

MATH 250a

Fall Semester 2007

Section 2 (J. M. Cushing)

Thursday, September 6

<http://math.arizona.edu/~cushing/250a.html>

Rational Functions

$$f(x) = \frac{p(x)}{q(x)}, \quad p(x), q(x) \text{ are polynomials}$$

Basic properties

- (1) Defined, continuous, differentiable at all points where the denominator $q(x)$ does not equal 0.

$$\frac{x^2}{1+x^2}, \quad \frac{1}{1-x^2}, \quad \frac{1+x}{1-x^2}$$

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Sometimes we can “remove” a discontinuity.

$\frac{1+x}{1-x^2}$ is discontinuous at $x = 1$ and -1 .

$$\text{For } x \neq -1: \quad \frac{1+x}{1-x^2} = \frac{\cancel{1+x}}{(\cancel{1+x})(1-x)} = \frac{1}{1-x}$$

Rational Functions

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For $x \neq -1$:
$$\frac{1+x}{1-x^2} = \frac{\cancel{1+x}}{(\cancel{1+x})(1-x)} = \frac{1}{1-x}$$

Defined and continuous at $x = -1$ and equals $1/2$ there !

Rational Functions

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Basic properties

- (1) Defined, continuous, differentiable at all points where the denominator $q(x)$ does not equal 0.

Sometimes we can “remove” a discontinuity.

$$f(x) = \begin{cases} \frac{1-x}{1-x^2} & \text{for all } x \neq \pm 1 \\ \frac{1}{2} & \text{for } x = -1 \end{cases}$$

now continuous at $x = -1$!

discontinuity at $x = 1$ cannot be removed (why not?)

Rational Functions

$$f(x) = \frac{p(x)}{q(x)}, \quad p(x), q(x) \text{ are polynomials}$$

Basic properties

(2) At non-removable discontinuities, $f(x)$ has a vertical asymptote.

$$y = \frac{1}{1-x} \begin{cases} > 0 & \text{for } x < 1 \\ < 0 & \text{for } x > 1 \end{cases}$$

$$\frac{dy}{dx} = \frac{1}{(1-x)^2} > 0 \Rightarrow \text{increasing}$$

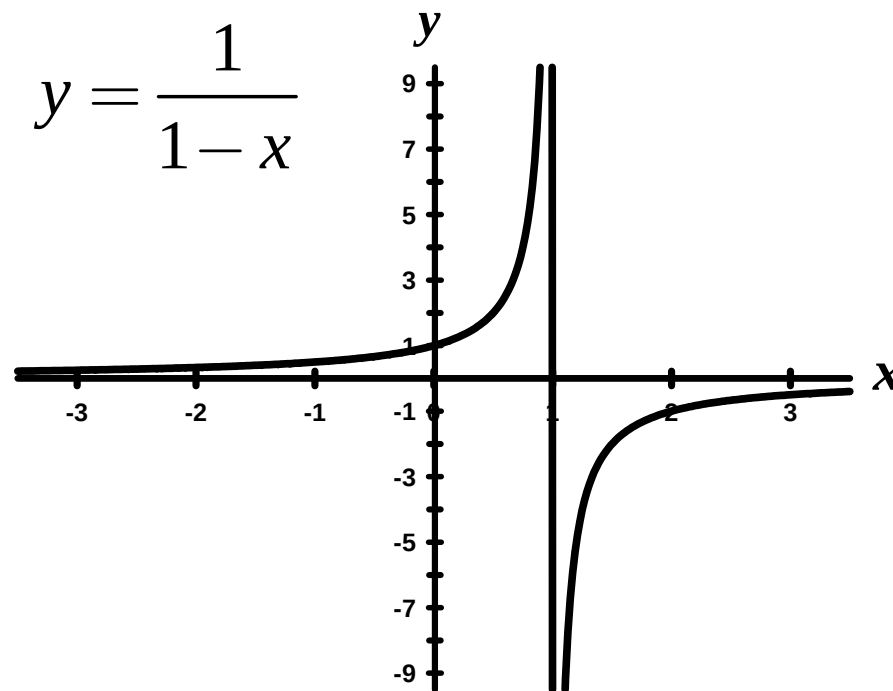
$$\frac{d^2y}{dx^2} = 2 \frac{1}{(1-x)^3} \Rightarrow \begin{cases} \text{concave up for } x < 1 \\ \text{concave down for } x > 1 \end{cases}$$

