

[student's have been de-identified]

Force/Acceleration in demonstrations

Sebastien

One of the first things that became apparent to me as I was observing the demonstrations was that each involved some degree of motion and displacement. Furthermore, in each experiment, the motion of fluid in question seemed to be the result of some unique forces. These forces were not always immediately apparent; the honey seemed to coil spontaneously, the motor oil almost magically climbed the motor shaft, and the tiny beads vibrated with no obvious intervention. However, we know from Newton's Laws that in order for these liquids to go from rest to motion, a nonzero net force must be applied.

It follows that with a nonzero net force, an acceleration occurs. Acceleration is the second derivative of displacement with respect to time. If we attempt to write a differential equation describing the motion of these liquids, we'll end up with a bunch of second-order (or perhaps higher) differential equations. Whether these are linear or nonlinear, autonomous or non-autonomous, or with constant coefficients is beyond me. I will research these phenomena further.

Juliette

If acceleration is the second derivative of position, then modeling the position over time is not even a differential equation! Sebastien however wonders how to model the movement of such materials and fluids. I would agree that to model such seemingly random phenomenon would be very complex and mathematically intense, but I think that it is fair enough to assume that the main factors affecting such motion are going to do with the materials used, their shape, and any frictional forces involved (including viscosities, as well as air and static. kinetic friction between materials). With these considerations we might be able to begin to put together a differential equation that models this motion.

Pascal

I think what Sebastien is saying is that the position could be modeled using differential equations. Position involves acceleration (2nd derivative), velocity (1st derivative), and initial position (Initial Conditions). So, modeling the accelerations and velocities of the particles of a given substance could be done through use of differential equations, and from these equations you could model the position of the particles.

Also, regarding the complexity of these equations, it seems that they could be any degree of complexity, depending on how accurately you model the behavior of the substance. It seems that to model real world situations exactly, you would need infinitely complex models, thus, all our mathematical models are simply approximations with varying accuracy.

Laser Tweezers

Sabine

I find the concept of lasers quite intriguing; in my genchem class, we recently learned about laser light and how it differs from ordinary light. Laser light is the result of stimulated emission (which differs from spontaneously emitted normal light in that this light is in a single phase and travels in a single direction). For lasers, population inversion must be maintained in order for this beam to be intensified and to have the properties of laser light. I learned how lasers work from a qualitative aspect, but not really from a quantitative one. I suppose laser light can be modeled using a system of equations..?

Pascal

I agree about the intriguing aspect of lasers. I don't know much about them, but I think I remember Bob saying that they can move and manipulate those little orbs he showed us with lasers. I'm not really sure how this works at all. I don't know how light could move anything. My only guess is that the energy from the laser interacts with the little orb as well as the substance in which it is suspended. How they can move it in any controlled fashion is beyond me. Perhaps this energy emitted by the laser light as well as it's interaction with the orbs can be modeled by differential equations which explain their movement.

Josephine

Well I know that light can move larger objects slightly via the photoelectric effect. That is, as photons come in, they interact with metals that have loosely-bound electrons and essentially "pop" them off. In fact, scientists have discussed using this principle to swerve potential comets out of the path of earth. Also, lasers create a relatively strong electric field at their center, so dielectric molecules feel attracted to the beam and essentially get trapped in the light, thus allowing the scientist to control the placement of the particle. Since light comes in waves, it seems natural that the force they exert on a molecule would oscillate. I see a connection to sinusoids again, like our last discussion, because light is such a model oscillator. Perhaps there's a connection between light and the oscillations of a spring or a pendulum? This is just speculation.

Henri

The photoelectric effect is a good demonstration of light acting as a particle. We also know it acts as a wave. I don't know which of these 2 aspects the laser tweezers use. Your description of attraction to the EM field of the laser beam below seems likely. With the particle aspect, I can imagine modeling the laser as a damping force on the bead. You might hit it from several directions with a force(intensity) and frequency tuned to damp the effect of the random brownian and hold it still. Each direction of motion acting as a separate mass on a spring. You could probably also tune it to be harmonic in one direction, and move the particle around that way.

Therese

The idea of the laser tweezers working by generating an EM field is interesting but I was thinking that it could also work by generating an electric field. Particles are somewhat charged so I was thinking that the laser tweezers could work by attracting small particles to the center of an EM or electric field generated by the laser.

Liquid Traveling Up a Rod

Tristan

One of the experiments which really confused me was how the liquid in the petri dish traveled up the metal rod more and more as the rod spun faster and faster. As I thought about it more, I realized that there had to be some force causing the liquid to overcome gravity and travel upwards. Well, the only force I could think of was friction being converted into kinetic energy in the liquid, which makes sense because as the rod spins faster and faster, more friction will be created, causing more kinetic energy to be transferred to the liquid, causing it to travel somewhere. In this case, the only surface that the liquid could travel on was the rod, which was upward. But how might you explain the waves that resulted when the rod spun too quickly? Well my thoughts are that the gravitational force would eventually overcome the upward kinetic energy of the liquid. Since gravity is a second order function of time (acceleration) and kinetic energy is only a first order function of time (velocity), gravity can eventually increase past the energy of the first order kinetic energy and cause the liquid to reach a point of stability on the rod. For example, the acceleration is decreasing (but still positive) as the rod spins faster and faster. Eventually, that acceleration will reach zero, and the liquid will cease to move up the rod any further. But since there is still a force or energy acting upon the liquid in an upward direction, the liquid will attempt to keep pushing up the rod, but will just fall back down, which explains the sinusoidal motion of the liquid at equilibrium. The liquid will continue to repeat the process of going upwards then falling back down upon itself. Imagine a person trying to pull a semi-truck and won't give up. They won't be able to move it, but they will keep pulling the semi-truck with quick bursts of energy. They won't just sit there and apply a constant force on the truck. Just like the liquid, which sends bursts of energy up the rod to attempt to move higher up the rod, but will fail and fall back, then try again and again.

