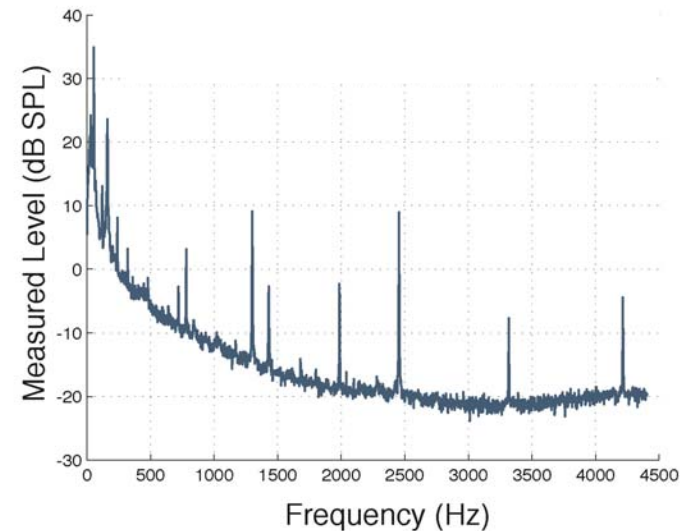
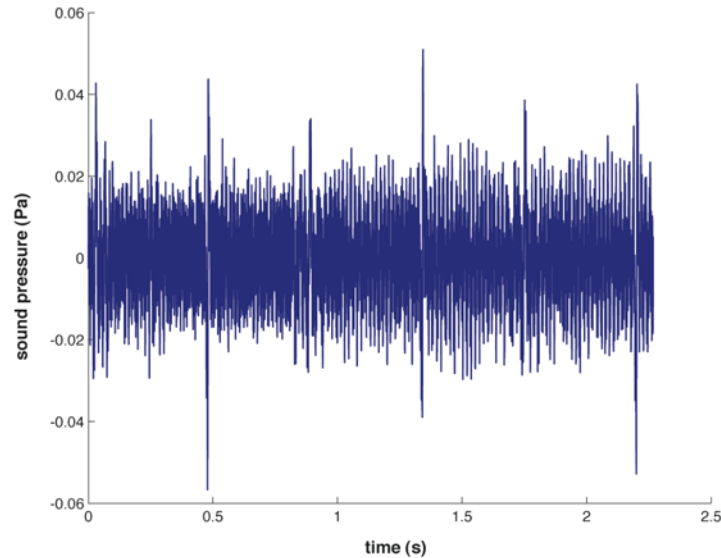


Why might the ear emit sound?

⇒ An Issue of Scale



NOTE: different vertical scales! (one is logarithmic)

$$\text{dB SPL} = 20 \log_{10} \left(\frac{\text{Pa}}{20 \times 10^{-6}} \right)$$

decibels (dB)

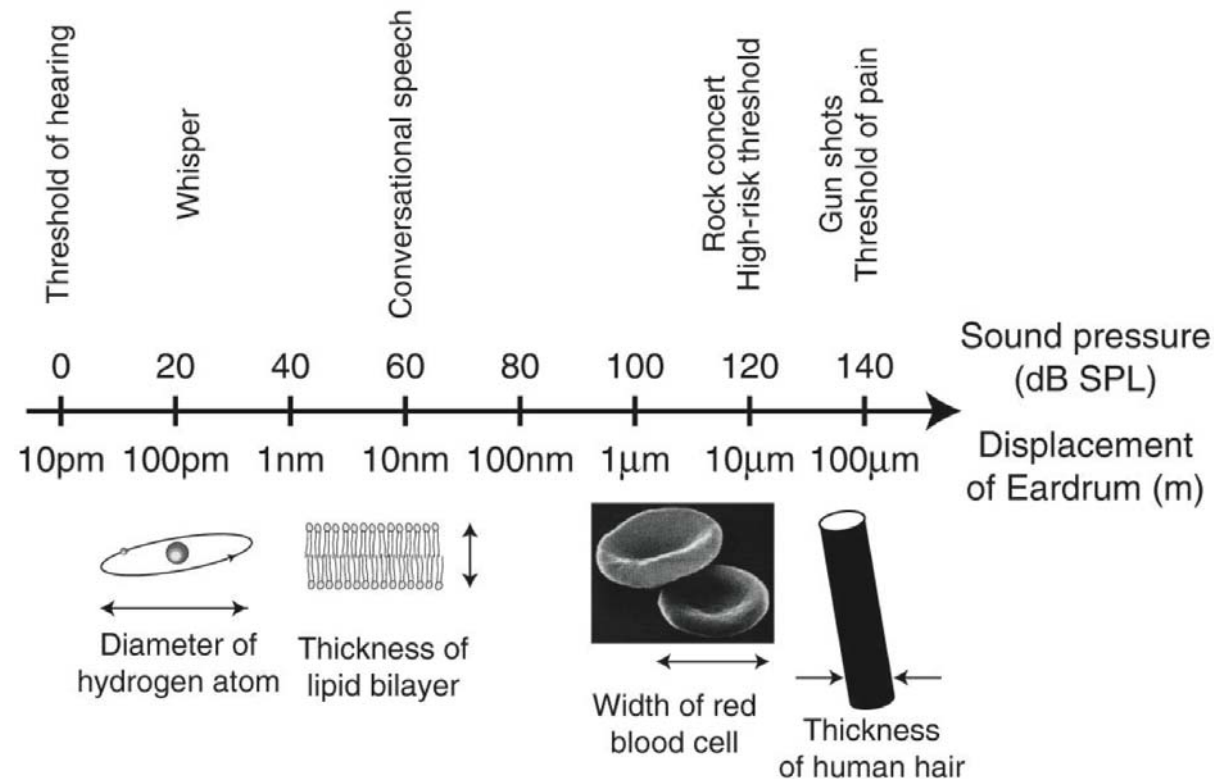
0 dB = x1
10 dB $\approx$ x3
20 dB = x10
40 dB = x100
60 dB = x1000
80 dB = x10000
100 dB = x100000