Rational Functions

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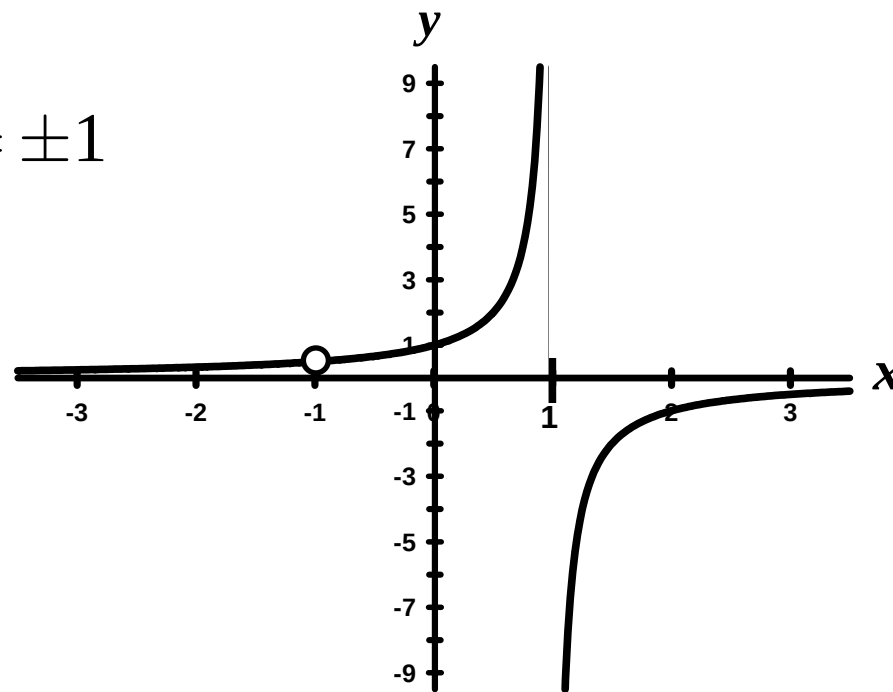
Rational Functions

$$f(x) = \frac{p(x)}{q(x)}, \quad p(x), q(x) \text{ are polynomials}$$

Basic properties

(2) At non-removable discontinuities, $f(x)$ has a vertical asymptote.

$$y = \frac{1+x}{1-x^2} = \frac{1}{1-x} \quad \text{for } x \neq \pm 1$$



Rational Functions

$$f(x) = \frac{p(x)}{q(x)}, \quad p(x), q(x) \text{ are polynomials}$$

Basic properties

(3) Has horizontal asymptote if and only if $\deg q \geq \deg p$.

$$y = \frac{1 + x^2}{1 - x}$$

Rational Functions

$$f(x) = \frac{p(x)}{q(x)}, \quad p(x), q(x) \text{ are polynomials}$$

Basic properties

(3) Has horizontal asymptote if and only if $\deg q \geq \deg p$.

$$\lim_{x \rightarrow +\infty} y = \lim_{x \rightarrow +\infty} \frac{1+x^2}{1-x} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x} + x}{\frac{1}{x} - 1} = -\infty$$

Rational Functions

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$$\lim_{x \rightarrow +\infty} y = \lim_{x \rightarrow +\infty} \frac{1+x^2}{1-x} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x} + x}{\frac{1}{x} - 1} = -\infty \quad \text{no horizontal asymptotes !}$$

$$\lim_{x \rightarrow -\infty} y = \lim_{x \rightarrow -\infty} \frac{1+x^2}{1-x} = \lim_{x \rightarrow -\infty} \frac{\frac{1}{x} + x}{\frac{1}{x} - 1} = +\infty$$

Rational Functions

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Basic properties

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$$y = \frac{1 + x^2}{1 - x^3}$$