Brigitte

You have some very interesting points and I like your analogies, but I have a question for you. Initially you say that the liquid has to overcome gravity to even travel upward in the first place. With that in mind, why would it be gravitational force that "would eventually overcome the upward kinetic energy of the liquid?" How would the gravity be overpowered in the first place and then come back and be too strong for the liquid later when there was an even greater acceleration of the rod? That just doesn't click in my head. It makes more sense to me to think the force exerted by the spinning rod at slower speeds is not enough to overcome static friction, so the liquid is able to climb up the rod without slipping down. But, at a certain speed, the rod exerts a force that overcomes static friction, which leaves the honey unable to hang on and climb further. So instead of climbing up, the honey falls down in waves, as more tries to climb up and more continues to fall down (the quick bursts you spoke of).

I'm not sure if any of what I am saying actually occurs, but it makes more sense to me than to say that gravity is overcome and then becomes too great later when the force of gravity exerted on the liquid by the earth should be constant if the mass of the honey doesn't change.

Emile

I agree with you Brigitte. Although Tristan's ideas are quite innovative, it seems a constant force cannot change from being the force that is overcome to the force that is so powerful that the entire structure caves in. However, one part still remains mysterious. If there was an optimum speed after which static friction was overcome that the oil, or other liquid, climbed up the rod, how can it be falling down in waves? Continuing at the same speed, it seems as if the force of static friction is overcome, the system would continue acting in the same manner, regardless of gravity pulling it down. Should we consider which force is larger in this case?

Yves

Both of you doubt how gravity can go from the force overcome to the overcoming force, but it's actually rather simple. First of all, the force of gravity is not constant. Force is mass time acceleration, in this case acceleration due to gravity. Acceleration due to gravity is constant, but unless the mass is constant, the force of gravity is not. As the rod spins faster and more liquid moves up the rod, there is more mass displaced along the rod. More mass means a greater force of gravity, and suddenly the upward forces are weaker than the downward force of gravity. As soon as the liquid pillar collapses, there is less mass again, so the upward forces are again stronger. Once this happens, the mass is large enough again so that the gravitational force is stronger yet again, and the liquid falls again. This process repeats as long as the rod continues to spin at this velocity, and is at least part of why we see these oscillations. Now I am not naive enough to claim this the full explanation, but it is part of it, and Andrew's point is a lot more accurate than he's been given credit for.

Sophie

First off, I'd just like to point out that through the entire thread we keep thinking of "the upward energy of the liquid" but we fail to realize that energy is a scalar quantity and that it does not have direction. In addition to this I believe that since we are dealing with a liquid a fluid dynamics phenomenon would offer a better explanation to what is happening here. I was intrigued by how the rising column initially looked like an upside vortex. This led me to look into how vortex's were formed and I eventually landed upon the Bernoulli Principle which states that the pressure exerted by a liquid will decrease in relation to an increasing velocity. With that in mind the fluid closest to

the shaft will have the lowest pressure and the fluid at the outer edges will have a higher pressure. This would cause a pressure difference in the liquid and as a result force the liquid up the rotating shaft. However as the shaft continues to spin faster and more fluid rises up there is less fluid present in the dish to maintain the pressure difference and the column collapses. This theory is analogous to an aircraft and how it gains lift from the pressure difference between the upper and lower surfaces of the wing. The shaft is the wing, the rotation of the shaft in the fluid is the air moving over the wing, and since the shaft can't move, the generated lift is represented by the rising fluid column. At higher speeds (higher rpm's for the shaft) there is a greater lift force and the column rises higher, but once it reaches a certain height the column "stalls" like an aircraft would at high altitudes and gravity takes over and the column falls down.

Gilles

i just want to point out that the liquid climbs up rod because of the motion is circular, which requires a centripetal force to maintain the entire circular motion; therefore, there will be a force pushing the liquid to the center, as long as it overcomes acceleration due to gravity. once the upward force cannot support the weight of the liquid that climbing up, it will start the oscillation like a wave. however, i consider the upward force would be the friction produced by centripetal force.

Sophie

So after conducting some more research on the matter, I came across an interesting phenomenon called the Weissenberg effect that seems to accurately describe what is happening to our solution. As it turns out friction does have a critical role in what's happening but is more involved at the molecular level within the solution. As we rotate our rod within the solution the molecules get tangled up with each other and a resultant stress/force is induced within the solution. To relieve this stress the molecules creep upward along the rod to where there are fewer molecules and as a result less stress. However depending on the viscosity of the solution the molecules will allow the solution to creep up the rod to different heights before the force of gravity is greater than the upward force and the column stops rising. An interesting youtube video illustrating this effect is <http://www.youtube.com/watch?v=npZzlgKjs0I>.

Sabine

I really liked this demonstration as well, although I would have liked to see what happens as the rod spins faster and faster. I'm surprised that there aren't more search results when I google this. But I did find a journal about the Weissenberg effect and how it's modeled by the Doi-Edwards theory. http://jcp.aip.org/jcp/a6/v88/i6/p4001_s1. This article says there's such thing as negative Weissenberg effect...I would definitely like to know under what conditions this actually occurs.