...

- a dB value is a comparison of two numbers
[dB= 20 log(x/y)]
 - A means to manage numbers efficiently
- ⇒ But why do we need to use a dB scale?

Dynamic Range

Humans hear over a pressure range of 120 dB [that's a factor of a *million*]



‘The ear is capable of processing sounds over a remarkably wide intensity range, encompassing at least a million-fold change in energy....’

- Peter Dallos

‘To appreciate this range ... we represent a similar range of potential energies by contrasting the weight of a mouse with that of five elephants.’



VS



⇒ Energy is related to the square of pressure ...

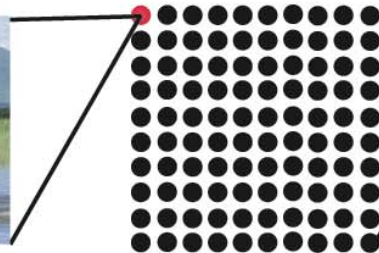
WRONG ANALOGY



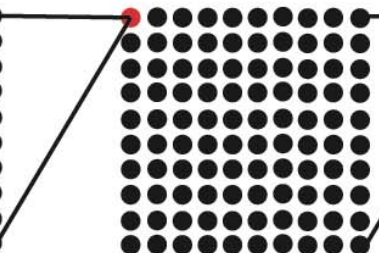
VS



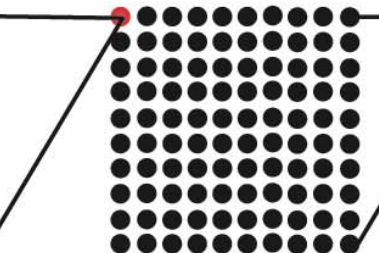
1 elephant



100 elephants



10,000 elephants

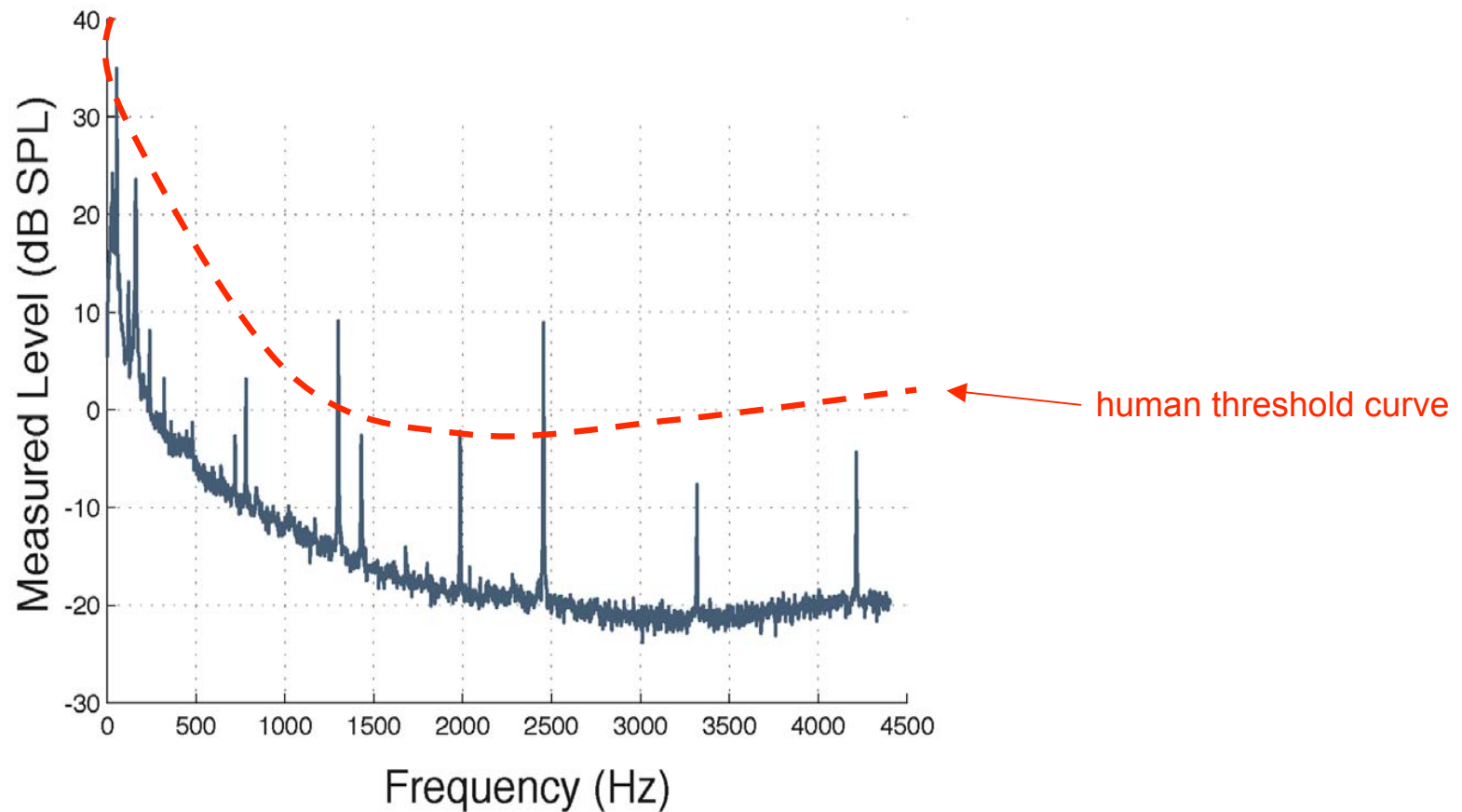


1,000,000
elephants



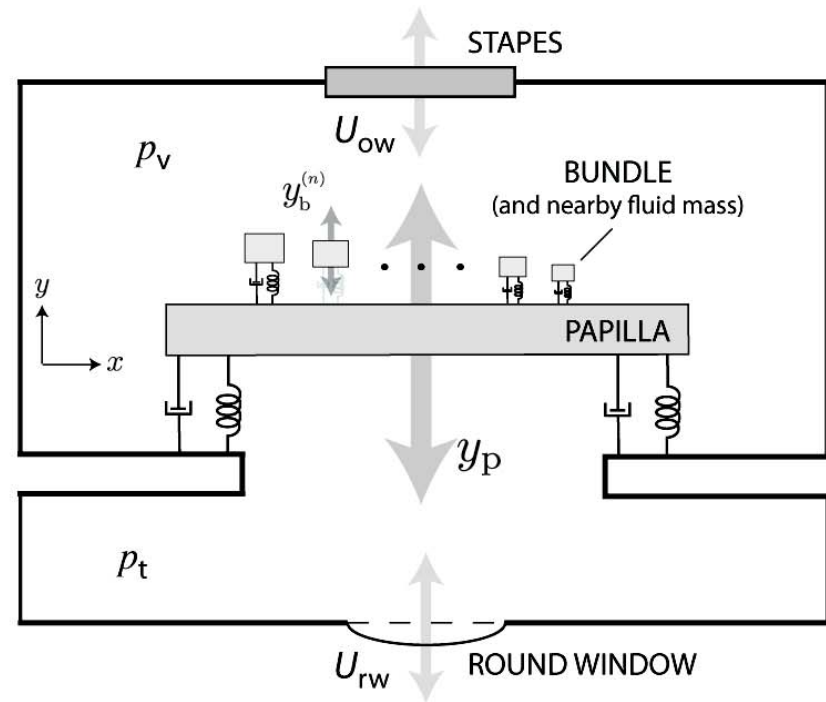
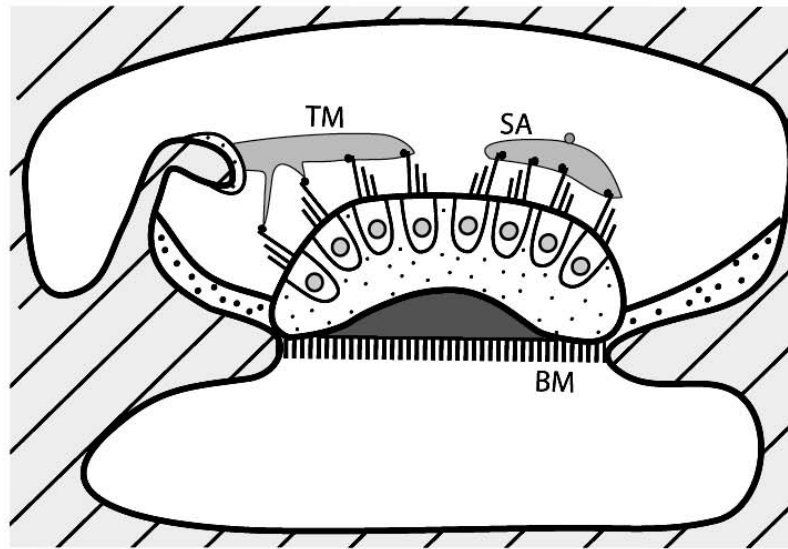
5,000,000
elephants!!!

SOAEs & Threshold



⇒ SOAEs byproduct of an *amplification* mechanism?

