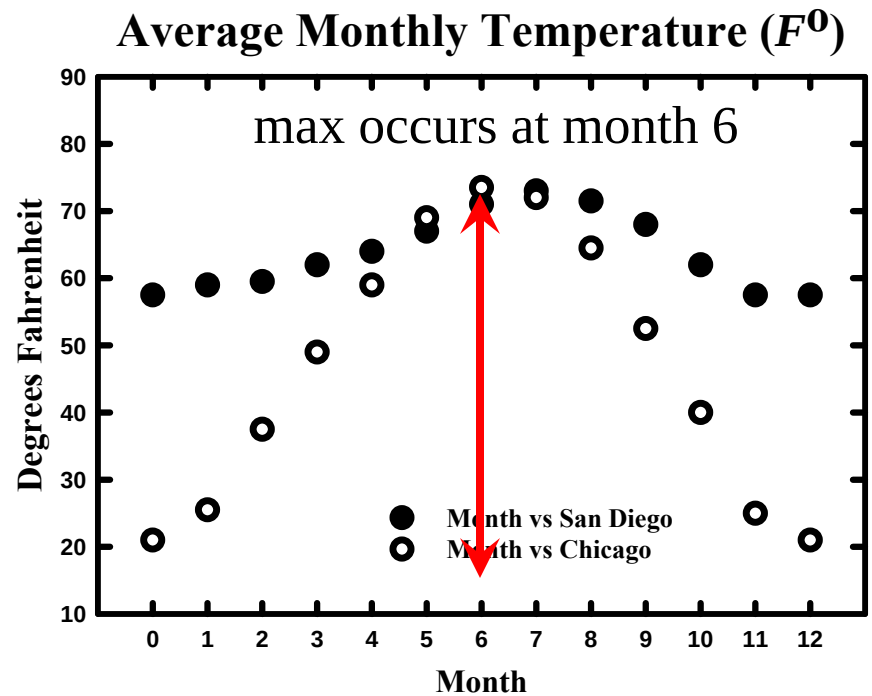


An Application

Average Monthly Temperatures (F°) $64.29 + 7.75 \cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for San Diego

$49.17 + 26.25 \cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for Chicago

Month	San Diego	Chicago
0	57.5	21.0
1	59.0	25.5
2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

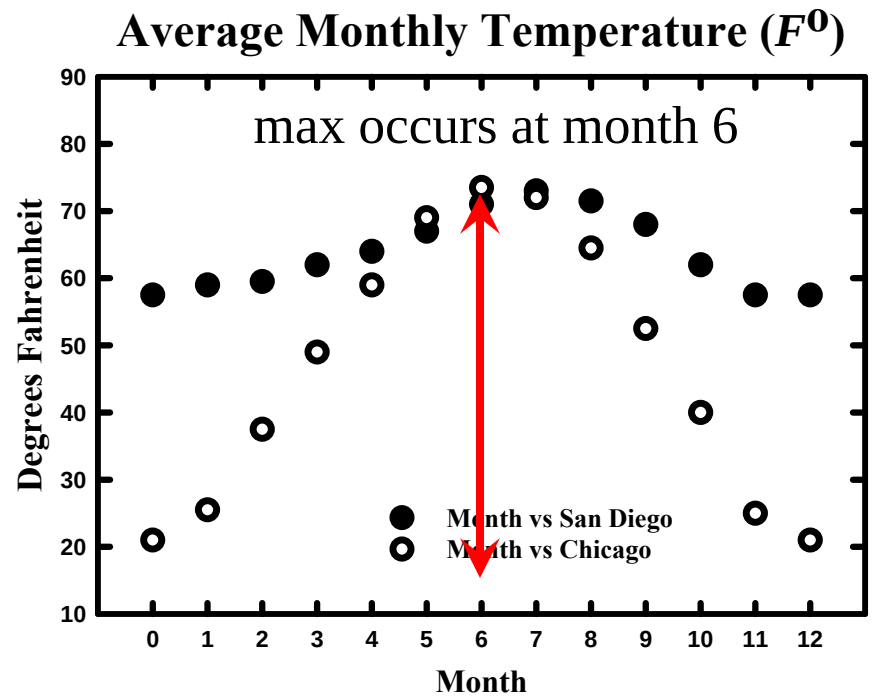


An Application

Average Monthly Temperatures (F°) $64.29 + 7.75 \cos\left(\frac{\pi}{6}(\theta - \varphi)\right)$ for San Diego

$49.17 + 26.25 \cos\left(\frac{\pi}{6}(\theta - 6)\right)$ for Chicago

Month	San Diego	Chicago
0	57.5	21.0
1	59.0	25.5
2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0

