

4/9/10 (cont.)

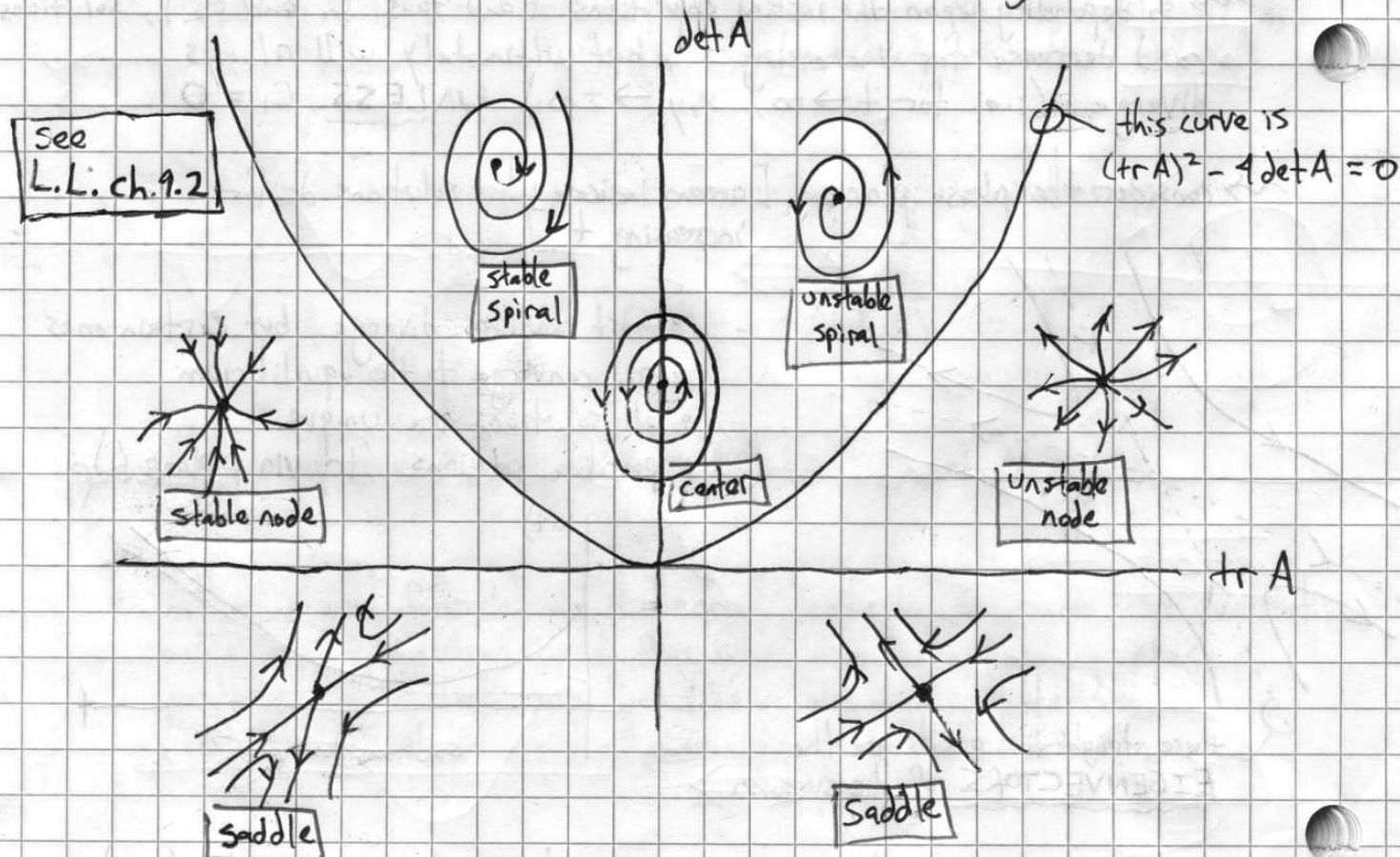
Theorem For a linear system of the form

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \bar{A} \bar{x}$$

the following statements are equivalent:

- the equilibrium is asymptotically stable
- all the eigenvalues have negative real parts
- $\det A = ad - bc > 0$ AND $\text{tr} A = a + d < 0$

□ another nice way to get at the qualitative behavior of solutions to a linear autonomous system is to consider the stability of the origin $[(0,0)$ in the phase plane] in the following context:



⇒ thus by simply determining $\text{tr} A$ and $\det A$ and using the guide above, you have a good sense what the solutions are going to do!

□ note that typically there is only one equilibrium point ($\odot (0,0)$), but there can also be a case where there is an infinite # of equilibria; for example, consider $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

→ what do solution curves do? (try using pplane to fiddle....)

[ex] Convert the following system to a single first-order ODE:

$$\dot{U} = U - V$$

$$\dot{V} = -U + 2V$$

$$\rightarrow \ddot{U} = \dot{U} - \dot{V} = \dot{U} - (-U + 2V) \quad [\text{but } V = U - \dot{U}]$$

$$= \dot{U} + U - 2(U - \dot{U}) = 3\dot{U} - U$$

$$\Rightarrow \ddot{U} - 3\dot{U} + U = 0$$

□ now convert this back to a system:

$$\text{let } \dot{U} = W \quad (\text{NOTE: } W \neq V!)$$

$$\text{then } \dot{W} = -U + 3W$$

→ but this is different from an original system! problem?

□ in converting from a system to a 2nd order ODE, we essentially lost information (which we never can get back); while the two systems are indeed different (noting $W \neq V$), they still qualitatively have the same behavior

→ the eigenvalues are identical between the two systems, but the associated eigenvectors are different (this is the information we lost!); try verifying this via pplane