

Math 250A (Fall 2009) - Quiz 4

SOLUTIONS

1. (5 points) Find the exact area under the curve of the function $f(x) = 3xe^{4x^2}$ between $x = 1$ and $x = \pi$.

To find the area under the curve, we must compute the following definite integral.

$$\int_1^{\pi} 3xe^{4x^2} dx$$

We see that we can solve this integral with substitution. Let $u = 4x^2$. Therefore, $du = 8xdx$. Now, we can rewrite our integral. Please note that we must also change our limits of integration when we make our substitution.

$$\begin{aligned}\int_1^{\pi} 3xe^{4x^2} dx &= \frac{3}{8} \int_1^{\pi} 8xe^{4x^2} dx = \frac{3}{8} \int_4^{4\pi^2} e^u du \\ &= \frac{3}{8} [e^u]_4^{4\pi^2} = \frac{3}{8} [e^{4\pi^2} - e^4]\end{aligned}$$

2. (5 points) Consider the following integral:

$$\int \theta \sin(\theta) d\theta$$

Evaluate the integral from $\theta = 0$ to where the integrand first crosses the θ -axis (i.e., the area under the first arch).

First, we must first find the upper limit of integration for our problem. To accomplish this, we must consider where the integrand is equal to zero. We know that θ is equal to zero only when it intersects the origin. Therefore, we only need to consider when $\sin(\theta)$ is zero. Since $\sin(\theta)$ is a periodic function, there are an infinite amount of zeroes which occur when θ is an integer multiple of π . Since we are only interested in the area under the first arch of the function, we can deduce that our upper limit of integration the first zero after the origin. We now set our upper limit to π .

To solve this integral, we must use the method of integration by parts. Since θ disappears after one iteration of integration by parts, we let $u = \theta$ and $dv = \sin(\theta)d\theta$. Therefore, $du = d\theta$ and $v = -\cos(\theta)$. We may now rewrite our integral.

$$\begin{aligned}\int_0^{\pi} \theta \sin(\theta) d\theta &= [-\theta \cos(\theta)]_0^{\pi} + \int_0^{\pi} \cos(\theta) d\theta \\ &= [-\theta \cos(\theta) + \sin(\theta)]_0^{\pi} = [-\pi \cos(\pi) + \sin(\pi)] - [0 + \sin(0)] \\ &= (-\pi)(-1) = \pi\end{aligned}$$