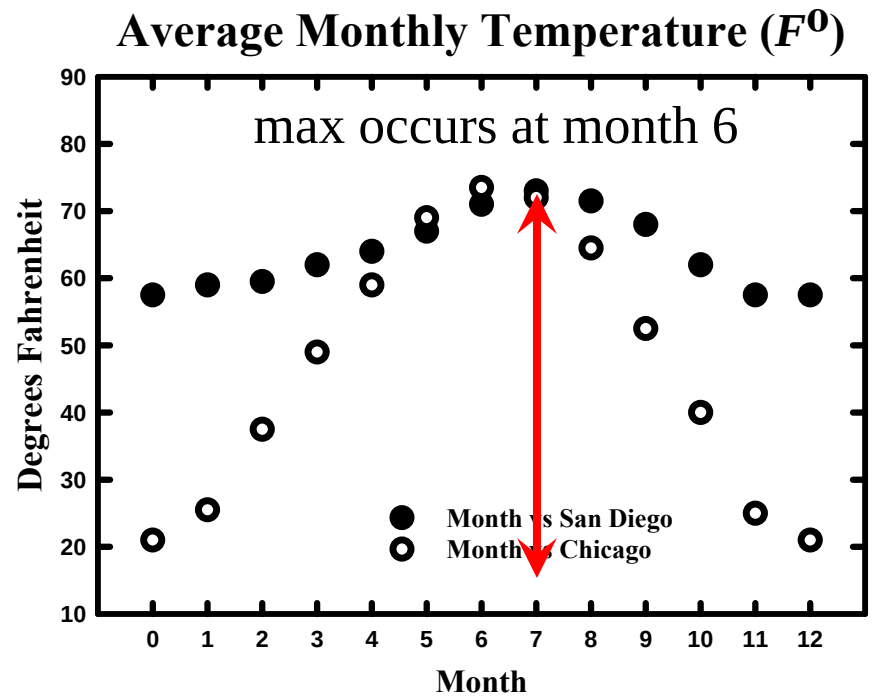


An Application

Average Monthly Temperatures (F°) $64.29 + 7.75 \cos\left(\frac{\pi}{6}(\theta - 7)\right)$ for San Diego

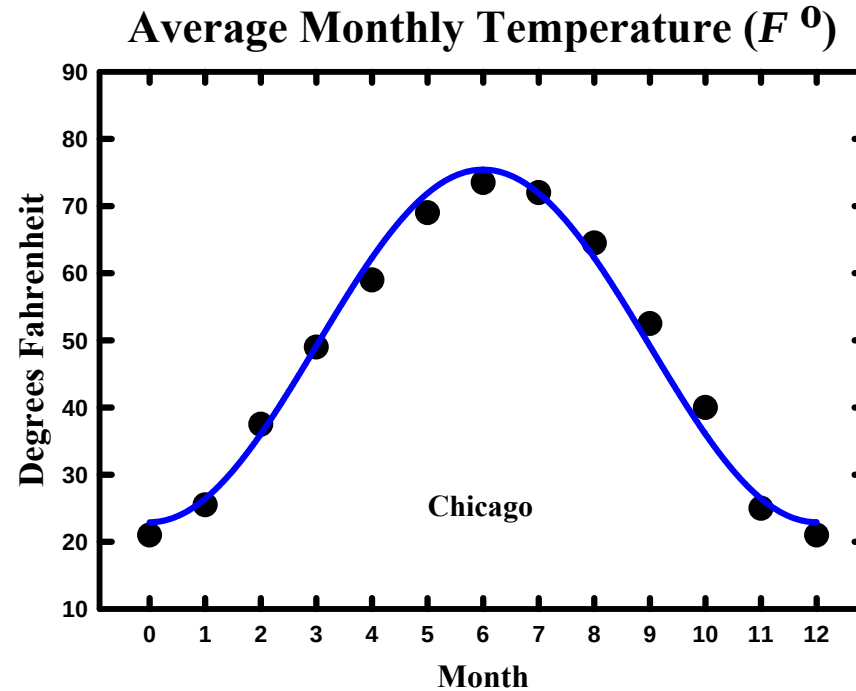
$49.17 + 26.25 \cos\left(\frac{\pi}{6}(\theta - 6)\right)$ for Chicago

Month	San Diego	Chicago
0	57.5	21.0
1	59.0	25.5
2	59.5	37.5
3	62.0	49.0
4	64.0	59.0
5	67.0	69.0
6	71.0	73.5
7	73.0	72.0
8	71.5	64.5
9	68.0	52.5
10	62.0	40.0



