

Math 250A (Fall 2009) - Exam I Review

Group 1

1. Find the derivative of the following function:

$$M(\alpha) = \tan^2(2 + 3\alpha)$$

SOLUTION: Using the chain rule, we have

$$\frac{dM}{d\alpha} = 2 \tan(2 + 3\alpha) \cdot 3 \cdot \frac{1}{\cos^2(2 + 3\alpha)} = 6 \frac{\sin(2 + 3\alpha)}{\cos^3(2 + 3\alpha)}$$

2. Find the general solution to the following differential equation:

$$\frac{dy}{dt} = k(y - A)$$

Use separation of variables, integrate and you end up with

$$y(t) = A + Ce^{kt}$$

where C is an arbitrary constant defined by your initial condition.

Group 2

1. Find the derivative of the following function:

$$g(x) = \tan^{-1}(3x^2 + 1)$$

SOLUTION: Need to remember that the general derivative for arctan (a sigmoidal function) is

$$\frac{d}{dx} \tan^{-1}(u) = \frac{1}{1+u^2} \frac{du}{dx}$$

Using this, we have

$$\frac{dg}{dx} = \frac{1}{1+(3x^2+1)^2} \cdot 6x = \frac{6x}{9x^4+6x^2+2}$$

2. Find the general solution to the following differential equation:

$$\frac{df}{d\theta} = ke^{-\theta/2}$$

Remember that the general solution for an ODE of the form

$$\frac{df}{d\theta} = g(\theta)$$

is given by

$$f(\theta) = \int g(\theta) d\theta$$

So computing the integral, we end up with the general solution

$$f(\theta) = C - 2ke^{-\theta/2}$$

where C is an arbitrary constant defined by your initial condition.

Group 3

1. Find the derivative of the following function (assuming a and b are constants):

$$g(u) = \frac{e^{au}}{a^2 + b^2}$$

NOTE: also consider how your answer would change if you did not necessarily know if a or b were necessarily constant

SOLUTION: Given that most quantities are constants, we simply have

$$\frac{dg}{du} = a \cdot \frac{e^{au}}{a^2 + b^2}$$

However, if a and b were functions of u , we would need to reapply the chain rule.

2. Show that any function of the form

$$x = C_1 \cosh(\omega t) + C_2 \sinh(\omega t)$$

satisfies the ODE

$$x'' - \omega^2 x = 0.$$

Hint: Can you express $\cosh$ and $\sinh$ in terms of functions you readily know how to take the derivative of?

Remember that

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

and

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

Thus one can readily show that $\frac{d}{dx} \sinh(x) = \cosh(x)$ and $\frac{d}{dx} \cosh(x) = \sinh(x)$. Thus

$$\frac{d}{dt} (C_1 \cosh(\omega t) + C_2 \sinh(\omega t)) = \omega C_1 \sinh(\omega t) + \omega C_2 \cosh(\omega t)$$

Carrying one step further and plugging back into the ODE, we see that the given function is indeed a solution.

Group 4

1. Find the derivative of the following function:

$$s(y) = \sqrt[3]{(\cos^2(y) + 3 + \sin^2(y))}$$

SOLUTION: A bit tricky. Seems like we would need to use the chain rule. But note that the argument inside the cube root contains $\cos^2(y) + 3 + \sin^2(y)$, which is just 1 no matter what the value of y is. Thus the problem is asking you to find the derivative of $\sqrt[3]{(4)}$, which is just zero.

2. The birth rate of a population is proportional to its size $x = x(t)$ with a constant of proportionality $b > 0$. The death rate of the population is proportional to the population size of a deadly virus $y = y(t)$ with a constant of proportionality $c > 0$. The virus population has a negative per unit growth rate $-r$.

a.) Write a differential equation and initial condition for the population sizes $x = x(t)$ and $y = y(t)$, assuming initial sizes x_o and y_o .

b.) Show that the following are solutions to the ODEs you obtained in the previous part:

$$x = \left(x_o - \frac{c}{r+b}y_o\right)e^{bt} + \frac{c}{r+b}y_oe^{-rt}$$

$$y = y_oe^{-rt}$$

Translating the problem, we have

$$\frac{dx}{dt} = bx - cy \qquad \frac{dy}{dt} = -ry$$

The second part of the problem merely requires one to take the derivative and do some algebra to show that the solution holds.

Group 5

1. Find the derivative of the following function (assuming a and b are constants):

$$w(r) = \frac{ar^2}{b+r^3}$$

SOLUTION: Using the product (or quotient) and chain rules, we have

$$\frac{dw}{dr} = \frac{2ar}{b+r^3} - \frac{ar^2}{(b+r^3)^2} \cdot 3r^2 = \frac{2abr - ar^4}{(b+r^3)^2}$$

2. If one pound of fertilizer covers 250 square feet, estimate the amount of fertilizer needed to fertilize the fairway. (Note: Use $n = 10$) Make sure to clearly explain your answer!

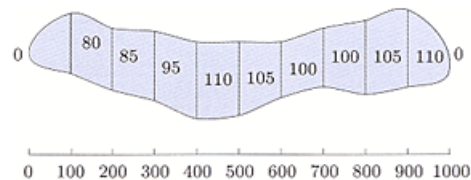


Figure 1: The width, in feet, at various points along the fairway of a hole on a golf course.

The idea here is to approximate the area of the course using Riemann sums. Using the **LEFT** rule, we have $\text{LEFT}(10) = (0 + 80 + 85 + 95 + 110 + 105 + 100 + 100 + 105 + 110) * 100 = 89000 \text{ sq. ft.}$ If one lbs. of fertilizer covers 250 sq. ft., then we would need **356 lbs.** of fertilizer.