

□ $S' = \text{inflow} - \text{outflow}$

[ditto I', R']

SIR
Model
Revisited

5/3/10

Susceptibles
 $S(t)$

↓ rSI

Infected
 $I(t)$

↓ aI

□ $S' = -rSI$

$I' = rSI - aI$

$R' = aI$

a - 'recovery' coefficient

r - 'infection' coefficient

□ assume $S + I + R = \text{const.}$, then $(S + I + R)' = 0$

→ $R' = -S' - I'$

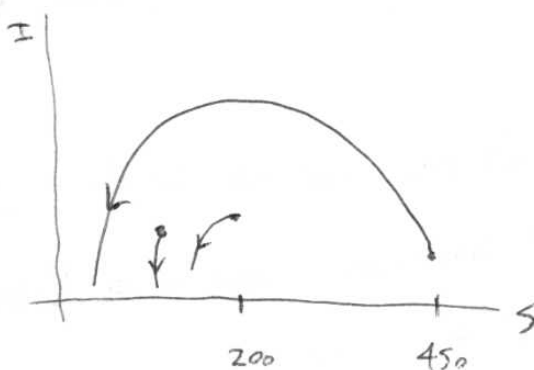
~~REDACTED~~

→ so if we know S and I , then we know R and thus the eqn. for R' is redundant

□ suppose there are some ICs: $S(0) = S_0 \geq 0$, $I(0) = I_0 \geq 0$

□ assume $a = 0.6$, $r = 0.003$

⇒ why? what is the underlying mechanism such that solutions behave this way?



→ increasing I means an epidemic, so $S \approx 200$ seems like a transition value re. whether an epidemic occurs or not

□ equilibria: $-rSI = 0$

$rSI - aI = 0$

→

$(S_{eq}, I_{eq}) = (0, 0), (S_e, 0)$

any value for S

□ stability: $J(S, I) = \begin{bmatrix} -rI & -rS \\ rI & rS - a \end{bmatrix}$

[cont.]

$$J(0,0) = \begin{bmatrix} 0 & 0 \\ 0 & -a \end{bmatrix} \quad \begin{array}{l} \text{Tr} = -a \\ \text{det} = 0 \end{array} \rightarrow \lambda^2 + a\lambda = \lambda(1+a) = 0$$

$$\lambda = 0, -a$$

$$\rightarrow s(t) = c_1 + c_2 e^{-at} \quad [\text{stable}]$$

$$J(S_e, 0) = \begin{bmatrix} 0 & -rS_e \\ 0 & rS_e - a \end{bmatrix} \quad \begin{array}{l} \text{Tr} = rS_e - a \\ \text{det} = 0 \end{array} \rightarrow \lambda^2 + (a - rS_e)\lambda = 0$$

$$\lambda = 0, rS_e - a$$

$$\rightarrow s(t) = c_1 + c_2 e^{(rS_e - a)t}$$

so if $rS_e - a > 0$ (i.e. $S_e > \frac{a}{r}$), the equilibrium is locally unstable

$\Rightarrow$ so $S_e \leftrightarrow \frac{a}{r}$ seems to represent some sort of transition point

□ higher level analysis can now be helpful (e.g. Poincaré-Bendixon theorem) but doesn't affect main idea: there is a threshold susceptible population for an 'epidemic' to occur.

□ what if there was an 'inflow' rate?

$$\boxed{19} \quad S' = -rSI + \alpha$$

$$I' = rSI - aI$$

where

$$a = 0.6$$

$$r = 0.003$$

$$\alpha = 10$$

$\rightarrow$ dynamics become more complicated, but basic idea highlighted above is still valid