

Problems

147. A function is defined by $f(t) = t^2$ for $0 \leq t \leq 1$ and $f(t) = 2 - t$ for $1 < t \leq 2$. Compute $\int_0^2 f(t) dt$.
148. (a) Find $\int (x+5)^2 dx$ in two ways:
 (i) By multiplying out
 (ii) By substituting $w = x+5$
 (b) Are the results the same? Explain.
149. Suppose $\int_{-1}^1 h(z) dz = 7$, and that $h(z)$ is even. Calculate the following:
 (a) $\int_0^1 h(z) dz$ (b) $\int_{-4}^{-2} 5h(z+3) dz$
150. The curves $y = \sin x$ and $y = \cos x$ cross each other infinitely often. What is the area of the region bounded by these two curves between two consecutive crossings?
153. (a) Find the average value of the following functions over one cycle:
 (i) $f(t) = \cos t$
 (ii) $g(t) = |\cos t|$
 (iii) $k(t) = (\cos t)^2$
 (b) Write the averages you have just found in ascending order. Using words and graphs, explain why the averages come out in the order they do.
154. What, if anything, is wrong with the following calculation?

$$\int_{-2}^2 \frac{1}{x^2} dx = -\frac{1}{x} \Big|_{-2}^2 = -\frac{1}{2} - \left(-\frac{1}{-2}\right) = -1.$$

155. Let $E(x) = \int \frac{e^x}{e^x + e^{-x}} dx$ and $F(x) = \int \frac{e^{-x}}{e^x + e^{-x}} dx$.
 (a) Calculate $E(x) + F(x)$.
 (b) Calculate $E(x) - F(x)$.
 (c) Use your results from parts (a) and (b) to calculate $E(x)$ and $F(x)$.
156. Using Figure 7.28, put the following approximations to the integral $\int_a^b f(x) dx$ and its exact value in order from smallest to largest: LEFT(5), LEFT(10), RIGHT(5), RIGHT(10), MID(10), TRAP(10), Exact value

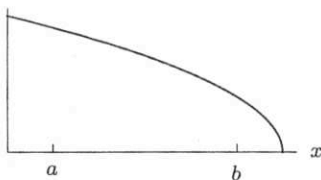


Figure 7.28

157. Given that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, find $\int_0^\infty x^2 e^{-x^2} dx$.
158. The moment generating function, $m(t)$, which gives useful information about the normal distribution of statistics, is defined by

$$m(t) = \int_{-\infty}^{\infty} e^{tx} \frac{e^{-x^2/2}}{\sqrt{2\pi}} dx.$$

Find a formula for $m(t)$. [Hint: Complete the square and use the fact that $\int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi}$.]

161. A patient is given an injection of Imitrex, a migraine medicine, at a rate of $r(t) = 2te^{-2t}$ ml/sec, where t is the number of seconds since the injection started.
- (a) By letting $t \rightarrow \infty$, estimate the total quantity of Imitrex injected.
 (b) What fraction of this dose has the patient received at the end of 5 seconds?

163. An organism has a development time of T days at a temperature $H = f(t)^\circ\text{C}$. The total the number of degree-days S required for development to maturity is a constant defined by

$$S = \int_0^T (f(t) - H_{\min}) dt.$$

- (a) Evaluate this integral for $T = 18$, $f(t) = 30^\circ\text{C}$, and $H_{\min} = 10^\circ\text{C}$. What are the units of S ?
 (b) Illustrate this definite integral on a graph. Label the features corresponding to T , $f(t)$, $H_{\min}$, and S .
 (c) Now suppose $H = g(t) = 20 + 10 \cos(2\pi t/6)^\circ\text{C}$. Assuming that S remains constant, write a definite integral which determines the new development time, T_2 . Sketch a graph illustrating this new integral. Judging from the graph, how does T_2 compare to T ? Find T_2 .

CHECK YOUR UNDERSTANDING

The left and right Riemann sums of a function f on the interval $[2, 6]$ are denoted by LEFT(n) and RIGHT(n), respectively, when the interval is divided into n equal parts. In Problems 1–9, decide whether the statements are true for all continuous functions, f . Give an explanation for your answer.

- If $n = 10$, then the subdivision size is $\Delta x = 1/10$.
- If we double the value of n , we make Δx half as large.
- LEFT(10) $\leq$ RIGHT(10)
- As n approaches infinity, LEFT(n) approaches 0.
- LEFT(n) - RIGHT(n) = $(f(2) - f(6))\Delta x$.
- Doubling n decreases the difference LEFT(n) - RIGHT(n) by exactly the factor $1/2$.
- If LEFT(n) = RIGHT(n) for all n , then f is a constant function.
- The trapezoid estimate TRAP(n) = (LEFT(n) + RIGHT(n))/2 is always closer to $\int_2^6 f(x) dx$ than LEFT(n) or RIGHT(n).
- $\int_2^6 f(x) dx$ lies between LEFT(n) and RIGHT(n).

NOTE Problems above came from Hughes-Hallet et. al. (4th ed.)
 Ch.7 Review