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Basic properties

(3) Has horizontal asymptote if and only if $\deg q \geq \deg p$.

$$\lim_{x \rightarrow +\infty} y = \lim_{x \rightarrow +\infty} \frac{1+x^2}{1-x^3} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x^3} + \frac{1}{x}}{\frac{1}{x^3} - 1} = \frac{0}{-1} = 0$$

Rational Functions

$$f(x) = \frac{p(x)}{q(x)}, \quad p(x), q(x) \text{ are polynomials}$$

Basic properties

(3) Has horizontal asymptote if and only if $\deg q \geq \deg p$.

$$\lim_{x \rightarrow +\infty} y = \lim_{x \rightarrow +\infty} \frac{1+x^2}{1-x^3} = 0$$

$$\lim_{x \rightarrow -\infty} y = \lim_{x \rightarrow -\infty} \frac{1+x^2}{1-x^3} = 0$$

$y = 0$
is an horizontal
asymptote
(in both directions)

Rational Functions

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$$y = \frac{1+x}{1-x}$$

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$$\lim_{x \rightarrow +\infty} y = \lim_{x \rightarrow +\infty} \frac{1+x}{1-x} = \lim_{x \rightarrow +\infty} \frac{\frac{1}{x} + 1}{\frac{1}{x} - 1} = \frac{1}{-1} = -1$$

Rational Functions

$$f(x) = \frac{p(x)}{q(x)}, \quad p(x), q(x) \text{ are polynomials}$$

Basic properties

(3) Has horizontal asymptote if and only if $\deg q \geq \deg p$.

$$\lim_{x \rightarrow +\infty} y = \lim_{x \rightarrow +\infty} \frac{1+x}{1-x} = -1$$

$y = -1$
is an horizontal
asymptote

$$\lim_{x \rightarrow -\infty} y = \lim_{x \rightarrow -\infty} \frac{1+x}{1-x} = -1$$

(in both directions)

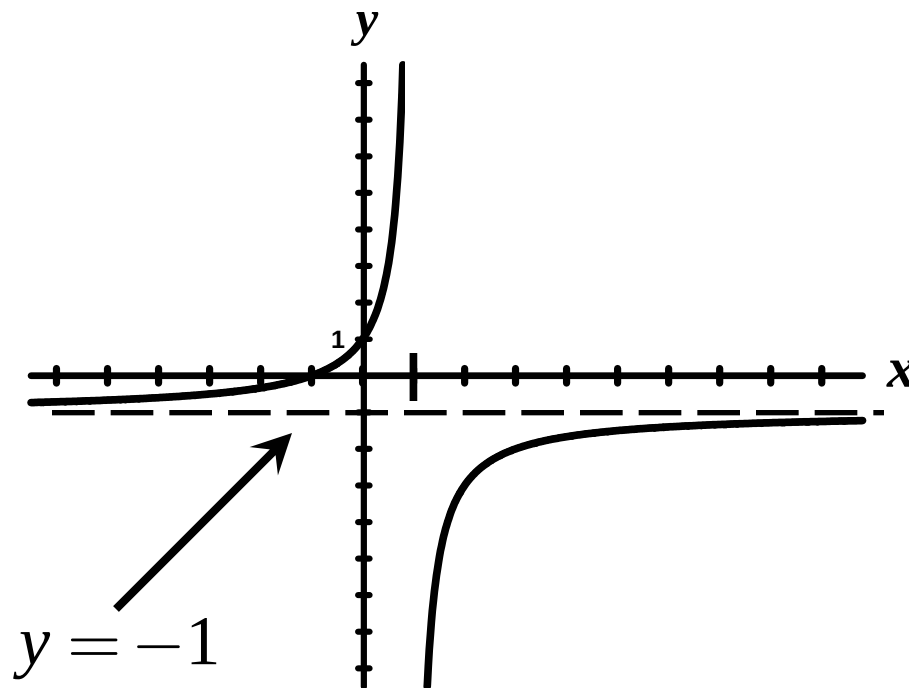
Rational Functions

$$f(x) = \frac{p(x)}{q(x)}, \quad p(x), q(x) \text{ are polynomials}$$

Basic properties

(3) Has horizontal asymptote if and only if $\deg q \geq \deg p$.