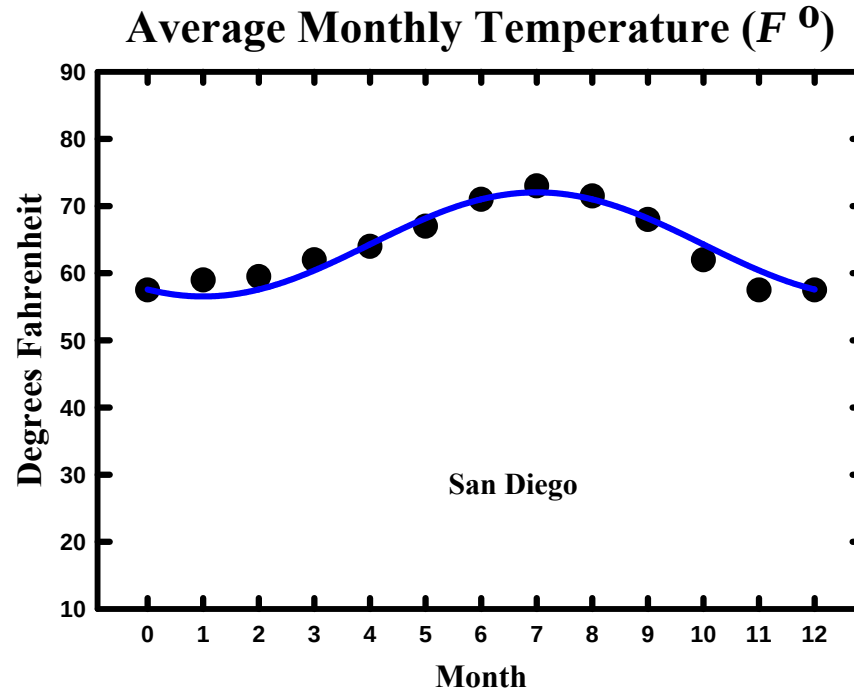
An Application

$$49.17 + 26.25 \cos\left(\frac{\pi}{6}(\theta - 6)\right) \text{ for Chicago}$$



An Application

$$64.29 + 7.75 \cos\left(\frac{\pi}{6}(\theta - 7)\right) \text{ for San Diego}$$



Some Basic Facts About Sine and Cosine

$$\sin x = 0 \quad \text{for } x = \pi$$

Some Basic Facts About Sine and Cosine

$$\sin x = 0 \quad \text{for } x = n\pi, n = 0, \pm 1, \pm 2, \dots$$

Some Basic Facts About Sine and Cosine

$$\sin x = 0 \quad \text{for } x = n\pi, n = 0, \pm 1, \pm 2, \dots$$

$$\cos x = 0 \quad \text{for } x = \frac{\pi}{2}$$

Some Basic Facts About Sine and Cosine

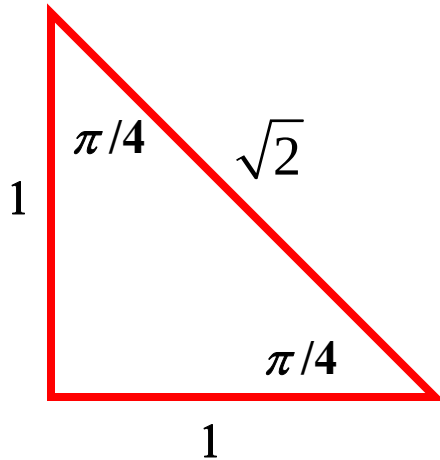
$$\sin x = 0 \quad \text{for } x = n\pi, n = 0, \pm 1, \pm 2, \dots$$

$$\cos x = 0 \quad \text{for } x = \frac{\pi}{2} + n\pi, n = 0, \pm 1, \pm 2, \dots$$

Some Basic Facts About Sine and Cosine

$\sin x = 0$ for (and only for) $x = n\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \dots$

$\cos x = 0$ for (and only for) $x = \frac{\pi}{2} + n\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \dots$

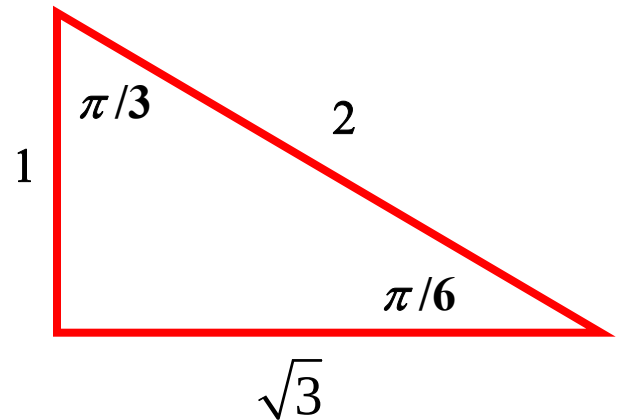
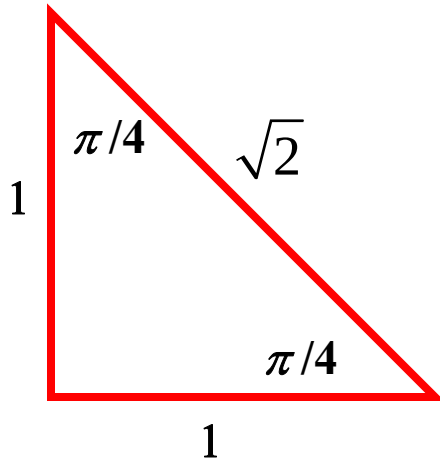


$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

Some Basic Facts About Sine and Cosine

$\sin x = 0$ for (and only for) $x = n\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \dots$

$\cos x = 0$ for (and only for) $x = \frac{\pi}{2} + n\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \dots$



$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}, \quad \sin \frac{\pi}{6} = \frac{1}{2}, \quad \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$