Mathematical Model: coupled resonators (2nd order filters)

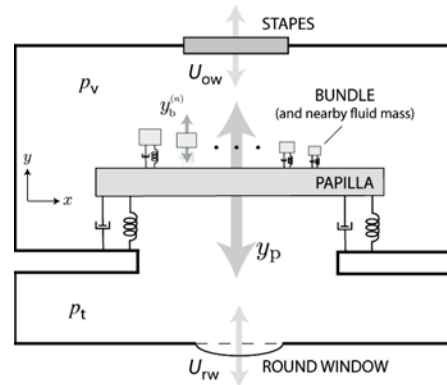


⇒ Each resonator has a unique tuning bandwidth $[Q(x)]$ and spatially-defined characteristic frequency $[\beta(x)]$

Equation of Motion

Assumptions

- inner fluids are incompressible and the pressure is uniform within each scalae
- papilla moves transversely as a rigid body (rotational modes are ignored)
- consider hair cells grouped together via a sallet, each as a resonant element (referred to as a bundle from here on out)
- bundles are coupled only by motion of papilla (fluid coupling ignored)
- papilla is driven by a sinusoidal force (at angular frequency ω)
- system is **linear** and **passive**
- small degree of irregularity is manifest in tuning along papilla length



Change of Variables

$y_b^{(n)} = y(x_n) = y_n$ bundle longitudinal location
(and similar notation for other parameters)

$$\omega_n = \sqrt{\frac{k_n}{m_n}} = \omega_{\max} e^{-x_n/\ell}$$

n 'th bundle characteristic frequency (CF)

$$\beta_n = \omega/\omega_n \text{ ratio of bundle resonant and stimulus frequencies}$$

$$Q_n = \omega_n/\gamma_n \text{ bundle bandwidth}$$

$$A_p(P_v - P_t) \text{ pressure difference across papilla is driving stimulus}$$

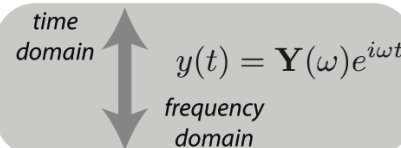
Papilla

$$m_p \ddot{y}_p = -k_p y_p - r_p \dot{y}_p + \sum_n k_b^{(n)} (y_b^{(n)} - y_p) + A_p (p_v - p_t)$$

Bundle

(n 'th bundle along papilla length)

$$\ddot{y}_b^{(n)} = -\omega_b^{(n)2} [y_b^{(n)} - y_p] - \gamma_b^{(n)} \dot{y}_b^{(n)}$$



Papilla:

$$\mathbf{Y}_p \left[-\omega^2 m_p + i\omega r_p + k_p + \sum_n k_n \right] = \sum_n k_n \mathbf{Y}_n + A_p (\mathbf{P}_v - \mathbf{P}_t)$$

Bundle:

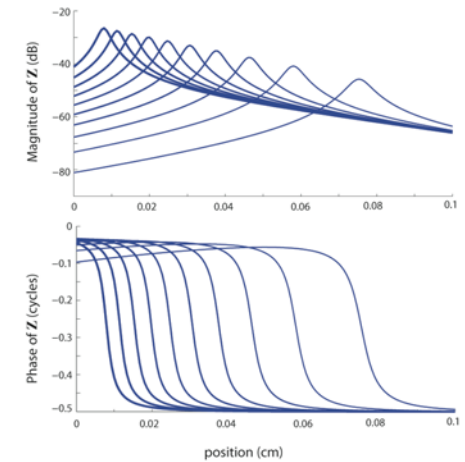
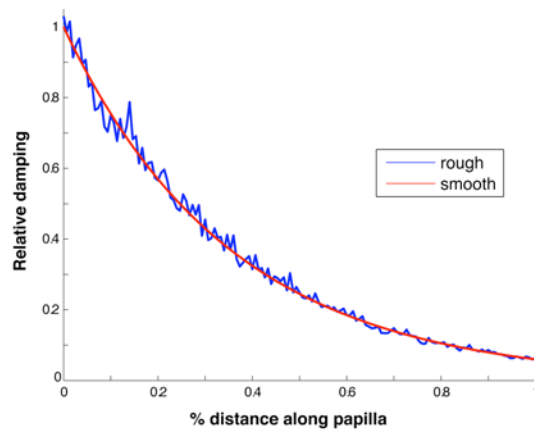
$$\mathbf{Y}_n = \mathbf{Y}_p \frac{1}{1 - \beta_n^2 + i\beta_n/Q_n}$$

An Emission Defined

$$\mathbf{Y}_p \left[-\omega^2 m_p + i\omega r_p + k_p + \sum_n k_n \frac{-\beta_n^2 + i\beta_n/Q_n}{1 - \beta_n^2 + i\beta_n/Q_n} \right] = A_p(\mathbf{P}_v - \mathbf{P}_t) \quad \text{combine both eqns. of motion in frequency domain}$$

Input Impedance

$$i\omega \mathbf{Y}_p A_p = \mathbf{U}_{ow} \quad \text{conservation of mass} \quad \rightarrow \quad \mathbf{Z} \equiv \frac{\mathbf{P}_v - \mathbf{P}_t}{\mathbf{U}_{ow}} = \frac{1}{i\omega A_p^2} \left[Z_p + \sum_n k_n \frac{-\beta_n^2 + i\beta_n/Q_n}{1 - \beta_n^2 + i\beta_n/Q_n} \right]$$



Model impedance (Z) for ten different stimulus frequencies (thicker lines for higher driving frequency). The plot shows contributions from the bundles, papilla terms are neglected. [150 bundles with CFs from 0.2-5 kHz; 10 stimulus frequencies linearly spaced from 0.6-4 kHz]. Bundle stiffness (k_n) was assumed to vary exponentially.