BZ Reaction

Alain

It was made clear by Dr. Bergevin that we missed a very interesting demonstration because the BZ Reaction could not be performed during our lab visit. Since I have an interest in chemistry, I decided to do some research online to see if I could find out more about this reaction. I was able to find many youtube videos, pictures of different reactions, and many articles. The Belousov-Zhabotinsky Reaction is a type of non-equilibrium system that oscillates far from the equilibrium point for a long period of time (plus there's amazing color patterns). As the reaction proceeds, concentric wave fronts form and create a series of target patterns. The collision of two of these waves leads to "mutual annihilation," in which the wave fronts from the touching portions disappear (kind of similar to when two bubbles touch and the inner wall disappears leaving one bubble). Even more interesting than the concentric circles is what happens when the wave front is broken. When this happens the system will proceed by spiraling in on itself, creating an entirely new set of patterns. The simple oscillation of concentric circles seems as if it could be modeled through the use of higher order differential equations (as we have begun to see in class). The ending results probably would end up containing complex exponentials or sinusoids. Since there is a large variation of possible patterns, to model this reaction mathematically seems to be very tedious. It would be necessary to model the concentric circles, the "mutual annihilation" of waves after collision, the spiraling effect when the wave front is broken, and even the eventual stabilization of chemicals that would cause the patterns to die out. (Source: http://www.scholarpedia.org/article/Belousov-Zhabotinsky_reaction)

Sebastien

I agree, it's a real shame we didn't get to see this reaction. It seemed like the one demonstration that Dr. Bergevin was the most excited about during the lectures. On the subject of modeling this beast somehow, I can see how this could get out of hand pretty quickly. I suppose it depends on what we actually want to model. Before we can start throwing around mathematical terms, we must have a clear notion of what we hope to accomplish. We'd probably be interested in finding an equation that describes the concentration of one of the reactants at a given point as a function of time.

Working under this assumption, I think it's fair to say that we'll be dealing with second-order or higher differential equations. My intuition would be that these are primarily homogeneous equations, because there is no forcing function. In this context, a forcing function might describe the addition of a reactant at some point in time; the BZ reaction is free-running, and does not require external intervention to sustain it.

Alain

I wasn't really thinking along the lines of all the different things that could be modeled in this reaction, but it obviously makes sense that there are many things that could be modeled mathematically. I was originally thinking more about modeling the visual oscillations of the patterns, but it makes sense to model the concentrations of the reactants as the reaction proceeds as well. To model the concentrations it would seem as if at least a second order differential equation would be necessary, if not a much higher order differential equation. I'm questioning the idea that it is a homogeneous differential equation though. I'm just wondering if the constant switching between reactant concentrations could be seen as a forcing function? What I was thinking was that the switch from a low concentration to a high concentration of one of the reactants could force the reaction to proceed and switch to a high concentration of one of the other reactants.

It's just a thought...

Henri

I think this one is fascinating too. My chem 152 teacher actually did a variation in class where it flashed from inky blue to clear and back to blue over and over. It ran for several minutes. Really cool. She said the initial concentrations were very important in that reaction, so they may be with the BZ reaction as well. The mixture then needed some kick to get it started. Those two things together might lead to differential equations with some initial conditions (concentrations) and a forcing component (activation energy) that might need to have a minimum value to get the reaction started and sustain it.

Tristan

Speaking of the chemistry demonstration with the flashing inky blue and clear solution, I imagined that the reaction could best be described using the chaos theory of mathematics, which is the study of the "behavior of dynamical systems that are highly sensitive to initial conditions" (Wikipedia). Just as Henri said, the initial conditions or concentrations of the reactants was very important, similar to the chaos theory. The Belousov-Zhabotinsky (BZ) reaction also changes colors in what seems to be a random or chaotic pattern, leading me to believe that the chaos theory can be applied to its behavior. The BZ reaction is almost impossible to predict logically or numerically, hence why the "widely diverging outcomes for chaotic systems, rendering long-term predictions impossible" are a perfect match for how the BZ reactions works.

Tristan

In addition, the BZ reaction has a circular pattern to it, leading me to think that maybe a polar coordinate system would produce a simpler result than trying to use a Cartesian coordinate system. That way, the function used to describe a BZ reaction would be more understandable and not so complex. However, since the BZ reaction may be a chaotic system, it still might be very complex or even impossible to formulate a function for the characteristics and patterns of a BZ reaction.

Monique

I actually question exactly how random the reaction is. As I was watching a film of the reaction, it seemed to follow a fairly consistent pattern, for the most part. Then, I saw a graph of the [voltage plot](#) for the reaction where the voltage given by the reaction was plotted against time and it looked like a fairly consistent "wave" of some sorts. This voltage, I believe shows the energy given by the reactions occurring and hence somehow indicates the progress of the reaction. Granted, it needed an initial boost and was only consistent for a few minutes, the reaction did not seem completely random or chaotic. The plot does seem to decrease in voltage as time goes on, which I would suspect to add some extra dimension to the equation(s) that would describe this reaction.

Coiling (I)

Yves

In the honey experiment, the first thing I notice is the honey's tendency to coil like a spring. It oscillates in a way that the honey falls circularly. I would assume that this behavior could be analyzed using linearly independent combinations of sinusoids, perhaps a Fourier series, similar to what we've recently been doing in class with the mass-on-a-spring problem. This, along with the behavior of the honey suggests and displays some sort of natural frequency and harmonic motion. It would be interesting to further analyze and understand this.

