

Math 250A (Fall 2009) - Quiz 12

SOLUTIONS

1. (4 points) Find the first six terms in the recursively defined sequence:

$$s_n = s_{n-1} + 2s_{n-2} \quad \text{for } n > 2 \text{ and } s_1 = 3, s_2 = 5$$

We are given the first two terms in the sequence.

$$s_3 = s_2 + 2s_1 = 5 + 6 = 11$$

$$s_4 = 11 + 2(5) = 21$$

$$s_5 = 21 + 2(11) = 43$$

$$s_6 = 43 + 2(21) = 85$$

2. (6 points) Baby formula can contain bacteria which double in number every half hour at room temperature and every 14 hours in the refrigerator. Suppose there are B_0 bacteria initially.

- a) Find the sequence, R_n , for the number of bacteria n hours later if the baby formula is kept at room temperature.

From the given information, the amount of bacteria quadruples every hour.

$$R_n = B_0 4^n$$

- b) Find the sequence, F_n , for the number of bacteria n hours later if the baby formula is kept refrigerated.

Our sequence, F_n , should be defined as follows.

$$F_n = B_0 a^n$$

Where a is a multiplying factor for the growth of the bacteria. Now, we only need to find the value of a . To accomplish this, we recall our given information.

$$F_{14} = 2B_0 = B_0 a^{14} \quad \Rightarrow \quad \ln(2) = \ln(a^{14}) = 14 \ln(a)$$

$$\Rightarrow \quad \frac{\ln(2)}{14} = \ln(2^{1/14}) = \ln(a) \quad \Rightarrow \quad a = 2^{1/14}$$

$$\Rightarrow F_n = B_0(2^{1/14})^n = B_0 2^{n/14}$$

- c) What is the ratio, Y_n , of the number of bacteria at room temperature to the number of bacteria in the refrigerator.

The ratio is simply the number of bacteria in room temperature divided by the number of bacteria in the refrigerator for any n .

$$Y_n = \frac{R_n}{F_n} = \frac{B_0 4^n}{B_0 2^{n/14}} = 2^{2n-n/14} = 2^{(2-1/14)n} = 2^{27n/14}$$

- d) How many hours does it take before there are a million times as many bacteria in baby formula kept at room temperature as in baby formula kept in the refrigerator?

$$1,000,000 = 10^6 = Y_n = 2^{27n/14}$$

$$\Rightarrow \ln(10^6) = 6 \ln(10) = \ln(2^{27n/14}) = (27n/14) \ln(2)$$

$$\Rightarrow \frac{6 \ln(10)}{\ln(2)} = \frac{27}{14}n \quad \Rightarrow \quad \frac{2 \ln(10)}{\ln(2)} = \frac{9}{14}n$$

$$\Rightarrow n = \frac{28 \ln(10)}{9 \ln(2)} \approx 10.3 \text{ hours}$$