$$y = \frac{1+x}{1-x}$$



Rational Functions

Enzyme-substrate reactions

The diagram illustrates the relationship between the reaction rate and substrate concentration in an enzyme-substrate reaction. It features the Michaelis-Menten equation, $V \approx \frac{a[S]}{b + [S]}$, centered at the top. Two arrows point towards this equation: one from the left labeled "reaction rate" pointing to the variable V , and another from the bottom right labeled "substrate concentration" pointing to the term $[S]$ in the numerator and denominator.

$$V \approx \frac{a[S]}{b + [S]}$$

reaction rate

substrate
concentration

Rational Functions

Enzyme-substrate reactions

$$V([S]) = \frac{a[S]}{b + [S]}$$

$$\frac{dV([S])}{d[S]} = \frac{ab}{(b + [S])^2} > 0 \quad \text{increasing}$$

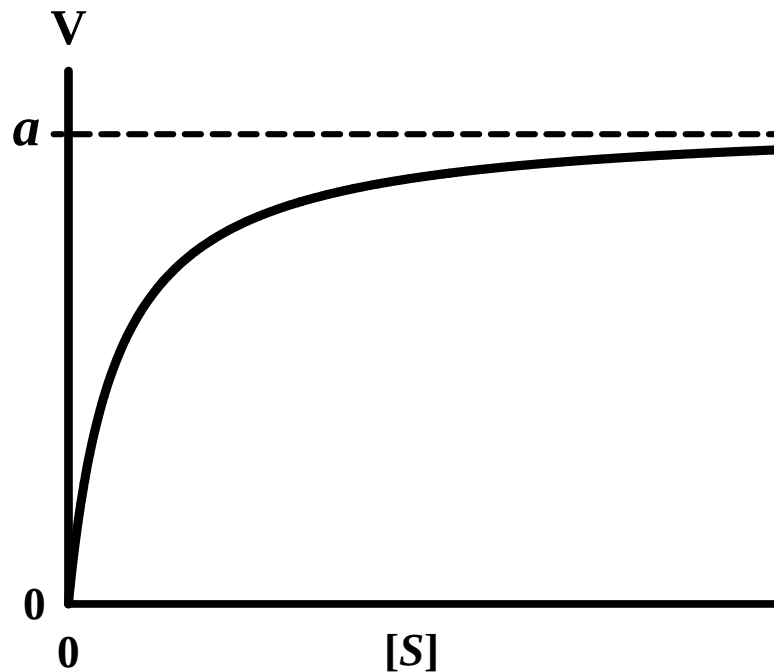
$$\frac{d^2V([S])}{d[S]^2} = \frac{-2ab}{(b + [S])^3} < 0 \quad \text{concave down}$$

$$\lim_{[S] \rightarrow +\infty} V([S]) = \lim_{[S] \rightarrow +\infty} \frac{a[S]}{b + [S]} = a \quad \text{horizontal asymptote}$$