Marie

I agree with the mass on a spring theory, however, and I might be wrong on this, but it just seemed to me like it was going to some sort of limit while it was coiling. If we set up an equation and somehow set it going to some sort of limit (ie the place where the coil begins to fall) then we could correctly figure out the sort of capacity of how many coils were actually possible. Just a guess though.

Brigitte

I agree with both of you, but I was thinking the same thing as you Marie. I found it very interesting that there seemed to be a point where the coil of honey collapsed down to the rest of the blob of honey. However it seemed like this point wasn't always caused by the same issue. If I remember the video correctly, it seemed like the honey sometimes coiled straight up vertically and toppled over when it got to a height that was too significant to be stabilized, but other times the coil was not as straight and firm and the coil fell simply because there was a lack of tight organization to keep building up. In other words, it seemed that the honey's behavior consistently coiled, but it didn't make consistent coils (if that makes sense). So then the point becomes, how would you describe something that tends to be random or spontaneous? Yes, the general harmonic analysis (similar to that of the mass-on-a-spring analysis) could be run and might be accurate to a certain extent, but is there anything else that could be introduced to account for a varying result in the coiling of the honey?

Julien

What I found interesting about this experiment was that the honey didn't always fall the same way. At some points in time, it fell in a coil like you all have previously stated but at other times it simply fell in layers but oscillated back and forth from one end to the other. I'm wondering how to go about mathematically describing these two different scenarios. I think the latter could be described by a Fourier series somehow which would describe the oscillations and I think the former could possibly be described by a Fourier series within another Fourier series. That may or may not work but it could perhaps show how the honey coils in circles and doesn't just oscillate.

Elise

To answer your question, Brigitte, I think you must take the physics of the whole situation into account. Most significant is probably the force of gravity. Because the flow of honey is constant and seems to be the same rate throughout, there is no randomness in its coiling. What happens is the piled up honey blends together to make a massive blob. Keeping in mind that honey is a liquid, though a very viscous one at that, it has a weak foundation. Thus, the force of gravity of the still coiled honey must at some point overcome the normal

force (force exerted upward) of the honey blob, and the blob caves in or is reshaped so the mass of the coiled honey cannot be supported. I'm sure a Fourier series would adequately model the flow of honey, but to accurately model the system as a whole, one must introduce other terms that affect it (like the damping term in the mass-on-a-spring problem).

Brigitte

Elise, that sounds like a good explanation, but that is only in the case where the honey coils up tightly. My question also concerned how to describe the case where, as Julien described, the honey simply oscillates back and forth and doesn't have the same organized look as it does when it coils nicely. I agree that the Fourier series could help describe the oscillating behavior of both the coiling and layering case, but how could we include something that could adjust to both cases occurring and perhaps even predict a pattern in which those cases occur? That's more of what I was getting at.

Emile

I was having a hard time seeing through the magnifying glass, but from what I saw, it seemed as if the honey fell in coils not layers. Could it be possible that the coils were "smushed" so to say that it looked like layers, essentially making the mathematical models for both the same? It's quite an interesting observation otherwise, as you mention Julien. Were there environmental changes (i.e. the air conditioning just turned on, etc.) to account for layers sometime and coils other times?

Marie

Yeah, I also found it interesting that it wasn't consistent. I mean to the naked eye, it obviously looks like it is not going to be a consistent coil, but it also doesn't even look like it is going coil to the naked eye. Under a microscope, however, it was interesting that it wasn't consistent, like I said. I'm curious to what determines how high the coil can go at any given time. It seemed like the coil got higher in the later stages of the experiment though, maybe that is because the base on the bottom of the petri dish had more mass to hold it up in the later stages? I'm not sure.

Emile

I don't know if this is sort of the answer you're looking for Brigitte, but in the past, we've shown random/spontaneous behavior with free variables. Perhaps something similar is at work here. Finally I understand the coiling vs. layering topics as is being discussed here, but seeing as how I still can't seem to comprehend the reason for these two different formations, I am unfortunately unable to provide a suggestion in where such a free variable, dependent on various parameters, may work.

Therese

I remember Bob (sorry I forgot his last name) mentioned something about one test where the the honey was coiling one way and then started to coil the other way. This kind of behavior along with the seemingly random behavior of the coiling led me to think that maybe multiple combinations of sinusoids would be required to describe such behavior. Perhaps some combinations are more prevalent at certain times.

Benoit

I agree that since the honey gave a coiling effect as it fell as well as repeated itself in an oscillating type pattern that it could be best described using a combination of linearly independent sinusoidal functions, or a Fourier series. The fact that the coiling seemed almost random, in that once a coil fell, the next coil did not necessarily repeat the exact same way, fell over after a different amount of time etc. has to do with the physics of the experiment. Different forces, gravity, friction etc. are acting upon the coil causing it to change in various ways. I think that each separate coil could be represented with a unique combination of linearly independent sinusoidal functions and then these series could be combined together to represent the entire experiment. The experiment with the oil (i think thats what it was but I'm not sure) with the rotating pole in the middle of the dish could also be represented in the same way. It also oscillates and repeats itself similarly to the coiling. The changes in motion caused by higher speeds of rotation could be represented with coefficients before the series, but the basic Fourier series for each of the frequencies would probably be the same.

Emile

That's a good insight; I wonder if it would be possible to simplify the process by taking averages; otherwise it could be quite tedious without a computer program. Also, the solidification experiment he showed us, with the liquid going to the solid seems like it could be approximated with Fourier series as well. Thought it isn't oscillatory as in the primary contexts we have seen them used, the repeating pattern (i.e. the leaf formation) suggests a Fourier series might be a possible mathematical model for the simulation.

