

Math 250A (Fall 2009) - Quiz 9

SOLUTIONS

1. (4 points) Find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x^2}$, and $\frac{\partial^2 f}{\partial y^2}$ for the following equation.

$$f(x, y) = 3x^5y^7 - 5x^4y^3 + 6xy$$

$$\frac{\partial f}{\partial x} = 15x^4y^7 - 20x^3y^3 + 6y$$

$$\frac{\partial^2 f}{\partial x^2} = 60x^3y^7 - 60x^2y^3$$

$$\frac{\partial f}{\partial y} = 21x^5y^6 - 15x^4y^2 + 6x$$

$$\frac{\partial^2 f}{\partial y^2} = 126x^5y^5 - 30x^4y$$

2. (6 points) Determine all values of (x, y) for which solutions to the following differential equation *both exist and are unique*.

$$\frac{dy}{dx} = \sqrt{|y - 5|}$$

We can see that our differential equation is written in the form $\frac{dy}{dx} = g(y)$ which is independent of x . Therefore, we can assume that all values of x are valid as long as the associated values of y satisfy our conditions for existence and uniqueness.

We can see that g is continuous for all y , and the conditions for existence are satisfied for any region around any initial value (x_0, y_0) . Therefore, solutions for our differential equation exist for all (x, y) .

In order to determine the uniqueness of solutions, we must analyze $\frac{dg}{dy}$. Without explicitly defining $\frac{dg}{dy}$, we can determine a singularity exists at $y = 5$ since we will have $|y - 5|$ in the denominator of our derivative. Therefore, uniqueness is not guaranteed within any region containing $y = 5$. However, $\frac{dg}{dy}$ is defined and continuous for all $y \neq 5$. So we can conclude that solutions to our differential equation exist and are unique for all (x, y) such that $y \neq 5$.