Rational Functions

Enzyme-substrate reactions

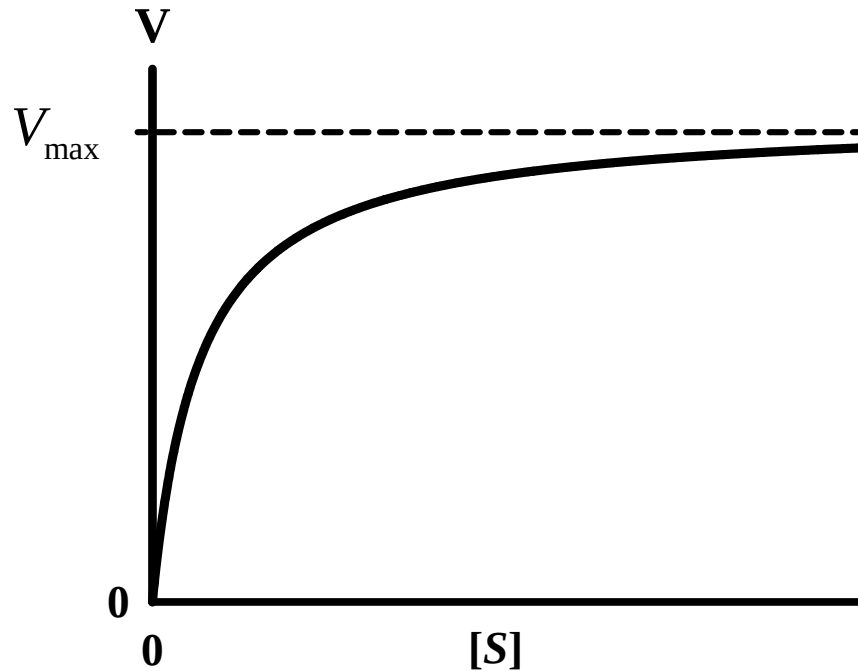
$$V([S]) = \frac{a[S]}{b + [S]}$$



Rational Functions

Enzyme-substrate reactions

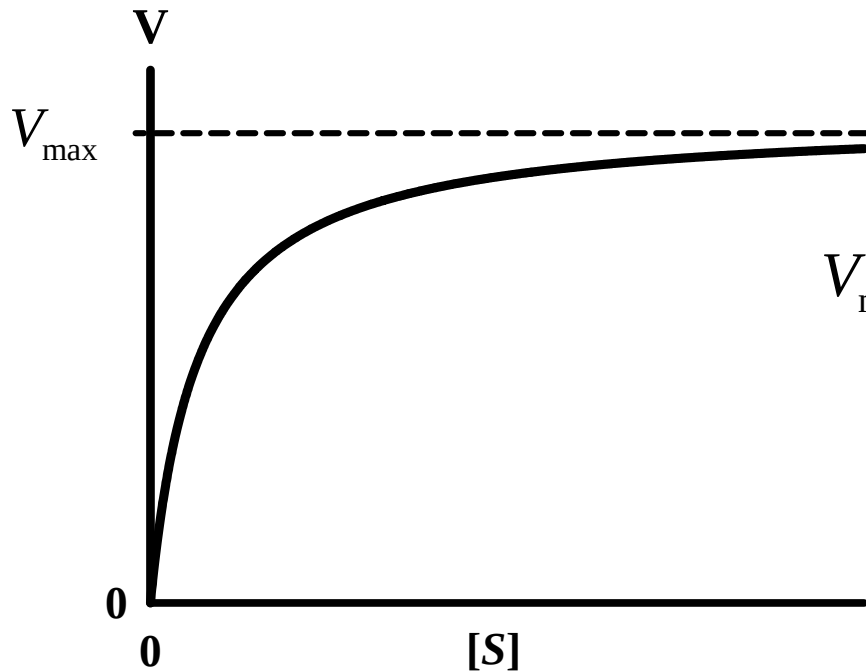
$$V([S]) = V_{\max} \frac{[S]}{b + [S]}$$