Christophe

I agree that sines or cosines could be used to accurately represent the x and y position of the coil. What I think would be really interesting is to set up two different honey streams in the same environment and to see if the coils coincide at all between the two coils. If they did that means the coils are dependant upon the environmental factors, if not then that means they are completely random.

Juliette

I never was able to attend the lab, but I was able to observe this same phenomenon on Youtube and I had a couple of variables to add to the discussion: Everyone seems to have come to the understanding thgat on a 2d representation of the coil, the system as a whole seems to oscillate similarly to a wave function (sine cosine etc.) but we don't know whether or not this is going to be consistent. What about the hieght from which the Hhoney is being dropped and the viscousness of the honey. Do different types of honies provide for diffrent thicknesses and amplitudes of the oscillations?

Emile

As stated in the presentations, there is a ton of literature on this subject, so I assume it's been tested with honey from different heights and with varying viscosities. Just as different masses and spring constants affect the natural frequency, I believe the altering types of honey also affect the "amplitudes" of oscillation and maybe even the frequency (i.e. how fast it comes to the ground and tumbles).

Pascal

I'm not sure if this has been brought up somewhere else (there are a ton of posts on this subject), but another factor would be the radius of the stream itself. I realize this moves us into the 3d aspect of the phenomenon, but it would certainly affect the behavior. Watching Bob perform the demonstration, he attempted to get the stream to a very small radius. It seems that as the stream expands past a certain

radius, this coil behavior will not longer occur, or at least not noticeably. This "critical radius" (made that up) would likely depend on the viscosity of the liquid you use.

Veronique

I am not sure if this has been brought up by anyone else, but it seems like the honey's coiling could be best modeled in a different coordinate system. Would this be easier to describe in polar coordinates than in cartesian coordinates?

Julien

I bet each individual property of the honey has everything to do with exactly how the honey coil turns out. For example, I think if the honey was dropped from a higher point, the honey would be traveling faster at the point where coiling begins. I think this would make for a tighter coil and also make it harder for the coil to switch directions because so much force has to be overcome to counteract the falling honey. I also think the thicker the honey is the bigger the coiling radius. I believe the coil can only be so flexible due to the molecules of the honey. If the honey was thinner, I don't think the coil could bend as easily compared to thin honey. This would produce a wider coil in my opinion.

Emile

I think that might be an interesting experiment to perform. However, I'm not sure the conclusions are such black and white. As Andre pointed out earlier, the external factors of air resistance are negligible, so as you suggest, the implication is that the coils occur randomly. But what if it wasn't random, and the coils were just too far apart. Wouldn't the optimum distance between the two streams have to be obtained in order to determine whether or not it were plausible for the two to touch in order to achieve accurate conclusions? Besides trial and error, how would you suggest we figure out such a distance? Or, in your opinion, will the distance not matter?

Sebastien

A bit of research on the "honey coiling phenomenon" yielded the following paper describing the viscous fluid's motion:
<http://www.jstor.org/stable/4143202?seq=1>

In this article, a system of 17 first-order differential equations is derived, with 19 boundary conditions required. The coiling frequency can be determined by solving these equations. It's immediately clear that this simple phenomenon is not at all trivial to solve! The article does not explicitly state why the honey coils, however. From what I gather, it is the combination of gravitational, inertial, and viscous forces that cause the phenomenon. The viscous forces acting within the body of the fluid seem to be responsible for the initiation of the coil.

Alain

What I found to be incredibly interesting was that the coiling of the honey could reverse direction (thought it isn't very common). The presenter mentioned this in passing, but I think it creates an interesting event. I don't remember if he said that the reversal was for a single coil and then it returned to normal coiling or if the reversal would propagate throughout further coils. I would imagine that this would create some problems when creating a mathematical model for the behavior of the honey coiling. There would have to be some way to show that the coil could reverse direction, but that it did not commonly occur. I have no idea how this would be done in a model but, I am assuming it's pretty complex.

Emile

This may be completely off, but I wonder if the reversal of the coil may be simulated in a similar manner as the mass on a spring. I realize the motion for the coiling isn't oscillatory like a block, but the block goes forward, reaches an equilibrium point (initial) and goes back. I wonder if it would be possible to consider only the portion going in the negative direction of the equilibrium position (although, after going through all the probable solutions, I don't see how this could be a possibility). The reversal may be linked to the other lab, where oil was traveling up a rod. Perhaps it could be modeled by taking the negative effect of gravity in the equations Sebastien brought up earlier.

Taking into account the gravitational, inertial, and viscous forces separately, it seems we'd be able to express this as a linear combination of a system of three first order differential equations, instead of 17. What do you think the other 14 differential equations are representing? Also, from a brief read through of the introduction, it appears that the biggest influence is that of compressive forces (as in columns). A simple second order differential equation would give a good starting point to understand this phenomenon instead of including 17 differential equations. This raises the question, how important are each of the differential equations in understanding the entire phenomenon?

Yves

I think it is very easy to see how 17 or 19 different equations would be needed for the system to properly describe the honey's behavior. Just starting with other forces involved, no one has mentioned the normal force, which I would think is a large reason for why it coils like it does. In addition, all the random behavior, from its occasional reversal as has been mentioned to the times when it doesn't even coil properly, instead just layering back and forth over itself, would be incredibly difficult to describe through differential equations.

However, even as I type this, I remember chaos theory, which Prof. Bergevin showed us early this year, which if I remember right can be displayed with only three equations. So I can also see how it would be reasonable to expect much fewer equations to be necessary.