Irregularity

$$\tilde{Q}_n = Q_n (1 + \epsilon_n)$$

ϵ_n is a small perturbation term

$$\Delta \mathbf{Z} = \tilde{\mathbf{Z}} - \mathbf{Z}$$

impedance difference between irregular and smooth conditions

$$\Delta \mathbf{P} \equiv \Delta \mathbf{Z} \mathbf{U}_{ow}$$

papilla contributions cancel out

[SFOAE is complex difference between 'smooth' and 'rough' conditions]

Phase-Gradient Delay

$$\tau_{\text{OAE}} = -\frac{1}{2\pi} \frac{\partial \phi}{\partial f} \quad \text{where } \phi = \arg(\Delta P) \quad \longrightarrow \quad \boxed{N_{\text{OAE}} = f \tau_{\text{OAE}}}$$

Analytic Approximation

-To derive an approximate expression for the model phase-gradient delay, we make several simplifying assumptions (e.g., convert sum to integral, assuming bundle stiffness term is approximately constant, etc.)

$$\Delta P \approx \frac{U_{\text{ow}} k_{\text{o}} \ell}{\omega A^2} \int_{\beta_0}^{\beta_L} \epsilon(x) \frac{A^2}{Q} e^{2i\theta} d\beta \quad \text{rewrite expression for emission in continuous limit with suitable change of variables}$$

$$\begin{aligned} \beta_L &\equiv \omega / \omega_{\min} \\ \beta_0 &\equiv \omega / \omega_{\max} \end{aligned} \quad x = \ell \ln \left(\frac{\beta}{\beta_0} \right)$$

$$\frac{1}{1 - \beta^2 + i\beta/Q} \equiv A(\beta) e^{i\theta(\beta)} \quad \text{transfer function for the harmonic oscillator}$$

express irregularity as composed of distinct spatial frequencies along length of papilla

$$\epsilon(x) = \sum_{\kappa} \epsilon_{\kappa} e^{i\kappa x}$$

frequency dependence of emission phase (ϕ) comes primarily from this term, requiring us to determine κ_{opt}

the amplitude is a sharply peaked function (Fig.3), indicating the value of the integral is relatively constant with respect to stimulus frequency (ω)

given the strongly peaked nature of the integrand and by analogy to coherent reflection theory, we expect that only spatial frequencies close to some optimal value will contribute

$$\Delta P \approx \frac{U_{\text{ow}} k_{\text{o}} \ell \epsilon_{\kappa_{\text{opt}}}}{\omega A^2} e^{-i\kappa_{\text{opt}} \ell \ln \beta_0} \int_{\beta_0}^{\beta_L} \frac{A^2(\beta)}{Q(\beta)} e^{i[\kappa_{\text{opt}} \ell \ln \beta + 2\theta(\beta)]} d\beta$$

Analytic Approximation (cont.)

for the integral to be maximal, we require the phase be stationary about the magnitude peak (i.e., $\beta = 1$), allowing us to solve for the optimal spatial frequency

$$\frac{\partial}{\partial \beta} [\kappa_{\text{opt}} \ell \ln \beta + 2\theta(\beta)] \big|_{\beta=1} = 0 \quad \longrightarrow \quad \kappa_{\text{opt}} = -\frac{2}{\ell} \frac{\partial \theta}{\partial \beta} \bigg|_{\beta=1} = -\frac{2\omega}{\ell} \frac{\partial \theta}{\partial \omega}$$

as shown previously for the harmonic oscillator

$$\frac{\partial \theta}{\partial \omega} \bigg|_{\omega_o} = -\frac{Q}{f\pi}$$

from above, our expression for the model phase-gradient delay will be

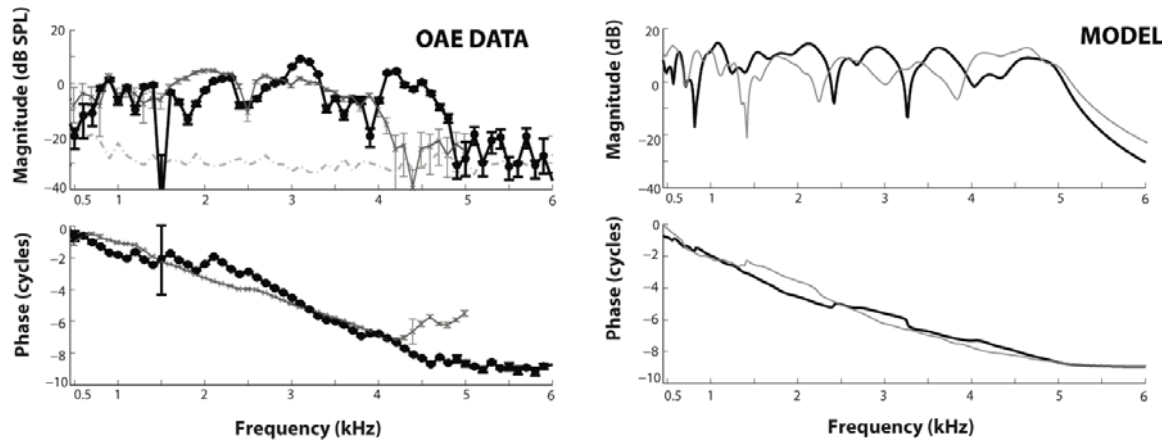
$$N_{\text{OAE}} = \frac{\kappa_{\text{opt}} \ell}{2\pi}$$

combining all our expressions we finally have....