Rational Functions

Enzyme-substrate reactions

$$V([S]) = V_{\max} \frac{[S]}{b + [S]}$$



$$V([S]) = \frac{1}{2} V_{\max}$$

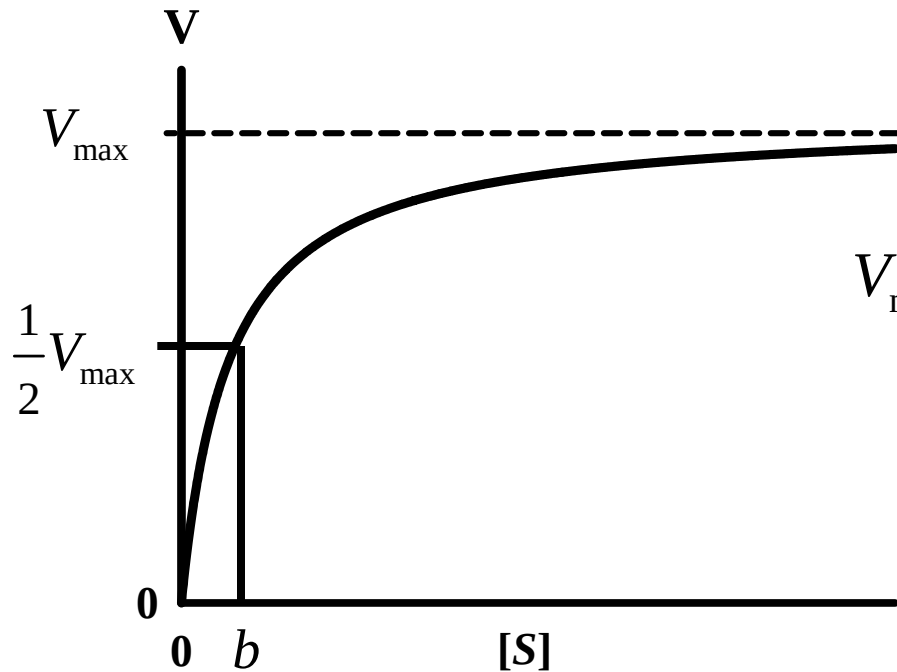
$$V_{\max} \frac{[S]}{b + [S]} = \frac{1}{2} V_{\max}$$

$$[S] = b$$

Rational Functions

Enzyme-substrate reactions

$$V([S]) = V_{\max} \frac{[S]}{b + [S]}$$



$$V([S]) = \frac{1}{2}V_{\max}$$

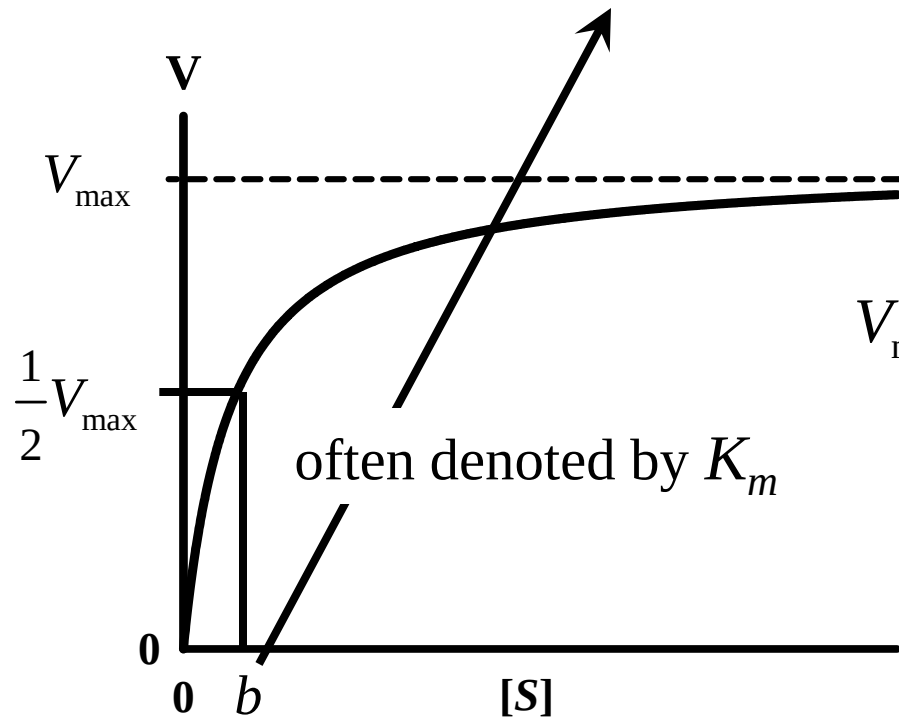
$$V_{\max} \frac{[S]}{b + [S]} = \frac{1}{2}V_{\max}$$

