

2/12/10

MATH 250B

□ one useful practice: getting used to express infinite series in 'sigma notation' and vice versa:

☐ → express the following power series in sigma notation:

$$p x + \frac{p(p-1)}{2!} x^2 + \frac{p(p-1)(p-2)}{3!} x^3 + \dots \Rightarrow \sum_{n=1}^{\infty} \frac{p(p-1)(p-2)\dots(p-n+1)}{n!} x^n$$

$$(x-1)^3 - \frac{(x-1)^5}{2!} + \frac{(x-1)^7}{4!} - \frac{(x-1)^9}{6!} + \dots \Rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{2n+3}}{(2n)!}$$

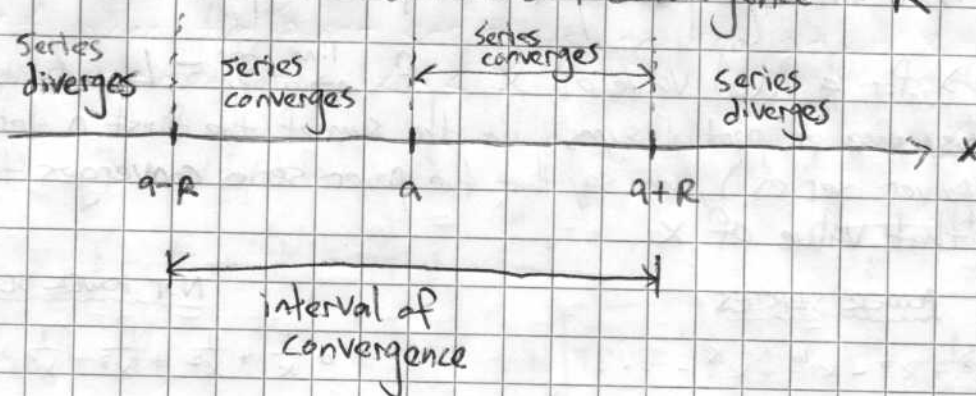
$$2(x+5)^3 + 3(x+5)^5 + \frac{4(x+5)^7}{2!} + \frac{5(x+5)^9}{3!} + \dots \Rightarrow \sum_{k=0}^{\infty} \frac{(k+2)(x+5)^{2k+3}}{k!}$$

□ Power Series Convergence

IDEA: depending upon what x is (relative to a), the power series may or may not converge

DEFINITION: if a power series converges for a given value of x , that value of x is said to be within the radius of convergence for that given power series

→ we call the radius of convergence $\equiv R$



□ there are a couple different cases:

1) Series only converges for $x=a \Rightarrow R=0$

2) Series converges for all $x \Rightarrow R=\infty$

3) Series converges for $x \approx a \pm R \Rightarrow R$ is the radius of converg. (a finite, positive #)

□ the power series ALWAYS converges when $x=a$

$$\leadsto \sum_{n=0}^{\infty} C_n (x-a)^n = C_0$$

□ when $x = a \pm R$, the series may or may not converge (and the behavior could be different on either end); in this case you might need to plug in $x = a \pm R$ and explicitly determine if the series converges or not

ex Is $x=4$ within the interval of convergence for $\sum_{n=0}^{\infty} n^2 (x-3)^n$?

• plugging in our value of x , we have

$$\sum_{n=0}^{\infty} n^2 (4-3)^n = \sum_{n=0}^{\infty} n^2 = 0 + 1 + 4 + 9 + 16 + \dots \Rightarrow \text{diverges}$$

$\leadsto$ therefore $x=4$ is outside the radius of convergence for this power series

ex Is $x = \frac{5}{6}$ within the radius of convergence for $\sum_{n=0}^{\infty} 2^n (x-1)^n$?

$$\sum_{n=0}^{\infty} 2^n \left(\frac{5}{6} - 1\right)^n = \sum_{n=0}^{\infty} 2^n \left(-\frac{1}{6}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(-\frac{1}{3}\right)^n = \frac{1}{1 + \frac{1}{3}} = \frac{3}{4}$$

$\rightarrow$ infinite geometric series; so $x = \frac{5}{6}$ is within the radius of convergence