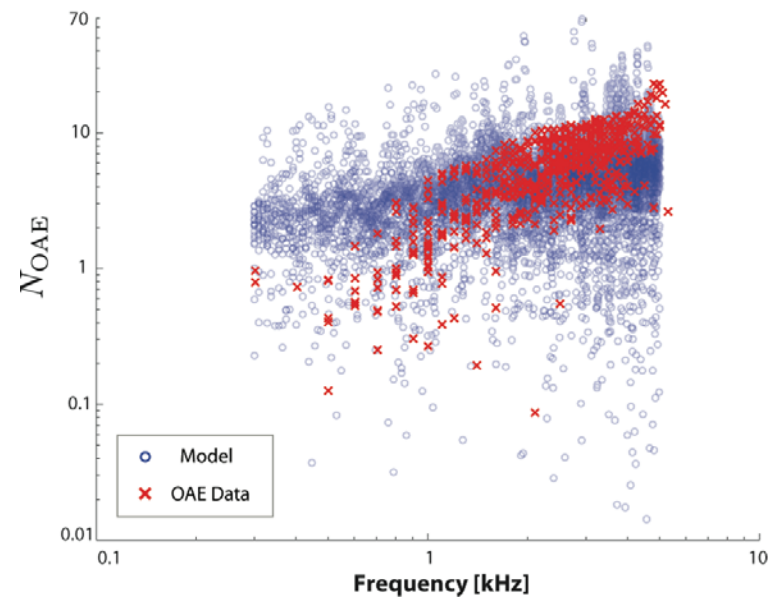
$$N_{\text{OAE}} \approx \frac{2Q}{\pi}$$

thus, the phase gradient delay is directly proportional to the sharpness of tuning

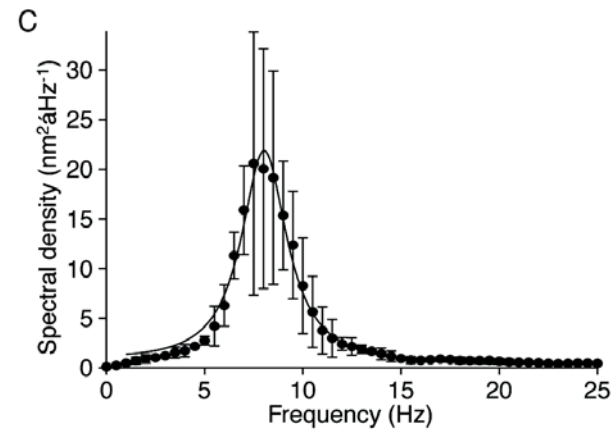
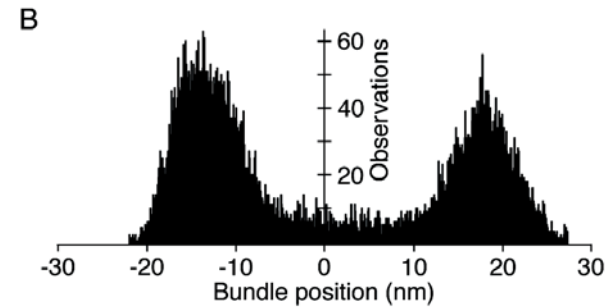
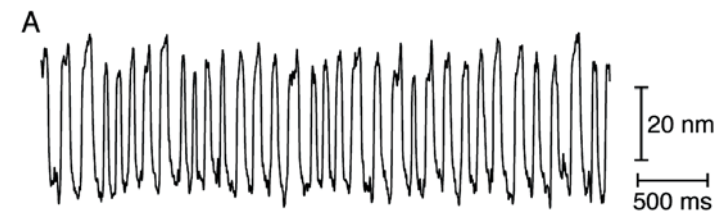
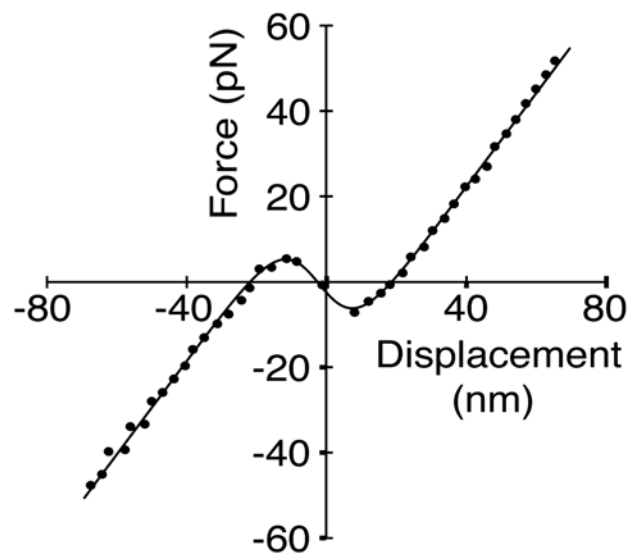
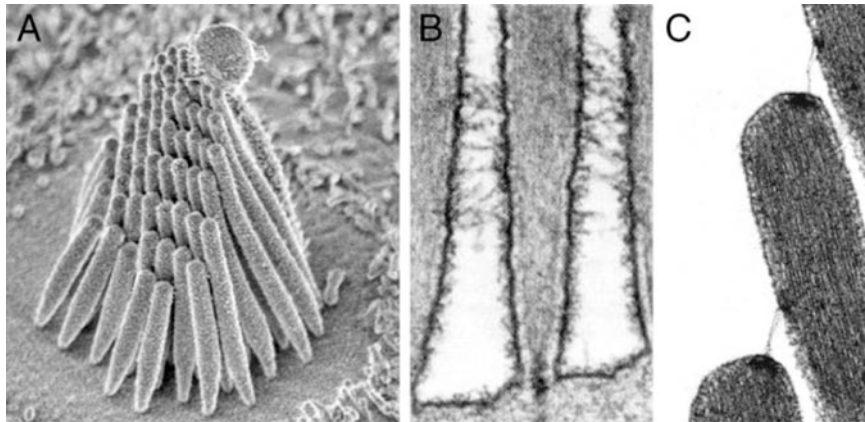
Model and Data Comparison



