

MATH 250a

Fall Semester 2007

Section 2 (J. M. Cushing)

Tuesday, August 28

<http://math.arizona.edu/~cushing/250a.html>

## Example

Deposit  $P(0)$  in a bank account at  $r\%$  interest (compounded annually).

How many years does it take for the deposit to double ?

Solve for  $t$  :

$$(1 + r)^t = 2$$

This requires the *inverse* of the exponential function,  
that is, requires the *logarithm function*

## DEFINITIONS

- (a) A function is *one-one* (invertible) if each element in the range corresponds to *only one* element in the domain.

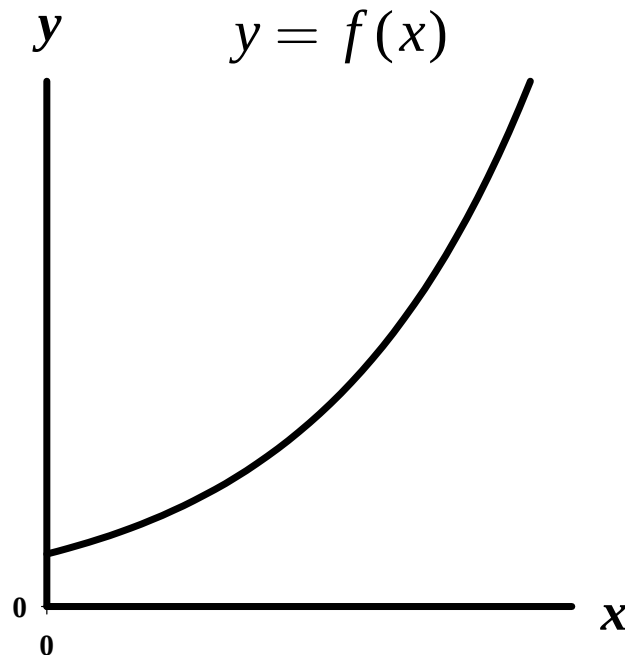
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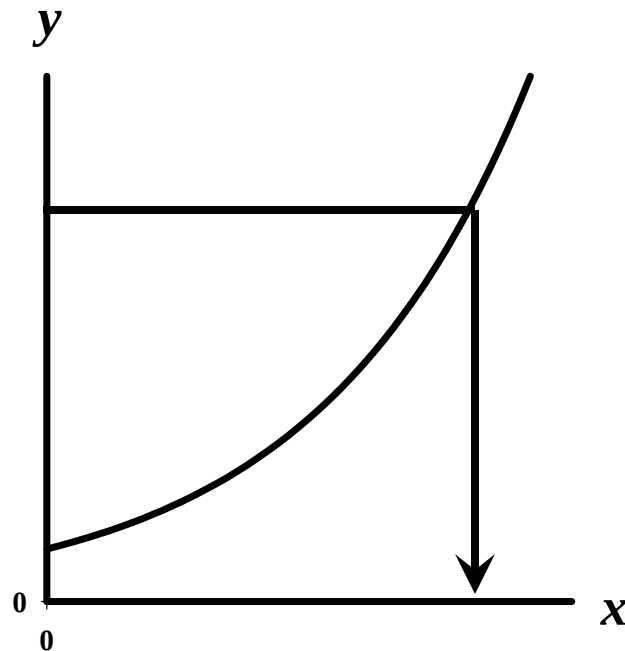


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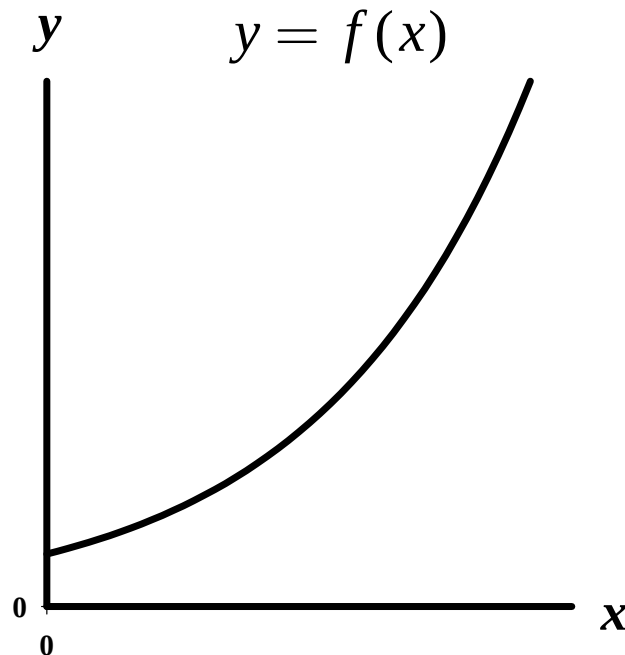
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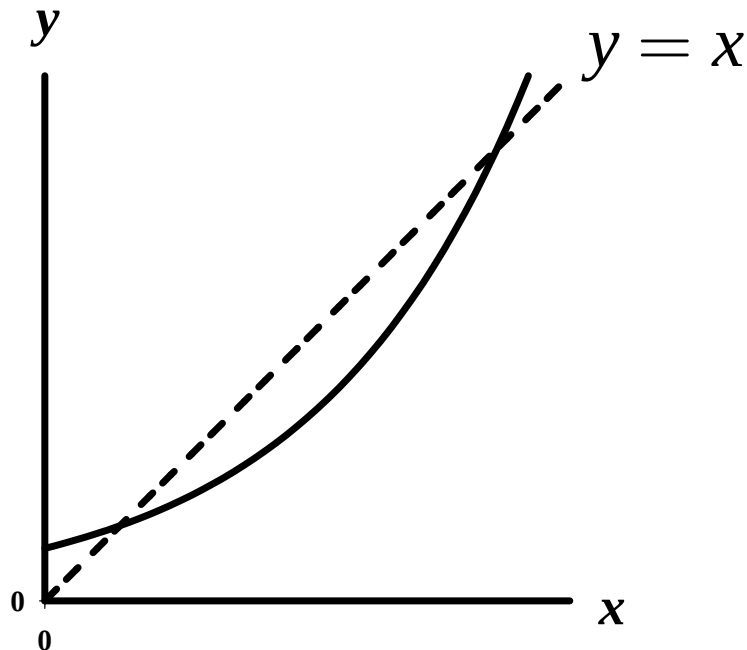


