

Math 250A (Fall 2009) - Quiz 8

SOLUTIONS

1. (5 points) Determine whether the following differential equations are linear and/or separable or neither. **Make sure to explain clearly so to justify your answer!**

(a) $9t = e^t \frac{dy}{dt} + y \ln(y)$

The differential equation is neither linear nor separable. It is not linear due to the $\ln(y)$ term since it deviates from the linear form $\frac{dy}{dt} + p(t)y = q(t)$. The equation is not separable since you cannot rewrite the equation in the form $\frac{dy}{dt} = f(y)g(t)$.

(b) $8r = \frac{dr}{d\theta} - \theta^5$

The differential equation is linear but not separable. It is linear since it can be written in the form described above. Similar to the previous differential equation, it is not separable.

2. (5 points) Find $y(x)$ for the initial value problem below. (**Hint:** Complete the square)

$$\frac{dy}{dx} = \frac{3x^2 + 6x + 1}{2y - 2} \quad y(0) = 2$$

First, we use separation of variables to obtain two integrals.

$$\int 2y - 2 \, dy = \int 3x^2 + 6x + 1 \, dx$$

$$\Rightarrow y^2 - 2y = x^3 + 3x^2 + x + C$$

Now we can apply our initial condition to solve for C .

$$2^2 - 2(2) = 4 - 4 = 0 = C$$

Since the left side of the equation is not in a simple form to solve for y , we can will complete the square by adding 1 to both sides of the equation to obtain a more manageable equality.

$$y^2 - 2y + 1 = (y - 1)^2 = x^3 + 3x^2 + x + 1$$

$$\Rightarrow y - 1 = \pm \sqrt{x^3 + 3x^2 + x + 1}$$

$$\Rightarrow y = 1 \pm \sqrt{x^3 + 3x^2 + x + 1}$$

Since our initial value of y is greater than 1, we take the positive root for our solution.

$$y(x) = 1 + \sqrt{x^3 + 3x^2 + x + 1}$$