⇒ Model can be used to help us better understand physiological processes at work in the ear giving rise to emissions, leading to new science and clinical applications



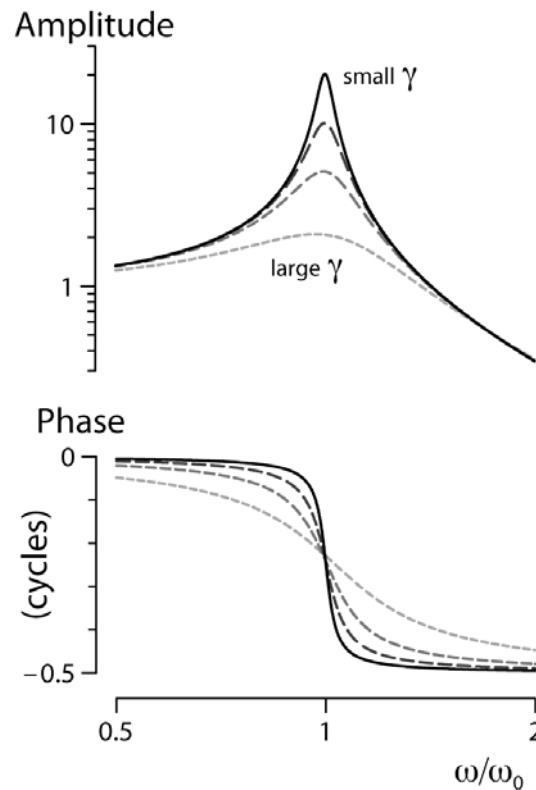
Bundle = Force Generator?



- bundle can oscillate spontaneously
- exhibits nonlinear and negative stiffness

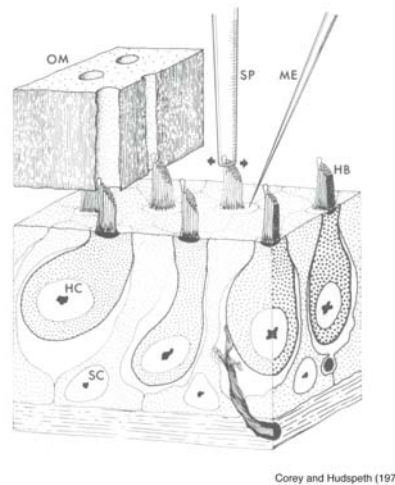
Simple Model to Explain SOAEs?: Part I

- ear is composed of resonant filters (e.g. a second-order filter such as a harmonic oscillator)
- consider just one of these filters: e.g., an individual hair cell or a particular location along the length of the basilar membrane

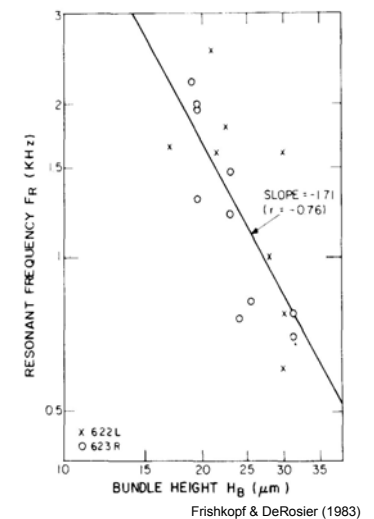


$$\ddot{x} + \gamma \dot{x} + \omega_o^2 x = A_o e^{i\omega t}$$

mass term
damping term
stiffness term
driving term



Corey and Hudspeth (1977)



Frishkopf & DeRosier (1983)

NOTE: quantities are complex so to describe both magnitude and phase

Simple Model to Explain SOAEs?: Part I (cont.)