Andre

Many of you have mentioned the physical properties of the honey and their effect on the coiling of the honey, viscosity, gravity, and inertia, when doing further research I found an article, *The Cook's Instability: Coiling a Thread of Honey*, that discusses a forth factor that also plays a significant role in the "liquid rope coiling" which is the inertia-gravitational aspect which demonstrates the resonance of the honey's "tail" or stream from the source. The resonance of the "tail" directly relates to the mass-on-a-spring problem as many of you have mentioned. The inertia-gravitational aspect is most prevalent through the various coiling states (i.e. going from coiling to oscillating more

in a line as Julien discussed) but eventually the inertial force takes over again. The article gives a long explanation with findings similar to the article Sebastien cited with a 17th order system of ODEs. The link is <http://documents.irevues.inist.fr/bitstream/handle/2042/16381/CFM2007-0084.pdf?sequence=1>. The article actually relates to the exact experiment we observed and gives a thorough explanation so it's pretty interesting!

Another article I found shows this phenomenon modeled mathematically using Euler's method. They use the name cylindrical and planar viscous jet buckling to describe the coiling of viscous liquids. They explain the process of the coiling "by considering that the liquid will prefer the path of least resistance. The falling liquid above and the viscous pile below apply opposing forces, but the surrounding air applies little or no resistance, causing the fluid to bend or bow out to one side." This process of using Euler's method is said to be easily performed and accurate.

The link is <http://people.cs.ubc.ca/~rbridson/docs/batty-sca08-viscosity.pdf> and this article is also the experiment we observed with a great explanation and new method to solving it.

Emile

I wonder if the honey experiences little/no air resistance because its mass is small or because the shape (basically a thin line) doesn't gather enough area for air resistance to have an effect. Consider jumping out of an airplane—before the parachute opens, you're pushed from above and below with gravitational forces and air resistance, you reach a terminal velocity based on whether your hands are open wide or if you are tucked in a ball formation, which makes you go slower or faster. Maybe the area affected for the honey is so small that the air doesn't push up on it quite as much. Although as Stephanie pointed out, we were told that the coiling could reverse direction, in which case it would seem that perhaps some external influence is at play here?

Wow, a really neat observation! If it would be possible to model the honey coiling in a similar manner as the spring, perhaps gravity and the inertial forces Sebastien brought up earlier could replace the spring constant. I wonder if we could neglect friction or if the air resistance/viscous force/air friction would be the substitute.

Michele

I'm very interested in this experiment. And I wonder is it rotate force to make holyup? And if we use water to do the some expenriment? Can we see the same phenomenon?

Paulette

I think that if you were to model the coiling action into any sort of equation, more likely than not differential. You could incorporate it into a much larger postulate that includes and compensates for the differing viscosity of any liquid for that matter and thus affects the degree of coiling or almost close to 0 (water for instance) which could bring us back into limits and that incredibly fun stuff.....perhaps even a $e^{(-kt)}$ term.....hmmm

Coiling (II)

Christophe

In the lab visit the coiling that occurred seemed to me to follow some general rules.

1. The liquid fell and depending on the initial landing of it, some liquid would build up on one side, and would be at a lower level on another side (of a circle-point is highest and one point is lowest, like a tilted disk kind of).
2. The liquid would then eventually land on the highest point of the "disk" and would start to roll down due to gravity to the lowest point of the disk, continuously, because there is always a lowest point to the disk, allowing for the liquid to always be flowing in one direction in circular motion.
3. Because the liquid was so viscous, meaning there were pretty strong bonds between the molecules in the liquid, when the initial liquid started flowing in one direction on the disk, it "pulled" all of rest of the liquid in that direction.
4. This pulling essentially causes also a reverse force that pushes on the tip of the liquid that is in contact with the disk, and so the point had not only the force of gravity pulling it to the lowest point, but also the rest of the liquid pushing it down to the lowest point.

Anyways, I think you could parameterize the liquid, use sine and cosine parameterization to describe the position of the tip of the liquid (such as $x=\sin(t)$ $y=\cos(t)$, depending upon the initial condition and the direction the coil takes), and some $z=Ct$ to describe the height of the liquid, and using this first and second, and perhaps higher, derivatives could be taken to describe the velocity and the acceleration. As Sebastien said, the acceleration could be used to describe the force.

The thing that really doesn't make much sense to me is why the coiling would randomly change direction during coiling. The initial direction just depends on the initial fall of the liquid, but to change direction the rate of flow must change and the point at which the liquid falls from must also change, but I'm not sure how they would fit into the differential equation as sources of error.

Julien

I think the reason the coiling sometimes changes has to do with the properties of the liquid. It would seem to me that a liquid as viscous

as the honey should be thought of as a single, uniformed unit. I think the molecules of the honey are bonded so well that you could think of the honey as a string, where the beginning of the coiling is still connected with the stream coming out of the bottle. I think the coiling sometimes changes direction because (keep in mind the honey is a single unit) the coil eventually migrates too far away from where the honey is being poured. When this occurs, the honey that is still falling will pull on the coil, causing it to change directions.

Perhaps this could be related to the "frequency" of the coiling. If the frequency became too high (the coil started to get too far away from the origin of the honey) then the coil shifts directions. Perhaps this aspect could be embedded into some type of Fourier series or some differential equation where the answer is in terms of sines and cosines (like we've been discussing in class).

