

4/9/10 (Cont.)

→ the eigenvalues  $\lambda$  must satisfy the corresponding equation:

$$|\bar{M} - \lambda \bar{I}| = \begin{vmatrix} \alpha - \lambda & \beta \\ -\beta & \alpha - \lambda \end{vmatrix} = (\alpha - \lambda)^2 + \beta^2 = 0$$

[this is just the characteristic equation! rearranging ...]

$$\lambda^2 - 2\alpha\lambda + \alpha^2 + \beta^2 = 0$$

$$\rightarrow \lambda = \frac{2\alpha \pm \sqrt{4\alpha^2 - 4\alpha^2 - 4\beta^2}}{2} = \alpha \pm i\beta$$

→ so our eigenvalues are complex and conjugates and provide us w/ the following solutions:

$$\begin{aligned} x_1(t) &= [C_1 \sin \beta t + C_2 \cos \beta t] e^{\alpha t} \\ &= A e^{\alpha t} \cos(\beta t + \phi) \end{aligned}$$

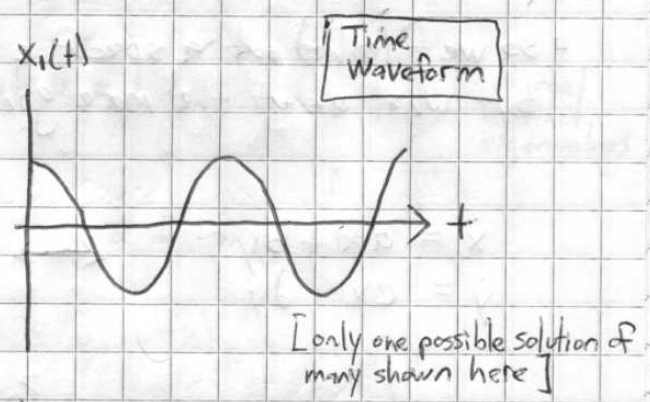
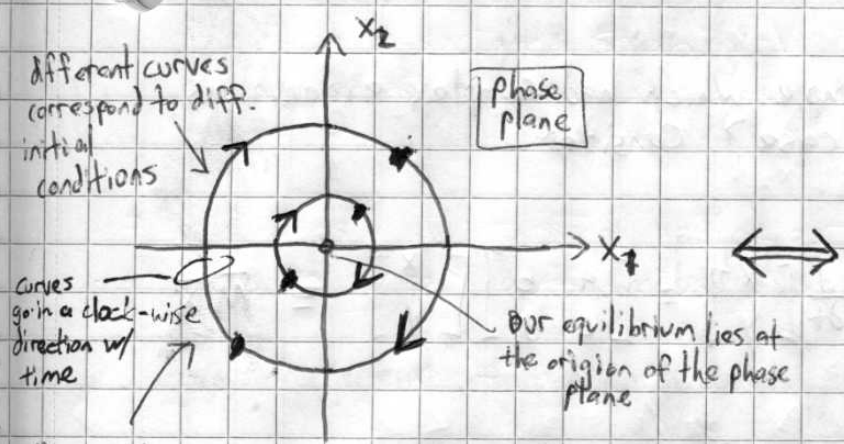
$$x_2(t) = -A e^{\alpha t} \sin(\beta t + \phi) \quad (\text{verify this!})$$

such that the consts.  $A$  and  $\phi$  will be uniquely determined by the initial conditions

$$A, \phi \in \mathbb{R}$$

□ Keeping in mind that  $(x_1, x_2) = (0, 0)$  is a fixed point, we can envision three different scenarios:  
[all of which are oscillations in some form or another]

$\alpha = 0$  (eigenvalue is purely imaginary)

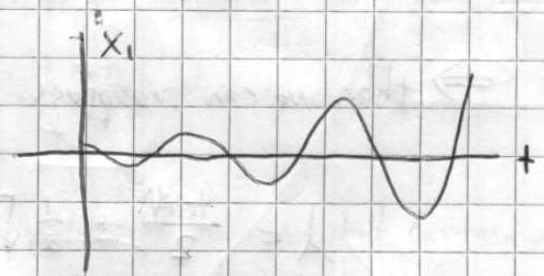
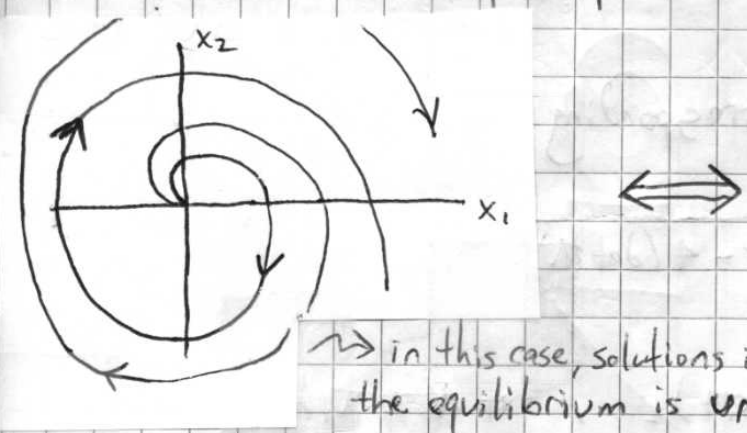


this is a phase portrait!

$\Rightarrow$  solutions are purely sinusoidal, which corresponds to a circle in the phase plane [NOTE: if you are stuck intuition-wise, think back to the harmonic oscillator -  $x_1$  is the position of the block and  $x_2$  is its velocity]

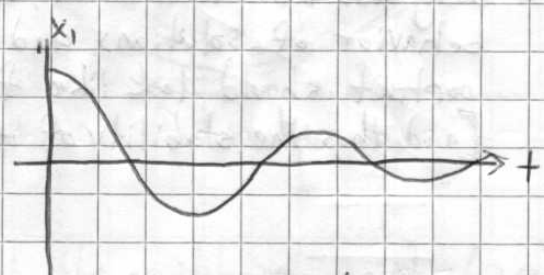
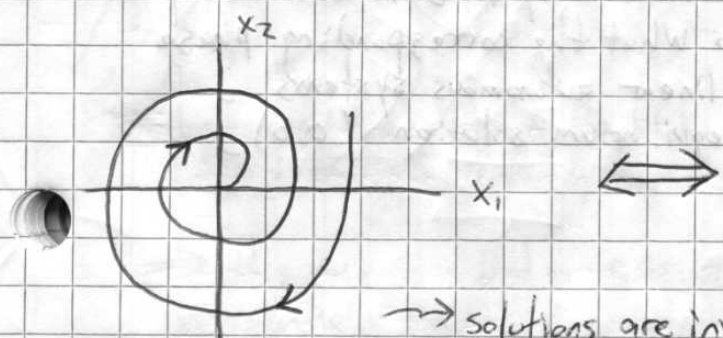
$\Rightarrow$  in this case, we classify the equilibrium point as a center

$\alpha > 0 \rightarrow$  real part is positive, so amplitude of oscillation grows for  $t > 0$



$\Rightarrow$  in this case, solutions in the phase plane are outward spirals, thus the equilibrium is unstable and we classify it as a source

$\alpha < 0 \rightarrow$  real part is negative, so oscillation's amplitudes decay w/ time



$\Rightarrow$  solutions are inward spirals, and thus the equilibrium is stable and we classify it as a sink