$$[S] = b$$

Rational Functions

Enzyme-substrate reactions

$$V([S]) = V_{\max} \frac{[S]}{K_m + [S]}$$



$$V([S]) = \frac{1}{2}V_{\max}$$

$$V_{\max} \frac{[S]}{b + [S]} = \frac{1}{2}V_{\max}$$

$$[S] = b$$

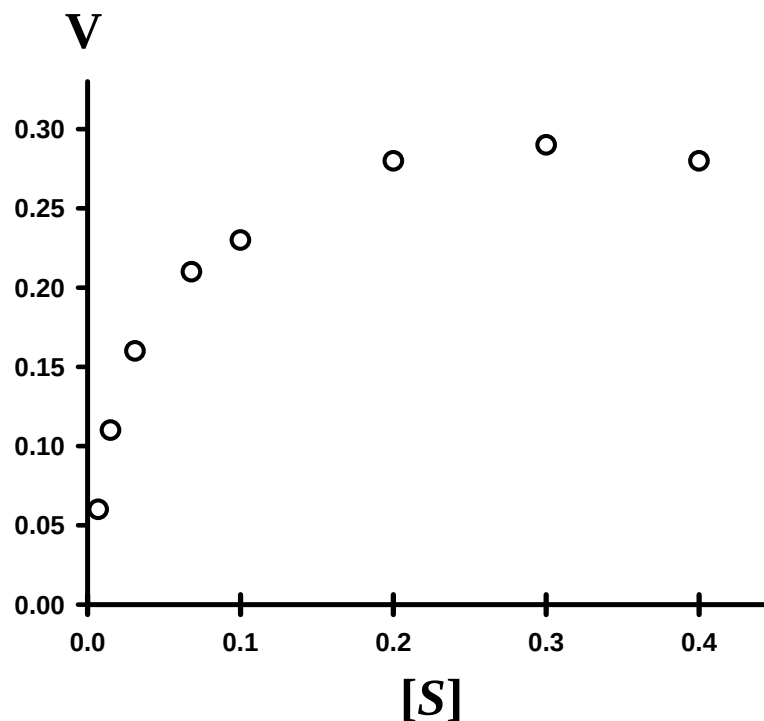
An Application

substrate = ethanol

enzyme = alcohol dehydrogenase (ADH)

product = acetaldehyde

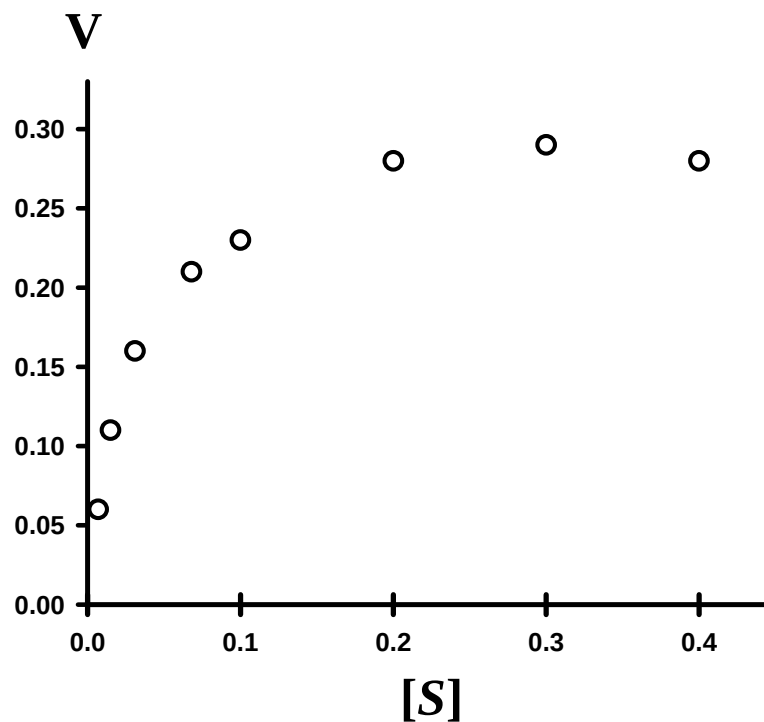
[S] (Mol/L)	V (Absorbance/min)
0.0070	0.0600
0.0150	0.1100
0.0310	0.1600
0.0680	0.2100
0.1000	0.2300
0.2000	0.2800
0.3000	0.2900
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An Application

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$$V = V_{\max} \frac{[S]}{K_m + [S]}$$



