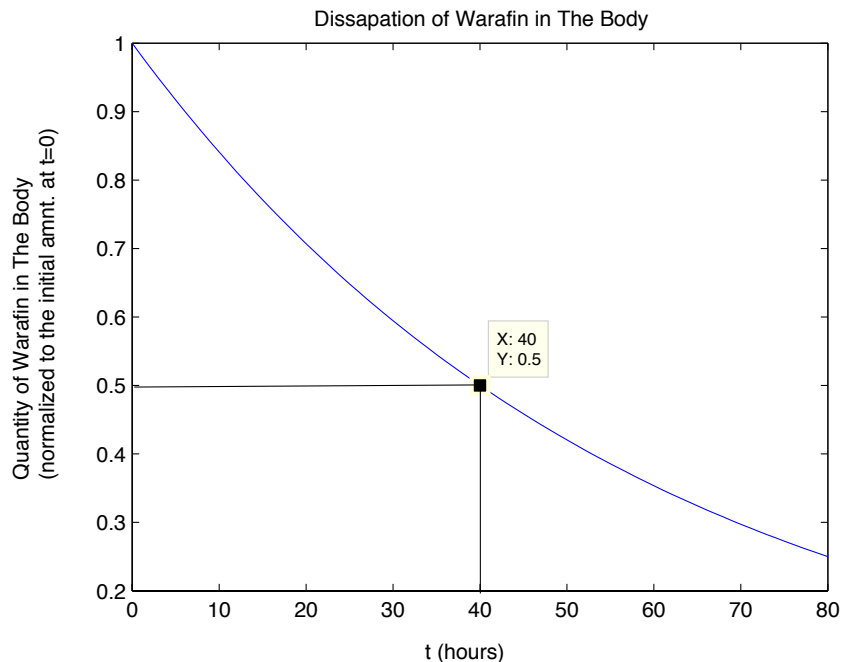


Math 250A (Fall 2009) - Quiz 3

SOLUTIONS

1. (5 points) Warfarin is a drug used as an anticoagulant. After administration of the drug is stopped, the quantity remaining in a patient's body decreases at a rate proportional to the quantity remaining. The half-life of warfarin in the body is 40 hours.

a. Sketch the quantity of Warfarin in a patient's body (Q) as a function of the time (t) since stopping administration of the drug. Mark the 40 hours on your graph.



b. Write a differential equation satisfied by Q .

We are given that the rate of change ($\frac{dQ}{dt}$) is decreasing proportional ($-k$) to the amount remaining in the body (Q). Therefore, we can construct our differential equation:

$$\frac{dQ}{dt} = -kQ$$

By using the technique of separation of variables, we can now find a general solution for Q .

$$\frac{dQ}{Q} = (-k)dt \Rightarrow \ln|Q| = -kt + C \Rightarrow Q(t) = Ce^{-kt}$$

Since $Q(0)$ is the initial amount of the drug in the body, we denote the constant, C , as Q_o . Now, we can solve for the decay constant, k , by using the given information on the drug's half-life.

$$Q(40) = \frac{1}{2}Q_o = Q_o e^{-40k} \Rightarrow \frac{1}{2} = e^{-40k}$$

$$\Rightarrow \ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2) = -\ln(2) = -40k$$

$$\Rightarrow k = \frac{\ln(2)}{40}$$

2. (5 points) Solve the following integral. Use differentiation to check your answer.

$$\int \left(\cos(\phi) + \frac{1}{\cos^2(\phi)} \right) d\phi$$

The first step is to rewrite $\frac{1}{\cos^2(\phi)}$ as $\sec^2(\phi)$. Now, we recognize that $\sec^2(\phi)$ is the derivative of $\tan(\phi)$. Therefore:

$$\int \left(\cos(\phi) + \frac{1}{\cos^2(\phi)} \right) d\phi = \sin(\phi) + \tan(\phi) + C$$

It is also important to remember the constant of integration in our solution.

To check our work, we differentiate each term of our solution.

$$\begin{aligned} \frac{d}{d\phi} [\sin(\phi) + \tan(\phi) + C] &= \cos(\phi) + \frac{\cos^2(\phi) + \sin^2(\phi)}{\cos^2(\phi)} \\ &= \cos(\phi) + \frac{1}{\cos^2(\phi)} \end{aligned}$$