

3/10/10

Math 250B

→ now that we have many of the key pieces, we are going to dive further into the theory of differential eqns.

DEFINITION(s)

An n 'th order linear differential equation has the form:

$$y^{(n)}(x) + a_{n-1}(x)y^{(n-1)}(x) + \dots + a_2(x)y''(x) + a_1(x)y'(x) + a_0(x)y(x) = h(x)$$

where $y^{(n)}(x) = \frac{d^n y}{dx^n}$ and $h(x), a_{n-1}(x), \dots, a_0(x)$ are functions of x only (i.e. they do not depend upon y or any of its derivatives)

- if $h(x) = 0$, the equation is **HOMOGENEOUS**
- if $h(x) \neq 0$, the equation is **NON HOMOGENEOUS**
- if $a_{n-1}(x), \dots, a_2(x), a_1(x), a_0(x)$ are all consts. (i.e. they do not depend upon x), we say this linear differential eqn. has **CONSTANT COEFFICIENTS**
- A **SOLUTION**, $y(x)$, is a function that is at least n -times differentiable and satisfies to ODE

3/10/10

DEFINITIONS (cont.)

- An **INITIAL CONDITION** specifies values of y and its $(n-1)$ st derivatives at a point x_0 :

$$\text{IC: } y(x_0) = y_0, \quad \frac{dy}{dx}(x_0) = y'(x_0) = y_1, \quad \dots, \quad \frac{d^{n-1}y}{dx^{n-1}}(x_0) = y^{(n-1)}(x_0) = y_{n-1}$$

where $y_0, y_1, \dots, y_{n-1}$ are just numbers

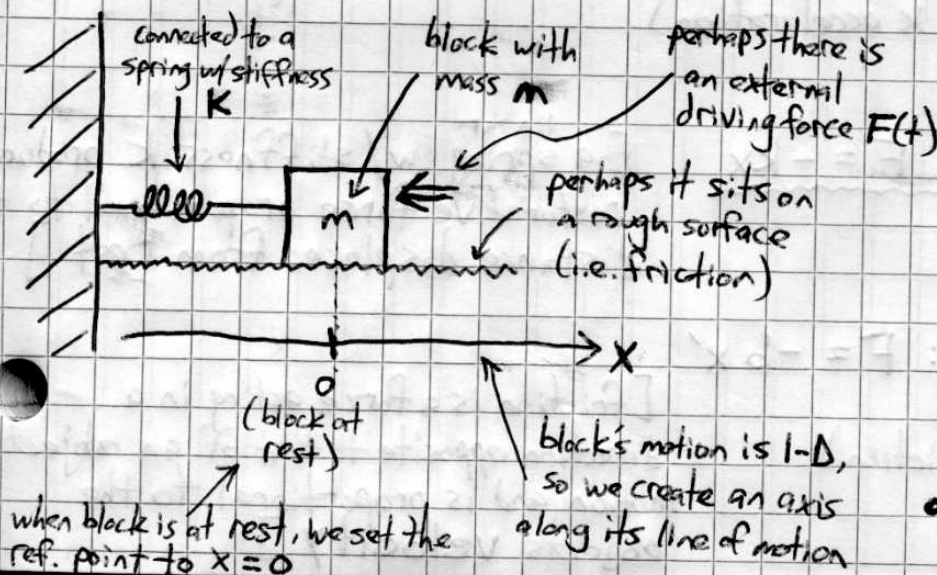
[these are also called 'initial value' problems]

- A **BOUNDARY CONDITION** specifies values of linear combinations of y and its derivatives at two (or more) different values of x

CONTEXT

⇒ let's provide a more concrete/intuitive example as a means to provide a foundation for this framework to rest upon:

⇒ Consider a 'mass-on-a-spring' (also called the **HARMONIC OSCILLATOR**)



Arises in a wide # of different physical situations:

- pendulum
- RLC circuit
- Helmholtz resonator (i.e. empty glass bottle)
- atoms/sub-atomic particles trapped in a potential well (quantum mechanics)
- sensory cells/structures of the ear