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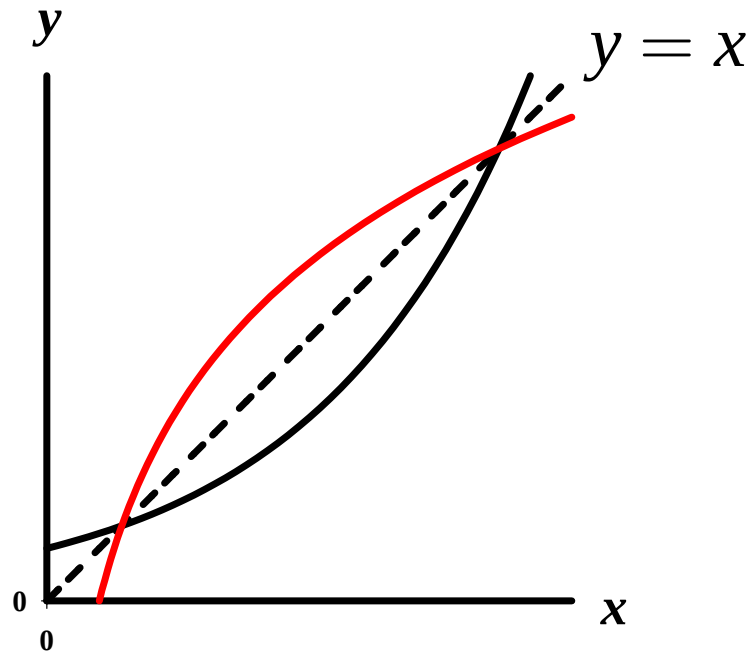
to get the graph of  
the inverse reflect  
through the line  
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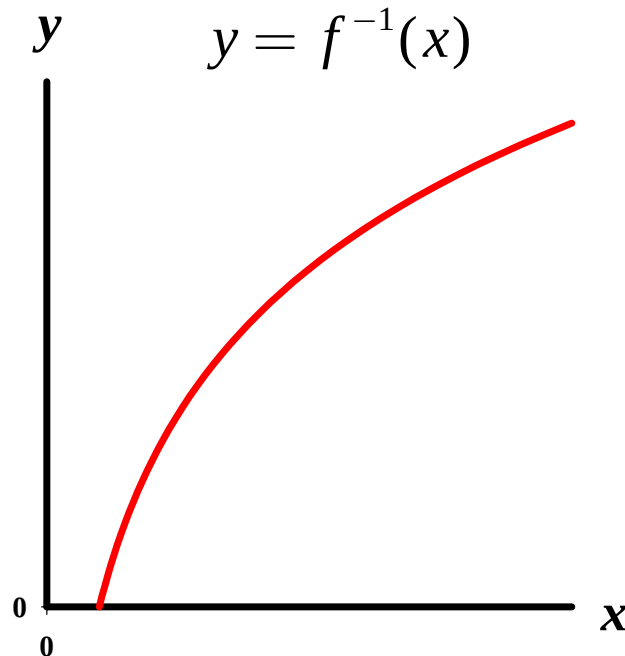
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The inverse of  $P(t) = a^t$ ,  $0 < a \neq 1$

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common mistakes :

$$\log(ts) \neq (\log t)(\log s)$$

$$\log(t + s) \neq \log t + \log s$$

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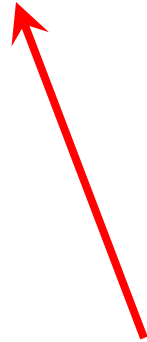


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If we want to work in base 10,  
we re-write this exponential  
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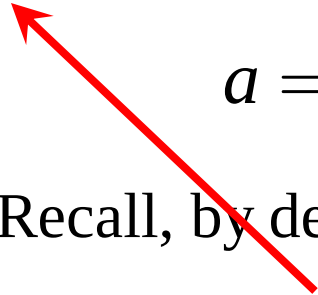
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Notation convention:  $\begin{cases} \log t = \log_{10} t & \text{common log} \\ \ln t = \log_e t & \text{natural log} \end{cases}$

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For example, for  $r = 0.05$

$$t = \frac{\ln 2}{\ln 1.05} \approx 14.21 \text{ (years)}$$

Next goal ....

The geometry (graphs) of exponential and log functions

## A Quick Review of Derivatives

Derivative of  $f(x)$  at  $x = a$

$$\left. \frac{df}{dx} \right|_{x=a} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

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$$(2) \text{ Geometrically: } \left. \frac{df}{dx} \right|_{x=a} = \begin{cases} \text{slope of tangent to } y = f(x) \\ \text{at point } (x, y) = (a, f(a)) \end{cases}$$

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Second derivative of  $f(x)$

$$\frac{d}{dx} \left( \frac{df}{dx} \right) = \frac{d^2 f}{dx^2}$$

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$$\frac{dx^m}{dx} = mx^{m-1} \quad (\text{power rule})$$

$$\frac{de^x}{dx} = e^x$$

$$\frac{d \ln x}{dx} = \frac{1}{x}$$

# A Quick Review of Derivatives

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