

Math 250A (Fall 2009) - Quiz 1

SOLUTIONS

1. (10 points) You are a burglar. You want to break into a certain house, but it is unfortunately surrounded by a fence that you can not climb directly (see figure). So you are going to need a ladder. But ladders are expensive! So you need to figure out what is the shortest ladder (of length L) you would need to get over the fence to the house. Your answer should depend upon d and h . **Make sure to clearly explain your thinking/methodology!**

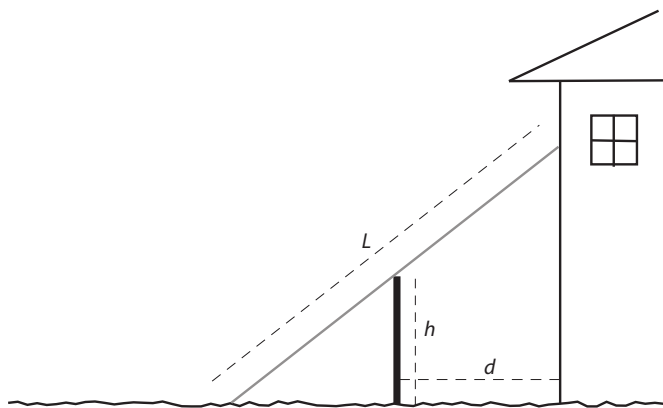


Figure 1: Schematic showing nature of problem. Note that fence is of height h and is at a distance d from the house. The ladder only need touch the house (i.e., the height of the window is irrelevant).

NOTE: There are likely quite a # of ways one could go about approaching this problem. Below outlines just one possible method (that has the added bonus of coming up with the correct answer!).

You need to essentially optimize L given the particular constraints d and h . Consider the ladder as comprised of two sections, one to the left of the fence (L_1) and one to the right (L_2). Thus $L = L_1 + L_2$. Now let θ be the angle the ladder makes with respect to the ground. Clearly we must have $0 \leq \theta \leq \pi/2$. When $\theta \approx 0$ or $\pi/2$, it follows that L will be very large, so there is presumably some angle in between where L is effectively minimized.

The first step is to derive an expression for L in terms of θ (i.e., find the function $L(\theta)$). Using trigonometric considerations, we have

$$L(\theta) = L_1(\theta) + L_2(\theta) = \frac{h}{\sin(\theta)} + \frac{d}{\cos(\theta)} \quad (1)$$

Now if we determine the critical point of the function $L(\theta)$ (i.e., where the derivative with respect to θ equals zero), we should find the minimum value of L . Or put another way, we wish to determine the value of θ (call it $\hat{\theta}$) such that $L(\theta)$ is smallest on the interval $0 \leq \theta \leq \pi/2$.

$$\left. \frac{dL(\theta)}{d\theta} \right|_{\theta=\hat{\theta}} = \frac{d \sin(\hat{\theta})}{\cos^2(\hat{\theta})} - \frac{h \cos(\hat{\theta})}{\sin^2(\hat{\theta})} = 0 \quad (2)$$

NOTE: Don't confuse the notation for d with respect to differentiation and for the distance of the fence from the house. In this case, we happened to use the same symbols, but the context should make the meaning clear. Note that we could have simply avoided potential ambiguity/confusion by labeling the fence distance w (or something other than d)!

We can now do some algebra to solve for $\hat{\theta}$, finding

$$\hat{\theta} = \arctan \left(\left(\frac{h}{d} \right)^{1/3} \right) \quad (3)$$

This value represents the angle (relative to the ground) at which the ladder length is effectively minimized for the given values of d and h . Plugging that back into our expression for $L(\theta)$, we have that the minimum ladder length is

$$\hat{L} = \frac{d}{\cos \left(\arctan \left(\left(\frac{h}{d} \right)^{1/3} \right) \right)} + \frac{h}{\sin \left(\arctan \left(\left(\frac{h}{d} \right)^{1/3} \right) \right)} \quad (4)$$