

1/27/10

Math 250B

Infinite Series (see H.H. 9.2 ff)

- a sequence and a series are related, but different in that the series represents a sum of terms

Sequence: $S_n = f(n)$ or $S_n = f(S_{n-1})$ (as defined recursively)

e.g. $S_n = \frac{1}{3n} + 4$, then ~~sequence~~ $S_{100} = \frac{1}{3(100)} + 4 = \frac{1201}{300}$
(a single value)

series: $S_N = \sum_{n=0}^{N-1} a_n$ where $a_n = f(n)$ (also called a 'partial sum')

e.g. $a_n = \frac{1}{n}$, then $S_4 = \sum_{n=1}^4 \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \approx 2.08$
(also a single value)

⇒ ultimately, both a sequence and a series represent a single value, but the key difference is that a series is comprised of the sum of a # of term

- in the limit $N \rightarrow \infty$, the series is said to be infinite (i.e. you are adding together an infinite # of terms, which may or may not add up to a finite #; this question deals w/ the issue of series convergence, as covered in H.H. 9.3-9.4)

□ why study series?

□ connection to calculus $\left\{ \begin{array}{l} \text{sequence} \leftrightarrow \text{function} \\ \text{series} \leftrightarrow \text{integral} \end{array} \right.$

- infinite series pop up in a large # of applied contexts in areas such as physics and chemistry
- an easy way to explicitly express irrational #'s:

ex Gregory-Leibniz series $\pi = 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \dots$

- provides a means for approximating functions via an infinite series of relatively simple 'basis functions'

- Taylor series $\rightarrow$ polynomials
- Fourier series $\rightarrow$ sinusoids

$\rightarrow$ such representations (or put another way, decompositions) are the backbone of many mathematical approaches to problems (whether it be in math, physics, biology, ...) as well as the underlying foundation to signal processing

Sequence Review

ex find the expression for the sequence S_n for $n \geq 1$ given that the first five terms are: 0, 2, 6, 14, 30

recursive: $S_{n+1} = 2S_n + 2$ (where $S_0 = -1$)
 functional: $S_n = 2^n - 2$ (starting @ $n=1$)

- a convergent sequence is bounded, i.e. one can find two #'s M and N such that $M < S_n < N$ for all n
- if a sequence is bounded and monotonic, then it converges

Geometric Series

• a special type of series is a geometric series, which has the form:

$$S_N = a + ax + ax^2 + ax^3 + \dots + ax^{N-1} = \sum_{n=0}^{N-1} ax^n$$

where a (scale factor) and x (common ratio) are const's.

$\rightarrow$ the unique property of the series that makes it 'geometric' is that the ratio of successive terms is just a const.

ex. $\frac{ax^k}{ax^{k-1}} = x \rightarrow$ hence x is called the 'common ratio'