An Application

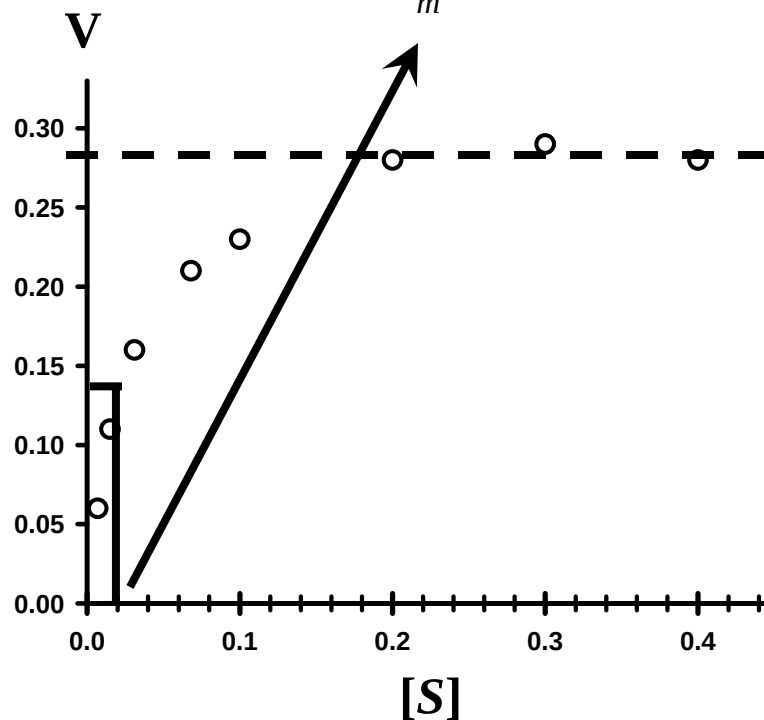
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$$V = V_{\max} \frac{[S]}{K_m + [S]}$$

“Eyeball” estimates

$$V_{\max} \approx 0.28$$

$$K_m \approx 0.02$$

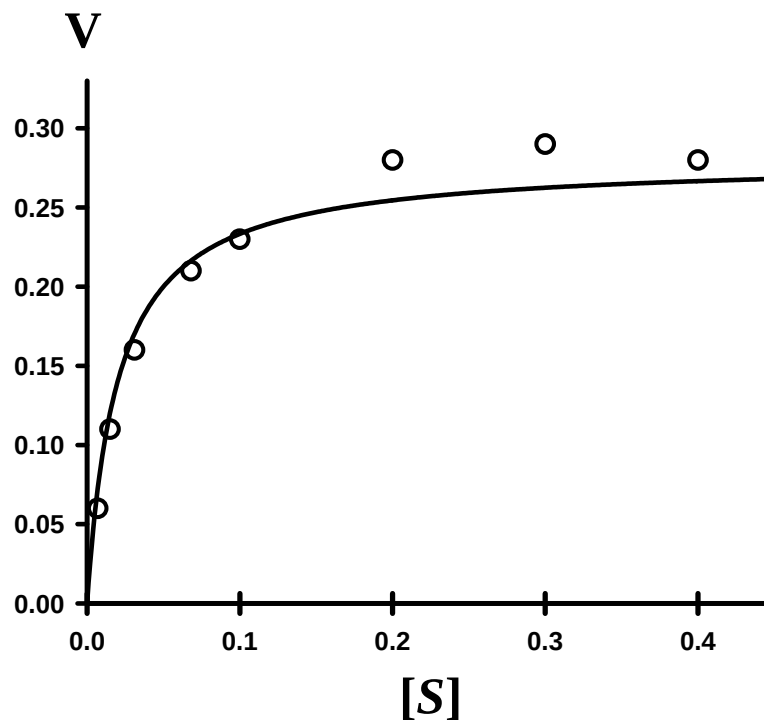


An Application

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“Eyeball” estimates



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$$V = 0.28 \frac{[S]}{0.02 + [S]}$$

“Eyeball” estimates

Fit not so good.

We can do better ...

