

Math 250A (Fall 2009) - Quiz 7

SOLUTIONS

1. (5 points) A cup of coffee made with boiling water is poured and left sitting in a room with an ambient temperature of 21°C . The coffee temperature (T) decreases at a rate proportional to the difference between the current temperature of the coffee and that of the room (see ODE below). If the temperature of the coffee is 90°C after sitting out for 5 minutes, how long will it take for the coffee to cool to 50°C ?

$$\frac{dT}{dt} = -k(T - 21)$$

After using separation of variables, we find the general solution for the temperature of the coffee is given with

$$T = Ce^{-kt} + 21$$

Since the coffee is boiling when it is initially brewed, we have an initial condition of $T(0) = 100^\circ\text{C}$.

$$\Rightarrow T(0) = 100 = C + 21 \quad \Rightarrow C = 79$$

$$\Rightarrow T = 79e^{-kt} + 21$$

Next, we find the value of k with our given information.

$$T(5) = 79e^{-5k} + 21 = 90 \quad \Rightarrow k = -\frac{1}{5} \ln \frac{69}{79}$$

We can now solve for the time taken for the coffee to cool to 50°C .

$$T(t_{50}) = 79e^{-kt_{50}} + 21 = 50 \quad \Rightarrow -kt_{50} = \ln \frac{29}{79}$$

$$\Rightarrow t_{50} = -\frac{1}{k} \ln \frac{29}{79}$$

$$\Rightarrow t_{50} = \frac{5 \ln \frac{29}{79}}{\ln \frac{69}{79}} \approx 37 \text{ minutes}$$

2. (5 points) Find all equilibrium solutions to the following differential equation and classify their stability. Sketch a slope field to illustrate your answer (be as clear as possible with your sketch).

$$\frac{dy}{dx} = y(y - 4)(y + 5)$$

To find all equilibrium solutions, we want to find all values of y such that $\frac{dy}{dx} = 0$. Therefore, our equilibrium solutions are $y = -5$, $y = 0$, and $y = 4$.

After creating a slope field, we can see that the equilibrium solution $y = 0$ is the only stable one since solutions that start near $y = 0$ stay near as x goes to infinity. The remaining two solutions are unstable because solutions do not stay near $y = -5$ nor $y = 4$ as x goes to infinity.