Renee

I'm not sure that the error you are trying to incorporate into the equations governing the coiling motion can be done. From what I observed the reason for the changes in coiling behavior is that after it had been in the coil for a certain time the honey flowed together and no longer provided a strong base for the coil and thus the coil would lean to one side and pull the falling honey with it. I think that it was this motion that causes the problems in the coiling motion and if the honey could be made to solidify quickly upon reaching the coil it could be created to an indefinite height as long as the source of the honey moved with it. Thus I think the motion is better described up to the point it becomes part of the coil. This time span still gives many interesting observations, when played slowly it appeared that there was a small curve in the stream of honey just above (~4mm) and about 120 degrees leading the coiling motion. I think that an equation could have some sort of damped helix pattern to it that would then just be rotated about its axis to create the coils we saw. Thus there is no equation for the coil created but the stream of honey creating it. Which is arguably a more interesting 3D shape.

Brownian Motion Lab and 'Mass on a spring problem

Lucie

I was thinking about the Brownian motion lab as similar to a mass on a spring problem. It seemed that in the basic state the little beads oscillated fairly consistently. But when the laser is introduced the bead gets trapped in the laser. I was trying to think of a way that this matches what we know about mass on a spring problems and I came up with an idea; first off I assumed that there was a driving force and a damping force in this problem. I know that we haven't learned the case with both of these included but I think we will later. What if originally just light is the driving force and the fluid acted as a damping force; when the laser is introduced it will have a much higher frequency than normal visible light, if the frequency of the laser is much further from the ω_0 then so the driving force is reduced (and the damping force may be great enough to reduce the oscillations relatively close to zero).

Elise

I think you got the idea of what's what wrong, or I do. What I think is that it is just like a mass-on-a-spring problem, and can most likely be modeled with the same equation. I don't think the fluid acted as a damping force; I'm pretty sure it is there to make as little friction as possible occur between the beads and petri dish (or whatever container), so the beads can oscillate freely. As for the driving force in the equation, I think it to be the laser, but there must be some reason the beads moved without the laser as well. My theory is that the system began with some initial condition. This initial condition had the role of jump-starting the system so the beads would not be at rest. Due to the low damping of the system (caused by a thin layer of water), the beads could keep oscillating at pretty much the same pace. Analogously to the mass-on-a-spring, it is like pulling or pushing the spring to a certain distance, then letting go. This is not a driving force, just some initial condition. Then the laser was introduced, acting as the driving force. The laser is probably a good driving force because, as compared to the other ambient light, it has a very high frequency beam. This frequency is directly proportional to energy, so the laser imparts much energy to the system, allowing the beads to exhibit such radical behavior. This explanation makes sense and seems to fill the gaps for me, but I could be wrong. Anyone have another theory, or even any ideas on what the initial condition for the system could be caused by?

Lucie

I don't buy the no friction idea. Think about it, I believe the lab tech said that the beads were about 5nm(5E-9m); it took a little digging online but I finally found an academic research paper at (<http://www.springerlink.com/content/xkdu73bk69dmrwnb/>) that stated that water molecules had a radius of 1.35Å(1.35E-10m). This means that the radius of the bead is only 37 times greater than that of the water molecules; this may sound like a large difference in radii but it is actually quite small when you consider that friction is a microscopic force (so most things causing friction are smaller than the objects being acted on by an extremely large amount). With the beads having to traverse through a field of water molecules significant resistance/friction would be imparted due to the size of the water molecules.

Henri

I am not sure if it is friction or not, but I think the motion of the beads is being caused by the motion of the water molecules. The water molecules (or whatever fluid it is) are vibrating and imparting kinetic energy to the bead. Since the bead is larger a lot of molecules have to hit the bead to make it move. The water molecules are moving due to having kinetic energy of their own. Heating the water should add kinetic energy to the molecules and cause the beads to move more. So at least one part of a description of the motion of the particles might be a simple differential equation depending on the temperature of the system.

Monique

I agree with this. When I first looked up Brownian Motion, I came up with the movement of smoke because the visible smoke is actually unburned carbon which "collides" with air molecules and hence moves somewhat randomly. This air can be seen as the liquid you are speaking of. I believe this is how this lab relates to class, especially since we slightly touched on reactions and the likelihood that certain molecules will collide. As for this laser that is spoken of, I agree that it would add energy to the surrounding molecules hence, making them move around much more rapidly and would force them to cover more of the area and collide with the "beads" much more frequently, not giving the beads as much time to move from one place to another.

Gilles

hmm...as i know that brownian motion can only correctly describe the irregular and completely random motion of pollen particles. theoretically, brownian motion can also be used to describe random movements like molecules. from what i learn in thermal dynamics, molecules are always doing some random movements like brownian motion depending on the temperature. containing more thermal energy, more rigid the motion would be. molecules would be frozen (no motion) under absolute zero, 0K, -273oC. i want to say that compare to mass on a spring problem, likely, the friction in the spring problem is just like the temperature in thermal dynamics reversely.

Freezing Water

Julien

I thought the freezing water experiment was the coolest one out of all of them. I just really liked the method in which the water froze and made all these different canals by doing so. It reminded me of a chaotic bifurcation diagram at first but after observing it for a while. I didn't think so anymore. This is because the chaos involved didn't seem apparent to me in the freezing water. What I saw was a thick "trunk" that split into smaller branches, but these branches didn't continue to split and split. Instead they stopped but the trunk continued to grow and produce more branches as the slide water continued to freeze. I would be curious to see the extent of the branches splitting because we were limited with the scope of the microscope. Is there a limit to the splitting of the branches and/or are there multiple think trunks that split and finally condense into one frozen lattice?

