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Summary of Topics Covered in 250 (both semesters)

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250A (Fall 2009)

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A1. Review (re high school): dealing w/ analytic expressions, differentiation, definite integrals, etc....

- definition of a function, limit, continuity, differentiability
- types of functions: linear, polynomial, exponential, logarithmic, rational, trigonometric
- using calculus to determine the behavior of functions (e.g., critical points, concavity, limits)
- basics of problem-solving & critical thinking
- methods of differentiation

A2. Introduce and motivate ODEs and what they can be used for, types of ODEs

- incorporate via the notion of 'word problems'
- expose to a diverse range of ODE applications to motivate
- different forms of 1st order ODEs:
 - $dy/dx = f(x,y)$: general form
 - $dy/dx = f(y)$: autonomous (general type we will approach early on)
 - $dy/dx = f(x)g(y)$: solvable via separation of variables
- 2nd and higher order eqns. (e.g., Newton's second law)
- systems of coupled ODEs
- expand upon linear vs nonlinear ODEs
- > our focus will be upon linear eqns. where we can do some degree of analysis
[nonlinear cases will come later, where for the most part we try to linearize!]

A3. Preliminary means to solve ODEs

- solving ODEs <---> integration
- separation of variables
- > motivates methods of integration

A4. Numerical methods to solve definite integrals (i.e., Riemann sums) **(LAB)**

- first exposure to using the computer as a programmable tool
- methods: LHR, RHR, Midpoint rule, trapezoid rule, Simpson's rule

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B1. Methods of Integration

- Integration by parts
- Integration via substitution
- Using the table of integrals
- long division, completing the square
- trig. substitutions
- partial fractions (motivate via solving the logistic equation)

B2. Integral applications

- arc length
- volumes
- solids of revolution

B3. Definite integrals within the context of solving ODEs

B4. Qualitative aspects of dealing with differential eqns.

- classifying systems (e.g. linear vs nonlinear, autonomous, etc...)
- slope fields **(LAB)**
- finding equilibrium points and identifying stability
- symmetry/behavior of solutions

B5. Analytical approaches to ODEs

- finding general solutions
- initial/boundary conditions (leads in exist. & uniqueness)
- separation of variables

B5. Logistic equation

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C1. Existence & Uniqueness of solutions to ODEs

C2. Phase line analysis & stability

- notion of bifurcations

C3. Euler's method to numerically solve ODEs **(LAB)**

C4. General method for solving first order ODEs

- homogeneous coefficients
- Bernoulli differential eqns.
- applications of ODEs (e.g., RL-circuits)

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D1. Introduce methods to solve linear ODEs (Lomen & Lovelock Ch.5)

D2. Examples of ODEs

- exponential growth/decay
- logistic growth

- D3. Metapopulations & the Levin's model
- D4. Relationship between discrete maps & differential equations
- D5. Logistic map & chaos (via period-doubling)
- D6. Linear regression

----- **Semester 1 Labs**

- Lab 1: Riemann Sums (via writing a Matlab script)
- Lab 2: Slope Fields
- Lab 3: Euler's method to solve 1st order ODEs (via writing a Matlab script)

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250B (Spring 2009)

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- A1. Improper integrals
 - evaluating
 - convergence
 - paradoxes associated with infinity and means to handle (e.g., cardinality)
 - A2. Sequences (e.g., convergence)
 - A3. Infinite Series
 - convergence
 - interval of convergence
 - geometric series

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- B1. Power series
 - radius of convergence
 - B2. Taylor polynomials (as a means of approximating functions)
 - B3. Taylor series expansions
 - B4. Using power series to solve ODEs/integral
 - B4. Notion of linear independence ---> Wronskian
 - B5. Fourier series
 - applications to hearing
 - as another means to solve ODEs
 - connection to complex #s
 - B6. Binomial series

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- C1. Complex #s
 - complex arithmetic
 - magnitude and phase <--> cartesian coordinates
 - C2. 2nd order ODEs (homogeneous and nonhomogeneous) & Systems of ODEs
 - classifying different types of ODEs
 - notion of the phase plane
 - general examples
 - **harmonic oscillator** [(un)forced, (un)damped, active/passive] ---> resonance
 - *connection back to Fourier series*
 - **Method of Undetermined Coefficients**
 - C3. Converting higher order eqns. into systems of 1st order eqns.
 - C4. Power series solutions to ODEs
 - C5. Cauchy-Euler Eqns.