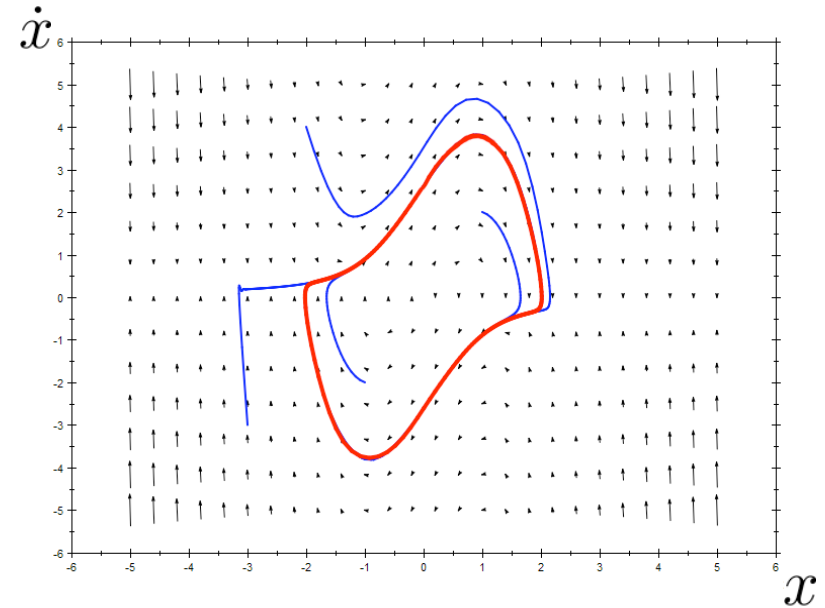
- need some sort of 'active' term for self-sustained oscillation (e.g. van der Pol)

$$\ddot{x} + \mu(x^2 - 1)\dot{x} + x = 0$$

- active term? negative damping

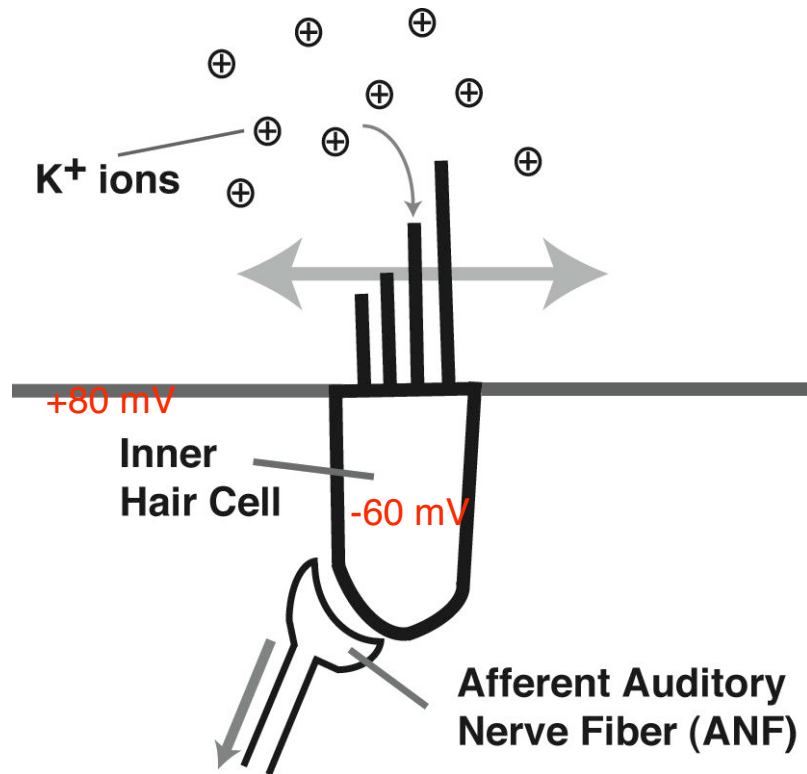
- nonlinear? yup!

- one can readily envision adding in a driving term (e.g. stochastic force due to thermal noise)

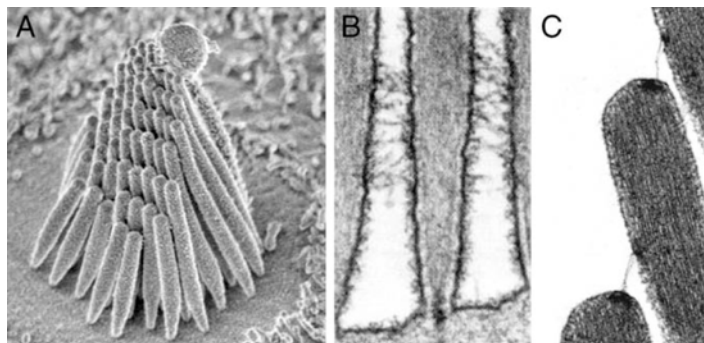


Model Idea I: SOAEs arise due to self-sustained oscillations of individual resonators (e.g. a limit cycle)

Hair cell = 'mechano-electro' transducer



1. Mechanical stimulation deflects bundle, opening transduction channels
 2. HC membrane depolarizes
 3. Vesicle release triggers synapsed neuron to fire
- ⇒ *non-linear* (saturation)



Martin (2008)