Josephine

I'd bet that as long as you have the water there and the means to freeze it, the "branches" of frozen water would continue to grow and bifurcate. I guess that I'd say there is one trunk, where the freezing begins, and that it just spawns a few branches, which in turn act as trunks for more branches later on. What interests me the most about this experiment, however, is that the growth of the branches width wise (as in getting wider on the screen) appeared to slow down as two branches began to widen closer to each other. This got me thinking that there's a good chance that the growth of the width of the branches could very easily be a differential equation where the growth rate of the width of each branch is inversly proportional to the distance between branches, so it would look something like $\frac{dw}{dd} = k/d$ where k is a proportionality constant and d is the distance between the edges of two branches. Of course, this probably isn't the most efficient or accurate method of modeling the growth of the branches, especially because it only deals with the width, and for each branch the equation appears to be different. But hey, it's a start.

Benoit

I thought the water freezing experiment was really cool also. Its weird how such a simple thing as water freezing can actually look and create such a complex and chaotic pattern. At first it looked like the main branch or trunk kept dividing but as the microscope moved along it actually seemed to travel and remain intact with branches off of the main stem. It would be really cool to see the water at a higher magnification to see how many more times the branches split. I also thought it was interesting how the branches not only split but spread out up against each other and yet never ran together, in other words kept the same shape. Im not sure what the best way to describe this pattern mathematically would be, but the bifurcation idea makes a lot of sense as well as the differential equation. The differential equation might work since it is specific and takes into account the rate of the motion. How would you determine the bifurcation solution or the parameter for the bifurcation to make sense of the chaos?

Andre

I also agree that it was very interesting to see the ice crystals form in the experiment. I thought the explanation for the structure of the ice crystals could be explained by further exploring fractals. Fractals can be described as "composition of exponential functions embedded in a multidimensional domain," in other words it is a never ending, repeating pattern. However in the case of ice crystals it reaches a limit where it stops reproducing. The fractals can be determined by a series of sinusoids at different frequencies. The article gives the equation $f = \sum (a_k \cos(k(\theta)) + b_k \sin(k(\theta)))$ which is very similar to the different types of equations we have been exploring in class. The axel pattern formed by water freezing has a lot to do with the surface it is freezing on, for example if I were to scrape my window in a certain pattern the ice crystals would freeze in the shape of that pattern.

The article I used to find information about the fractals was <http://www.ecs.soton.ac.uk/publications/rj/1995-1996/ccg/mem/gena.htm>.

Elise

Andre, you said "the fractals can be determined by a series of sinusoids at different frequencies. The article gives the equation $f = \sum (a_k \cos(k(\theta)) + b_k \sin(k(\theta)))$ which is very similar to the types of equations we have been exploring in class." First off, this is the definition of fourier series, only in a different form than we learned in class or in the textbook. Second, I don't see how, using a fourier series, you can have sinusoids at different frequencies. The argument for all sines and cosines in the series would be $k \cdot \theta$, according to your article. This reminds me very strongly of the mass-on-a-spring problem, where the arguments for the linear combination of sine and cosine trial solution was $w_0 \cdot t$, where w_0 was the frequency. This should apply to the freezing water case as well, since it involves oscillations like everything else in the lab and what we've been talking about. Thus k is analogous to w_0 , but k is going to be constant in all these fourier series terms, so there must be a problem in your article or my logic.

Renee

Actually I think that the 'k' is the step counter in an sum of the fourier series terms given in the above posts. Thus there are different frequencies because increasing 'k' increases the frequency of that terms sinusoid.

On a side note I believe the professor giving the tour mentioned that the crystals slowed down their growth as they grew close to each other because they had used all the liquid available in the local region. nonetheless it may be governed by a differential equation as stated earlier, I just wanted to clarify what was hapening on the slide.

On a related note is the subject of crystal growth in a more general capacity, i thought it was interesting that one of his students was using microgravity aboard the ISS to test gravity's effect on metalurgical crystal growth and its practical aplications. the first i thought of was the production of turbine blades which is currently done through a complex, costly and wasetfull process. each fin is made in a ceramic mold and slowly cooled in a specially controled oven that maintains a specific temperature gradient along its surface. thus one end of the mold can be below the metal aloys melting temperature and the other above and the point of solidification controled by moving the mold through the oven, almost exactly like in the experiment witht the hot and cold plates and actuator. a helix in the botom of the mold alows only the crystal whose growth plane is oriented perfectly perpindicular to the axis of the helix makes it through (the rest are blocked by the walls of the helix) and grows through the actual fin mold to make a fin out of one continuous crystal of metal, substantially increasing its strength at temperatures. This was just the first thing i thought of when i saw the experiment thought someone might be interested in it.</tangent>.

Brigitte

Elise, what Renee pointed out was exactly correct. The k you referred to is the index number for the sequence and therefore each term in the sequence would have a differencnt value for k. Therefore, the terms would all have different frequencies. This should make sense since we saw this process in action when Dr. Bergevin showed us the square wave and walked us through what happens as you add more terms. The more terms that Dr. Bergevin added, the more oscillations the Fourier estimation had within a given period, so logically the frequency would be changing and in this case increasing.

Emile

I like your idea, Josephine. Another factor to consider when discussing the width of the branches and when they started to narrow is when the "branches" got near the end of the formation. I'm not sure how to factor this into the differential equation though...Maybe as with nested differential equations for populations (i.e. birth and death terms), there can be a "death" term describing the termination of formation for that particular branch. This could then give us a coupled system, like the ones we've been talking about in class...Don't know if this a huge, incorrect leap....