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- D1. Coupled ODE Applications
 - *application to Michaelis-Menten kinetics (LAB)*
 - population dynamics
 - D2. Nonlinear systems
 - notion of orbits and relationship to critical points & stability
 - Eigenvalues from linearized system to classify stability
 - Bifurcation parameters
 - Phase plane analysis (e.g., pplane)
 - D3. Applications of nonlinear ODEs
 - Synchronization of fireflies
 - van der Pol Oscillators
 - Epidemiological models (SIR)
 - Tacoma bridge revisited
 - D4. Higher order systems and chaos (e.g. Lorenz eqns.) ---> phase space

----- **Semester II Labs**

- Discussion 1: Lunar Motion
- Discussion 2: Applied Mathematics/Biophysics Lab Visit
- Lab 1: Fourier Analysis
- Lab 2: Michaelis-Menten kinetics

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Key Concepts Students Should Come Away With from 250A&B (2009-2010)

- Comfort dealing with analytic representations of quantities/functions
 - o linear, polynomial functions
 - o logarithms
 - o exponential functions
 - o trigonometric functions (including hyperbolic versions)
 - o plotting functions (critical points, inflection points, limits, etc....)
- Methods of differentiation
- Methods of integration
 - o definite integrals
 - o improper integrals
 - o using trig. substitutions
 - o integration by parts
 - o numerical approaches via Riemann sums (**LAB**)
- Complex #s
 - o complex arithmetic
 - o magnitude and phase $\leftrightarrow$ cartesian coordinates
 - o connection back to hyperbolic trig. functions
- 'Word problems'
- Using ODEs as a means to describe phenomena
- Qualitative approaches to ODEs
 - o classifying systems (e.g. linear vs nonlinear, autonomous, etc...)
 - o slope fields (**LAB**)
 - o finding equilibrium points and identifying stability
 - o phase plane/space
- Methods for solving ODEs
 - o relation back to integrals (for separable eqns.)
 - o implementing numerical means to solve via Euler's method (**LAB**)
 - o series approxs. (see below)
 - o eigenvalues (including complex cases)
- Sequences
- Series
 - o infinite series
 - o nuances associated with infinity
 - o convergence, intervals of convergence
 - o geometric series
 - o power series
- Taylor series approximations
 - o notion of linear independence
 - o application to ODEs
- Fourier series (**LAB**)
- Notion of Linear Independence
 - o linear combinations of functions
 - o Wronskian
- Linear ODEs
 - o Systems of linear ODEs
 - o 2nd Order ODEs & Methods to Solve
 - o interplay between systems and 2nd order eqns
 - o Method of Undetermined Coefficients
 - o general solutions and eigenvalues
 - o phase plane analysis
- Introduction to nonlinear systems
 - o visualizing orbits in the phase plane
 - o relating orbits to critical points & their associated stability
- Applications to problems in various physical/biological contexts:
 - o harmonic oscillator/RLC circuits
 - o SIR models
 - o predator-prey dynamics (e.g., Lotka-Volterra eqns.)
 - o van der Pol oscillators
 - o auditory periphery (broad overview, e.g., Fourier analysis)
 - o application to Michaelis-Menten kinetics (**LAB**)
- Chaotic systems
 - o logistic map and period doubling
 - o Lorenz eqns. and the 'butterfly' attractor in phase space
- Linear regression (covered briefly, will come up in Math 363)