

Math 250A (Fall 2009) - Quiz 10

SOLUTIONS

1. (5 points) Determine whether the following differential equation is linear or nonlinear, and whether it is separable or non-separable. Make sure to explain!

$$\frac{d\phi}{dm} = m \ln(\phi^{3m}) + \pi m^2$$

If it is separable, specify the integrals you would need to solve in order to find a solution (but don't solve them!).

First, we need to simplify our differential equation using a property of the logarithm function.

$$\frac{d\phi}{dm} = m \ln(\phi^{3m}) + \pi m^2 = 3m^2 \ln(\phi) + \pi m^2 = m^2[3 \ln(\phi) + \pi]$$

Now, we can see that the differential equation is in the form $\frac{d\phi}{dm} = p(m)q(\phi)$ and is subsequently separable. The differential equation is also nonlinear due to the $\ln(\phi)$ term, which does not fit the linear form for a first order differential equation.

Since the differential equation is separable, the integrals needed to find a solution are as follows.

$$\int \frac{d\phi}{3 \ln(\phi) + \pi} = \int m^2 dm$$

2. (5 points) Find the solution to the differential equation and initial condition.

$$\frac{d\theta}{dx} + 3\theta = 40 \quad \theta(0) = 40$$

First, let us rewrite our differential equation.

$$\frac{d\theta}{dx} = 40 - 3\theta$$

Now, we can use separation of variables to solve the differential equation.

$$\begin{aligned} \int \frac{d\theta}{40 - 3\theta} &= \int dx \quad \Rightarrow \quad -\frac{1}{3} \ln|40 - 3\theta| = x + C \\ \Rightarrow \quad \ln|40 - 3\theta| &= -3x + D \quad \Rightarrow \quad 40 - 3\theta = Ae^{-3x} \\ \Rightarrow \quad \theta &= Be^{-3x} + \frac{40}{3} \end{aligned}$$

Now, we apply our initial condition.

$$\begin{aligned} 40 &= B + \frac{40}{3} \quad \Rightarrow \quad B = 40 - \frac{40}{3} = \frac{80}{3} \\ \Rightarrow \quad \theta(x) &= \frac{1}{3}[80e^{-3x} + 40] \end{aligned}$$